Joseph W. Dauben, "Georg Cantor y el Papa León XIII: Las matemáticas, la teología y el infinito," *Mathesis*, **6** (1990), pp. 46-74.

Reviewed by

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The English original of this paper appeared in 1977 in the *Journal of the History of Ideas*, **38**, pp. 85-108, under the title "Georg Cantor and Pope Leo XIII: Mathematics, Theology, and the Infinite." The paper discusses Cantor's endeavour to show that his ideas were not incompatible with Catholic dogma and thus keep his theory of transfinite numbers out of the reach of Catholic criticism.

Here Professor Dauben presents two aspects of Cantor's personality which account for the progressive agreement between his views and those of certain Catholic thinkers of his time in the spirit of Pope Leo XIII's encyclical *Aeterni Patris*. One of them is Cantor's belief in the freedom of mathematics. Cantor's concept of actual infinite and his theory of transfinite numbers were the target of bitter attacks by his fellow mathematicians and he was therefore very much concerned to avoid any repetition of this from the influential Catholic theologians. Cantor thought that mathematics should not be held back by any kind of constraint arising out of mathematics itself. Neither Philosophy nor Theology had, in his opinion, anything to say about the truth and correctness of a mathematical theory. Mathematics has its own mechanism to avoid error and so its freedom is not a threat to the discipline although any external contention might be very harmful indeed. Cantor's ideas on the existence of mathematical objects are treated in paragraph 8 of his work *Grundlagen einer allgemeinen Mannigfaltigkeitslehre* (1883) in which he distinguishes between *immanent* and *transsubjective* reality and also in a footnote in which he explains the process which a newly-defined notion must undergo to be accepted as a suitable mathematical notion referring to an existent entity.

The second aspect of Cantor's personality is his deep-rooted religious feelings. Admittedly, disheartened as he was by the treatment he and his

theory had received from the mathematical world, Cantor apparently turned to Philosophy and Theology. But let us not forget that he had felt from the very beginning a calling, a mystical vocation almost, to devote his life to mathematics, as he told his father in a letter of 1862 cited by Professor Dauben on p. 68. He followed his vocation and considered himself "chosen" and it is clear therefore that his interest in religion was there from the beginning of his career. He succeeded in convincing theologians that transfinite numbers were harmless for Catholic beliefs and was also concerned in developing the implications of his theory for Christian Philosophy and Theology. As Dauben points out on p. 64, he did not hesitate to use theologian arguments to support some of his theses or introduce some distinctions, such as *Transfinitum vs. Absolutum* (p. 63), to banish Gutberlet and Franzelin's doubts about the pantheism supposedly suggested by his thesis of the existence of an infinite in the created world. The deep relationship between his mathematical work and his religious beliefs can be also seen, as Dauben concludes (p. 67), at the beginning of his major work on set theory, ***Beiträge zur Begründung der transfiniten Mengenlehre,*** in which he cites the following aphorism from the Bible: "Veniet tempus, quo ista quae nunc latent, in lucem dies extrahat et longioris aevi diligentia," or in Dauben's translation (English original p. 107): "The time will come when these things which are now hidden from you will be brought into the light." Dauben interprets (p. 67) this aphorism as showing not only to what extent Cantor's mathematics and theological ideas were related but also his belief in the "revealed" nature of his theory.

We should not however suppose that Cantor's theory of the Infinite was developed for theological or philosophical reasons. His work is essentially mathematical, but his faith in God sustained him against the aggressive reactions of mathematicians and gave the theory enough time to become well known and so survive in its own right. In Dauben's words (English original p. 108, Spanish translation p. 68):

> Later generations might forget the philosophy, smile at his abundant references to St. Thomas and the Church fathers, overlook his metaphysical pronouncements, and miss entirely the deeply religious roots of Cantor's later faith in the veracity of his work. But these all contributed to Cantor's resolve not

> to abandon his transfinite numbers. Instead, his determination seems actually to have been strengthened in the face of opposition. His forbearance, as much as anything else he might have contributed, insured that set theory would survive the early years of doubt and denunciation to flourish as a vigorous, revolutionary force in scientific thought of the twentieth century.

I would not like to conclude this review without remarking on Dauben's treatment of Cantorian views about mathematical existence. The way Dauben expounds Cantor's position might lead us to understand that Cantor supported a view like the one later defended by Hilbert but this would be a mistake. Professor Dauben entitles section 2 or his paper (p. 47) *Consistency: the importance of Cantor's formalism* and he would seem to maintain (p. 48) that Cantor accepted consistency as the only criterion for mathematical existence. The same misleading impression might be drawn from the term "formalism" in the title of this section. Cantor did not embrace formalism but was a realist in a very strong sense. He not only requires consistency but also what we might call *extra-systematic coherence* for a notion to be able to name an existent entity. On occasion, he expresses himself as if consistency were everything any such notion needs to become part of mathematics but it is not because of his supposed formalism, but rather because of the neoplatonic features of his thought. I have developed this point in my "Notas sobre la evolución del realismo en el pensamiento de G. Cantor," Análisis Filosofico 11 (1991) No. 1, pp. 39-67; I shall not go into it here therefore. I do not wish to suggest that Professor Dauben is confused about Cantor's philosophy of existence but his way of expressing himself is unfortunate perhaps as it may prompt in some readers the idea of Cantor as a formalist *à la* Hilbert. Nonetheless, apart from this nuance in Dauben's terminology, this is a very clear and well-informed paper on the relationship between mathematical and philosophical ideas in Cantor's thought, as anybody who has an interest in his philosophy already knows. And Spanish readers are very lucky to have this translation available to them.