

THE FLY IN THE BOTTLE\*  
(In Memory of Jean van Heijenoort)

JEAN-YVES GIRARD  
Institute Henri Poincaré  
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"I pondered my doubts, and for several years the study of mathematics was all that allowed me to preserve my inner equilibrium. Bolshevik ideology was, for me, in ruins. I had to build another life." These simple and moving lines end Jean van Heijenoort's book of recollections.<sup>1</sup> That other life was, among others, activity as a philosopher and historian of mathematical logic.

To a mathematical logician Jean van Heijenoort is primarily the author of the monumental *From Frege to Gödel*;<sup>2</sup> this book offers a unique panorama of the stammering of logic, from the *Begriffsschrift* of Frege (1879) to Gödel's theorem (1931). It is made up of a selection of articles by the leading logicians of that half century, carefully annotated and above all provided with introductions allowing the disentanglement of various facts which would obscure their meaning for a reader knowing what happened "later." Along the same line one must clearly mention his edition of the logical writings of Jacques Herbrand<sup>3</sup> and his very recent participation in the editing in progress of the works of Kurt Gödel. Jean van Heijenoort's work has contributed quite a bit to the modification of our

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\* From the French original, "La mouche dans la bouteille (En Mémoire de Jean van Heijenoort)", The Paris Logic Group (editors), *Logic Colloquium '85* (Amsterdam, North-Holland/Elsevier Science Publishers, 1987), 9-12, by permission of North-Holland and the Author. English translation by Thomas Drucker, approved by the Author.

<sup>1</sup> Jean van Heijenoort, *With Trotsky in Exile: From Prinkipo to Coyoacán* (Cambridge, Mass., Harvard University Press, 1978), p. 149.

<sup>2</sup> Jean van Heijenoort (editor), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, Cambridge, Mass., Harvard University Press, 1967.

<sup>3</sup> Jean van Heijenoort (editor), *Jacques Herbrand, Écrits logiques*, Presses Universitaires de France, 1968.

image of the founding fathers. putting in their proper place pivotal works like those of Herbrand and Löwenheim. To judge his work as historian of logic, let us turn to his elegant article on "The logical work of Jacques Herbrand",<sup>4</sup> translated into English.<sup>5</sup> there one finds a very precise description of the works of Herbrand together with their links with the (earlier) works of Löwenheim and Skolem or the (later) works of Gentzen. Where this work distinguishes itself from other studies of the same type is Jean van Heijenoort's mathematically dominating the subject from above, so that the work of Herbrand is analyzed with clarity and rigour: he always goes to the essential, not losing time by going on about aspects which have become obsolete. Behind his taste for exact detail one does not sense a frozen admiration, making a fetish of the document: Herbrand's system of ideas is presented to us for what it without doubt was, a synthesis of Brouwer and Hilbert, the result of readings incompletely digested by a young man aged 23. In short, it is a stirring text, which one can consider as a model of the genre. As for what is of scientific interest in these historical studies, let us not forget that many of these questions more or less resolved fifty years ago are fires not wholly extinguished, that our grandfathers lived in a logical universe where many more possibilities were open, and that some doors seem closed to us because we no longer knock at them.... In 1976 Jean van Heijenoort said to me something like this: "In logic one has the impression of being the fly in the bottle." To develop this image, if the fly wished to leave today, it would have to return to the old plans, those from before 1930 when the boundary was not recognized, and including certainly the enormous addition of the last fifty years. The works of Jean van Heijenoort, and I do not limit myself here to his historical texts, but am thinking also of his more philosophical works on vagueness and on the sense about sense (!) in Frege, offer us visible testimony to the great questions which will only stop when thought does.

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<sup>4</sup> L'oeuvre logique de Jacques Herbrand et son contexte historique, in J. Stern (ed.), *Proceedings of the Herbrand Symposium, Logic Colloquium '81* (Amsterdam, North-Holland Publishing Co., 1982), 57-85.

<sup>5</sup> Jacques Herbrand's work in logic in its historical context, in J. van Heijenoort, *Selected essays* (Naples, Bibliopolis, 1986; copyright 1985), 99-121.

One of the major ideas which emerges from the philosophico-historical studies of Jean van Heijenoort is the opposition between absolutism and relativism. *Absolutism* is the belief in *one* logic, *one* geometry, etc., while *relativism* is rather more opportunistic. The history of logic in its first years was dominated by absolutism, e.g. Frege and Russell. It was they who made the conceptual breakthrough, especially the first. The theory only became operational, however, thanks to relativism, as represented by Löwenheim. Relativism, abandoning the immoderate ambition of the founding fathers, opens itself (in admitting *several* logics, *several* interpretations...) to possibilities which absolutism condemns with horror, when it can even imagine them. History has decided: 'The failure of absolutism in logic is that of realism, that is, of a conception for which experience is trans-muted into a reality independent of all processes of knowledge. This is certainly not a conception which the historical development of science seems to favor. The organization of knowledge does not proceed by piecemeal addition, but by an unceasing reorganization, in which concepts are replaced by others. This climate of science is much more congenial to relativism than to absolutism. Systems change according to needs. ...Human knowledge has not reached a stage of completion and stability that would allow us to fit it into a *logica magna*.'<sup>6</sup> History has decided, but the fascination of absolutist ideas remains; in addition, at the end of the twentieth century some sort of return of absolutist ideas is within the order of possibility. Although the distinctions between the traditional schools (formalists, intuitionists, etc.) have evaporated with their protagonists, the interpretation of the history of the subject about the absolutism/relativism axis, an essential distinction in the work of Jean van Heijenoort, seems more current than ever.

There is no need to insist on the clear hiatus between the two periods in the life of Jean van Heijenoort; although this life presents two quite contrasted parts, one is not forbidden to seek in it certain deep continuities. There is scarcely need to go on at length about the affinity between social upheaval and scientific upheaval – the birth of modern logic between Frege and Gödel, to come back to the title of the work already cited. After all,

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<sup>6</sup> P. 83 of J. van Heijenoort, *Absolutism and relativism in logic*, in J. van Heijenoort, *Selected essays* (Naples, Bibliopolis, 1986; copyright 1985), 75-83.

the beginning of the twentieth century was rich in scientific upheavals, of which logic did not necessarily constitute the deepest. A pivotal text, although marginal in Jean van Heijenoort's output, is his study of 'Engels and mathematics'. He shows us Engels pontificating about a domain which he poorly understands with the sole objective of having emerge, at all costs, the dialectical aspect: "Engels now stands as a man full of prejudices, unable to freely enter the competition of ideas. He would like to have his own 'dialectical' science aside from what he calls the 'ordinary metaphysical' science, that is, purely and simply science."<sup>7</sup> This severe judgement ends an analysis which makes evident the empiricist side of Engels, for example his ideas for improving geometry, but above all Engels appears there as an *absolutist* empiricist: mathematics exists by itself, as an unconscious act of nature and so forth, but even here one has a single geometry! With a priori philosophy and politics diametrically opposed, Engels finds himself at the end in the company of Frege, even if their works are of very unequal quality. One can wonder if, from Marxism to logic, the hidden unifying trait of the activity of Jean van Heijenoort is not the fascination of these theories which believed briefly that they could contain the world with the help of formulae.

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<sup>7</sup> P. 151 of J. van Heijenoort, *Friedrich Engels and mathematics*, in J. van Heijenoort, *Selected essays* (Naples, Bibliopolis, 1986; copyright 1985), 123-151.