I. Grattan-Guinness
The Search for Mathematical Roots 1870-1940
(Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel)
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REVIEW

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The history of modern logic is a difficult subject and we are still far from having a good general vision of it. This book surely helps greatly in that direction. At the present time there are many different books focusing on special topics and/or authors: Boole, Cantor, Frege, Russell, the Vienna Circle, the Lvov-Warsaw School, Set Theory, the Incompleteness theorem, indecidability, *etc.* But before Ivor Grattan-Guinness (IGG hereafter), the only man courageous enough to draw a general view was Jean van Heijenoort (JvH hereafter), he did so through his famous book *From Frege to Gödel*, a fine commented edition of some crucial logical papers and also in a series of papers. The work of JvH became a reference work for any historian of modern logic. However as we know nowadays it has severe drawbacks.

Although IGG makes only one explicit reference to JvH (p.228), the ghost of JvH haunts his book. The subtitle of IGG's book is a reformulation of JvH's one and it looks like a correction of JvH's views. The whole story would have started with Cantor rather than with Frege, and Russell would have played a key role. In fact IGG's story started even before Cantor since his first chapter (after the introductory one) dedicates about 60 pages to "Algebraic logic and mathematical analysis up to 1870". Let us note *en passant* that the expression "Algebraic logic" is an anachronism here. In the XIXth century people were talking about "Algebra of logic", the expression "Algebraic logic" was introduced only in the 1950s by Haskell Curry.

IGG's subtitle is in some sense strongly misleading. IGG is not telling us the story whose happy end (or dead end, as you wish) is Gödel's theorem, a story whose main plot would be oriented towards

this final climax. The book is rather a description of the development of several trends in "logics, set theories and the foundations of mathematics" during the period 1870-1940. So what is the main plot? In fact it seems that there is no plot and this lack of plot is the main defect of the book. Of course, one can argue that in reality there was no plot, and that the reconstruction of this piece of history according to a plot would be necessarily artificial. But on the other hand, without any plot the author takes the risk of falling into the accumulation of information, enumeration of facts, anecdotism and descriptivism. In some sense this is what happens.

In particular, IGG presents an extensive description of Schröder, Peano and Whitehead-Russell's work including, for example, the table of contents of Schröder's *Vorlesungen über die Algebra der Logik* and *Abriss der Algebra der Logik*. Among many anecdotes, he reports one about Gentzen: "In an extraordinary irony, one of the very few photographs taken by the Germans of the Enigma encoder shows it being operated by him (Gentzen) in uniform" (p.545). Outside this and the fact that Gentzen introduced the symbol ∀ for the universal quantifier, IGG says nothing about Gentzen's work, which is so important, especially relative to Gödel's work: we know Gödel studied Gentzen's work carefully and from this arose his "dialectica" consistency proof. (Note that there is a mistake in the index: after the name of Gentzen no pages are mentioned).

It seems that IGG wants to talks about everything, and he makes short references, for example, to works of people like Woodger and Piaget. About Piaget he comments that "his work played a role in the 'new mathematics' educational idiocy of the 1960s onwards" (p.567). In a footnote he adds, "Around that time Quine told me that when he had heard that set theory was being used in mathematical education, he had thought that he was being told a joke" (p.567). Again we have here an interesting anecdote but without any relevant philosophical comments.

Despite these defects, the book has many great qualities: it is written in a very lively tone, IGG doesn't hide his point of view and also, more importantly, he gives us a vision of the history of logic which doesn't give the wrong idea of researchers working apart from everyday life, in a world not connected with common problems. It is interesting, for example, to learn the financial difficulties surrounding the publication of the work of Schröder or *Principia Mathematica*. On Schröder he writes: "Maybe it was a pity that he paid for publication himself; had Teubner picked up the bill, they might have asked for a much tighter text" (p.176).

One of the best features of IGG's book is to put Frege in his right place. JvH promoted a very ambiguous picture of Frege, as a founding father of everything (modern logic, analytic philosophy, theory of relativity, ...). IGG rightly insists on the existence of two Freges: "Much commentary is available on an analytic philosopher of language writing in English about meaning and its meaning(s), and putting forward some attendant philosophy of mathematics. The historical record, however, reveals a different figure: Gottlob Frege (1848-1925), a mathematician who wrote in German, in a markedly Platonic spirit, principally on the foundations of arithmetic and on a formal calculus in which it could be expressed" (p.177). IGG calls the first one of these Freges, Frege$'$ according to the following reason: "I shall name him "Frege$'$" with the prime used in the spirit of the derived function "$f'(x)$" in Lagrange's version of the calculus" (p.178). About Frege$'$, he writes: "that philosopher of language and founder of the Anglo-Saxon analytic tradition; most of the massive Frege industry, especially in English, is devoted to him and his development." (p.177).

About the influence and import of the work of Frege (the real one), IGG notes that: "Russell's claim to be his (Frege) first reader after publicizing him in 1903 is ridiculous" (p.177), and that Russell didn't promote at all the work of Frege because he showed that his calculus was inconsistent and "chose to pursue childish polemics" (p.177). The relatively short account dedicated to Frege in IGG's book is in fact proportional to his role in the history of modern logic. On the other hand, a large part of the book (more than one half) is organized around Russell's achievement, and the related ideology of logicism, so that the book could have born the subtitle "Russell *et al.*" One can wonder if this is really justified or if this is an idiosyncrasy of IGG. Anyway, due to the Peano-Russell axis promoting investigations on Peano, IGG has nearly 50 pages on "The formulary of mathematics" (Chapter 5). On the other hand, this tends to undervalue the Hilbert-Bernays-Gödel-Gentzen's axis on which relatively little is said.

But probably the worst defect of IGG's book is one common to JvH's work: the downplay of the Polish school. Especially through the work of Tarski, this school has dominated the logical researches of the 20th century, so it would have been interesting to talk about the origin of this school. Moreover the investigations of the Polish school were also fundamental for Gödel's incompleteness theorem. In a letter from Tarski to Neurath published about ten years ago[1] (this correspondence

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[1]Tarski, A., 1992, "Drei Briefe an Otto Neurath", *Grazer Philosophische Studien*, **43**, pp.1-32.

does not appear in the bibliography of IGG's book), Tarski explains that Gödel and people from Vienna understood the distinction between language and metalanguage only after some discussions they had with Poles, and that this distinction was made explicit by Łukasiewicz in the early 1920s.

In conclusion, IGG wrote a very huge book, which is a rich source of information (in particular it includes an excellent bibliography), pleasant to read and very helpful. It is a useful guide to the jungle of the prehistory of modern logic, which is very hard to explore on one's own. But despite this rather long trip through the jungle, the reader could also feel frustrated not to have had closer encounters with some famous wild animals. And he also may have the impression that he has understood little in this jungle. But after all, perhaps this jungle is incomprehensible.

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