

James Gasser (editor)

A Boole Anthology.

Recent and Classical Studies in the Logic of George Boole

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REVIEW

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James Gasser's *A Boole Anthology* is a collection of seven classical and ten recent studies on George Boole's work, its background, and its intellectual frame of reference. Two of the classical articles were published during the 1860s, with the rest appearing during the latter half of the 20th century. The recent ones are based on lectures given at Lausanne on the occasion of the conference entitled "Boole 1997: One Hundred and Fifty Years of Mathematical Analysis of Logic". Gasser's anthology is a valuable contribution to the renaissance experienced by Boole studies since the publication of Desmond MacHale's biography of Boole in 1985. In addition to MacHale's book, three recent new editions of Boole's ground-breaking *The Mathematical Analysis of Logic* [6, 7, 8], an important selection of his manuscripts on logic and its philosophy [10], and the present volume are telling examples of the current lively interest in the life and thought of George Boole. Indeed, in the editor's words, "the present anthology constitutes an attempt to capture some of the 'buzz about Boole'" (p. vii).

Gasser's anthology begins with a brief Preface by the editor (pp. vii-viii). Given the topic and the nature of the work it is surprising to read in the very first line of Gasser's opening remarks that "centuries of stagnation in the study of logic were followed by an explosion of progress in the late nineteenth century" (p. vii). This statement carries an unfortunate echo of the misguided standard evaluation of the time between Leibniz and Frege as a period of stagnation or even of decline in the history of logic. Even though it is true that the currently fashionable way of understanding logic merely as the doctrine of the laws of correct inference would not have appealed even to most 19th

century logicians, it is still not the case that, save Boole's achievements, there is nothing worth discussing in the development of logic during the modern classical period. *A Boole Anthology* itself contains several contributions which testify against the editor's evaluation of the historical background of the break-through of modern mathematically-oriented logic at the turn of the 20th century.

The first classical article in the anthology is Samuel Neil's memorial essay on George Boole, "the earnest student, the successful toiler, and the noble thinker" (pp. 1-25). This brief account of Boole's life is welcome, especially at the beginning of this anthology. However, the reader should take note of the fact that this memorial essay completely refrains from criticism and is far from objective. At the end of the paper (p. 25) there is an interesting short note by Gasser to the effect that Neil's essay was first published in *The British Controversialist and Literary Magazine* in August of 1865, in a series entitled "Modern Logicians". Other subjects in the series included Immanuel Kant, G. W. F. Hegel, Richard Whately, William Whewell, William Hamilton, John Stuart Mill, and William Thomson. According to Ivor Grattan-Guinness, the article on Boole was by far the most significant in the series (*loc. cit.*).

The second classical contribution is George Paxton Young's concise critical account (pp. 27-43) of Boole's mathematical theory of the laws of thought as presented in his second major monograph, entitled *An Investigation of the Laws of Thought*. As opposed to Neil's mainly biographical account, Young approaches Boole from a systematic point of view. Young mirrors Boole's theory against its Scholastic-Aristotelian background. He is particularly concerned with Boole's famous statement that scholastic logic is "not a science, but a collection of scientific truths, too incomplete to form a system of themselves, and not sufficiently fundamental to serve as the foundation upon which a perfect system may rest" [4, p. 241]. Even though Young is happy to give credit to Boole's new method, he dissents from much that is contained in his theory of the laws of thought and believes that he has "fully refuted [Boole's] opinion that there are problems in the science of inference which the Aristotelian logic is incapable of solving" (p. 34). Further, he concludes that "as a contribution to philosophy, in the strict sense of the term, [Boole's *Laws of Thought*] does not possess any value" (p. 43).

Unfortunately this article is not accompanied by any editorial comment. On account of its general rejection of Boole's theory as a contribution to philosophy, it needs comment far more urgently than Neil's harmless piece of writing. Young's censure clearly tells us something

important about the reception of Boole's work during the mid 19th century. Indeed, Boole's work attracted very little attention, even in his own native England, until William Stanley Jevons supplemented and revised his theory in the 1860s.

Luis M. Laita's "The Influence of Boole's Search for a Universal Method in Analysis on the Creation of his Logic" (pp. 45-59) discusses Boole's most important early mathematical contributions. Laita aims to show that Boole's work on differential equations had an important influence on the creation of his algebra of logic. Laita's arguments are careful, well documented and convincing. This essay, which was first published in 1977 in the *Annals of Science*, is a valuable treatise on Boole's work during the 1840s culminating with the first presentation of his algebra of logic in 1847.

Theodore Hailperin's "Boole's Algebra isn't Boolean Algebra" (pp. 61-77) provides the reader with a detailed account of those developments in the field of the abstract algebra, within which Boole originated his own algebra of logic. As the title of his paper promises, Hailperin points out essential differences between Boole's theory and the more recent Boolean algebras. This important paper was originally published in 1981 in *Mathematics Magazine*.

Michael Dummett's review of the first volume of Boole's *Collected Logical Works: Studies in Logic and Probability* (1952) and of the jubilee anthology *Celebration of the Centenary of "The Laws of Thought" by George Boole* (1955) was published originally in 1959 in the *Journal of Symbolic Logic*. Volker Peckhaus asks, in the title of his contribution to the present volume (p. 271), if George Boole really was the father of modern logic. As far as Dummett is concerned, the only candidate for this honour was Gottlob Frege. According to Dummett: "The discoveries which separate modern logic from its precursors are of course the use of quantifiers . . . and the concept of a formal system, both due to Frege and neither present even in embryo in the work of Boole" (p. 79). In Dummett's opinion Boole deserves credit for having taken *any* step forward after a long fruitless period in the field of logic rather than for having taken a significant step. Moreover, in his opinion anyone who is not already acquainted with Boole's results will be disappointed to find out "how ill-constructed his theory actually was and how confused his explanations of it" (*loc. cit.*). Such statements were characteristic of many 20th century analytic philosophers who willingly applied logical tools in their philosophical work and typically underestimated the historical evolution of their own tradition. However, although historically insensitive, Dummett's review (together with Hailperin's contribution) is a good reminder that several theoretical discoveries were yet to be

made when Boole was working. Let it be noted that Dummett's severe criticism concerns not only Boole but also Rush Rhees, the editor of his *Collected Logical Works*. Dummett closes his review by lamenting that "the Centenary volume contains no article on Boole's work in probability theory" (p. 85). The same observation might be applied to Gasser's volume.

In the sixth classical article, "A Reassessment of George Boole's Theory of Logic" (pp. 87-99), James W. Van Evra introduces Boole as an important contributor to the 19th century transitional period in the history of logic and defends him against many historically insensitive attacks, such as Dummett's 1959 review. Indeed, Dummett is one of the explicit targets of his criticism (p. 98f.). Van Evra opens his paper, first published in the *Notre Dame Journal of Formal Logic* in 1977, with the following still more or less valid account of Boole's reputation:

George Boole's theory of logic has not fared well at the hands of the commentators While there is general agreement that his work occupies an important place in the history of logic, the exact nature of that importance remains elusive. On the one hand, he has been called the originator of mathematical logic, but on the other, that claim has been pointedly disputed. On the one hand, his logic does differ significantly from traditional syllogistic logic, and for this he has been applauded. But on the other, Frege's introduction of quantification theory forms such a complete barrier between paleo- and neologic that any lasting influence from Boole's work . . . seems permanently obscured. (p. 87)

Van Evra concentrates on Boole's theory as set forth in his *Laws of Thought* and draws attention to the following three points: (1) claims to the effect that Boole's system of logic included logically uninterpretable expressions are misguided; (2) it is unfair to suggest that Boole's successors in the field of the algebra of logic corrected his mistakes; and (3) much of the late 20th century criticism treated Boole as if he were a contemporary logician (p. 88). Van Evra believes to have shown that "although there are rough spots in it, and although it defies systematically clear understanding, Boole's logic does not merit much of the criticism aimed at it" (p. 94). He is not unjustified in this belief. His contribution is a masterful example of a dispassionate, historically sensitive, well organized, and carefully argued scientific piece of writing.

John Corcoran's and Susan Wood's joint article, "Boole's Criteria for Validity and Invalidity" (pp. 101-128), closes the section of classical studies. It provides the reader with a detailed comparative analysis of criteria for validity in Aristotle's *Organon* and in Boole's first monograph *The Mathematical Analysis of Logic* (1847). According to the authors, Boole was aware of only some of the fundamental differences between his own logical system and that of Aristotle. In their view "Boole's system was a step forward in one respect and a step backward in another; [for example] logical axioms were added but indirect deductions were dropped" (p. 107). Sriram Nambiar makes the same observation in his paper on Boole's philosophy of logic (p. 237).

Corcoran and Wood provide a list of apparent inadequacies in Boole's system of 1847. They point out, for instance, that Boole did not give laws specifically governing the null set and the universal set. Moreover, Boole did not provide axioms involving mere additions nor did his system include the so-called De Morgan's laws. However, Corcoran and Wood do not forget to remind the reader that nowhere does Boole reject any of these laws.

According to Corcoran and Wood, Boole's work contrasts with that of Aristotle most markedly in regard to the production of formally impeccable deductions and experimentation. They point out that "Boole did not *use* his own system and he did not expend any energy studying it prior to publication. Indeed, in the whole of [*The Mathematical Analysis of Logic*] there is not one example of a deduction in the above system" (p.109). This shortcoming is partly remedied by Corcoran and Wood as they present a couple of succinct deductions in Boole's system (pp. 110-111).

At the beginning of their paper, Corcoran and Wood emphasize that it is not their intention to determine Boole's proper rôle in the historical development of logic (p. 102). However, this is, in effect, exactly what they do. By pointing out pros and cons of Boole's logic in relation to the Aristotelian traditional wisdom and by showing systematic inadequacies in his system of deductions, they subscribe to Van Evra's view of Boole as a mediator between the Scholastic-Aristotelian tradition and contemporary logic, *i.e.*, a man who "stands with a foot in each of two logical epochs" (p. 97). All in all, their paper, which was published originally in the *Notre Dame Journal of Formal Logic* (vol. 21, 1980), is a rewarding and illuminating contribution. It is one of the very best articles in Gasser's volume.

The latter section of the book, which consist of articles based on talks given at Lausanne in 1997, begins with two essays which excavate Boole's background: Theodore Hailperin's "Algebraic Logic: Leibniz

and Boole” (pp. 129-138) and Marie-José Durand-Richard’s “Logic versus Algebra: English Debates and Boole’s Mediation” (pp. 139-166). Hailperin’s paper provides the reader with a brief comparison of Boole’s and Leibniz’s ideas about the foundations of the algebra of logic. Hailperin also discusses some of Boole’s noteworthy contemporaries, *e.g.*, Richard Whately, Duncan F. Gregory, and Augustus De Morgan.

Durand-Richard’s paper is an attempt to understand Boole’s original motivation for creating his algebra of logic by clarifying early 19th century British discussions about the nature of knowledge (p. 139). Her aim is to connect Boole’s work to various early 19th century philosophical and mathematical debates in England. She begins her paper with a longish account of the social and intellectual academic climate in Cambridge and Oxford during the first half of the 19th century (pp. 139-158). This part of the essay is highly interesting but not directly related to Boole. His position is taken up only on the last five pages of the article (pp. 159-163). There Durand-Richard gives a list of several 19th century British mathematicians who had, according to her, “a close professional relationship” with Boole. However, she does not specify the nature of these relationships in any detail, but does emphasize the influence of George Peacock’s symbolical algebra on Boole’s algebra of logic.

In an essay entitled “The Mathematical Background of George Boole’s *Mathematical Analysis of Logic* (1847)” (pp. 167-212), Maria Panteki attempts to track down the stimulus that led Boole to think about his algebra of logic. Unlike Durand-Richard, however, she probes into Boole’s own early contributions prior to *The Mathematical Analysis of Logic*. She pays particular attention to Boole’s prize-winning treatise “On a General Method in Analysis” (1844) and concludes that his discovery of a general method was inspired in an important way by Robert Leslie Ellis’s studies of differential equations in connection with a theory of the shape of the earth (p. 167).

Panteki gives an account of developments that took place within formal logic in England during the first decades of the 19th century. She argues that “when Boole entered the scene in the mid-1840s the climate was already favourable to a fruitful application of mathematical methods to logic” (p. 190). She finds evidence for an indisputable revival in the field of logic from the works of Kirwan, Whately, Bentham, De Morgan, Solly, Hamilton, and, of course, Boole. If we add to this list the names of some of the most notable philosophers and logicians who took part in the lively 19th century reform discussion of logic on the other side of the English Channel, *e.g.*, Herbart, Drobisch, Hegel, and

Trendelenburg, Gasser's reference to this period as one of stagnation in the field of logic (p. vii) seems unjustified.

Panteki's article offers a careful and detailed account of the developments in the field of logic that took place in England during the 1840s. One detail might be added to round out the picture from the point of view of the work being done at this time in Germany. On page 193 Panteki claims that "the symbolical expression of syllogistic laws was Solly's unique novelty [in 1843]". However, Moritz Wilhelm Drobisch had presented an "Algebraic Construction of the Simplest Forms of Judgment and Derivation of Inferences Founded Thereupon"—*i.e.* the foundations of an algebraic syllogism calculus—several years earlier in an appendix to his *Neue Darstellung der Logik* [9]. Solly most likely did not know about Drobisch's results and *vice versa*. As Volker Peckhaus has shown, it was not until the 1870s that the English and German lines of development in the field of the algebra of logic really came into contact with one another (see, *e.g.*, [15]; also p. 279 in Gasser's volume). Durand-Richard's and Panteki's articles complement one another in illuminating the complex background of Boole's ground-breaking first monograph.

Ivor Grattan-Guinness's outline "On Boole's Algebraic Logic after *The Mathematical Analysis of Logic*" (pp. 213-216) provides the reader with a convenient account of the most important modifications and extensions to Boole's logic after the publication of his first monograph.

In an essay entitled "The Influence of Aristotelian Logic on Boole's Philosophy of Logic: The Reduction of Hypotheticals to Categoricals" (pp. 217-239), Sriram Nambiar compares Boole's conception of logic with the Aristotelian view presented in Richard Whately's *Elements of Logic* (1826). Nambiar concludes that even though Boole's work is "often taken to be a break from the Aristotelian tradition in logic" in turns out that Aristotle's logical theory "influenced his work in important ways" (p. 236). I do not see any problem in accepting that Boole, whose background was Aristotelian and who was greatly influenced by Aristotle's work, took an important step forward in the history of logic. Scientific discoveries always have their background and in Boole's case this background was, for obvious reasons, Aristotelian. In Gérard Bornet's words, "when we look at the contribution of Boole to the science of logic and assume it to be an evolution rather than a revolution, we do not expect to find a refutation of his predecessors but an extension of their results" (p. 264).

The latter part of Nambiar's essay engages in more speculative considerations. Nambiar criticises severely the traditional interpretation of

Boole's plus-operation as analogous to the modern exclusive disjunction (pp. 228-231). He also suggests that Boole's Universe (or "1") "contains all possible worlds and 'x' concatenated with '1' denotes the class of all possible worlds in which X is true" (p. 232) and hence, "'x = 1' translates to 'X is true in all possible worlds' and 'x = 0' to 'X is true in no possible world'" (p. 234). It is hard to see how the use of the expression "possible worlds", instead of the more familiar "possible interpretations of general names", facilitates our understanding of Boole's limiting notions of the Universe and Nothing.

Nambiar's essay is followed by Batrice Godard-Wendling's detailed discussion of the conceptualization of time in Boole's algebraic logic (pp. 241-255).

G rard Bornet takes a more general look at Boole's algebra of logic. In his paper, "George Boole and the science of logic" (pp. 257-270), Bornet argues that much of what modern logicians find peculiar about Boole work arises from different conceptions of science. According to him, Boole's conception of science allows a view of the world only from *inside* science, *i.e.*, it is restricted in the sense that it allows one to construct a science "only relative to a group of people who share evidence" (p. 258). Bornet focusses his attention to Boole's central notions of "unity", "harmony", and "order". His essay is an important and insightful, historically sensitive piece of writing. Bornet closes his paper with powerful conclusions concerning the progress of science:

If 150 years after *The Mathematical Analysis of Logic* we ask ourselves how far the science of logic has advanced since, then according to one's viewpoint there are *two* answers. We can say that the progress is overwhelming if we speak about the power of expression of our artificial languages and the multitude of available calculi. But we should be modest if with progress is meant a better understanding of ourselves as rational beings – and this is what Boole was concerned with. (p. 267)

In the late 1980s, Desmond MacHale and W. V. Quine had a disagreement as to whether George Boole really was the father of modern logic. Volker Peckhaus has taken this question as the starting point for his survey of Boole's place and role in the history of logic (pp. 271-285). His contribution is divided into two parts. Peckhaus starts with a brief description of the debate between MacHale and Quine, and distinguishes between two alternative perspectives of priority with regard to scientific fatherhood. The "originator interpretation" concentrates on

the first formulations and initial occurrences of novel ideas. The “initiator perspective” emphasises a father’s responsibility for conducting a new development (p. 271). In the latter part of his essay Peckhaus introduces a third, more genetic perspective. He concludes that “the story of the development of logic can only be written as one of multiple creations, independent of one another, and in most cases independent from the development of traditional logic” (p. 281). And for this reason it is, in his opinion, best to stop looking for the father of modern logic (p. 282). His contribution is elaborate, insightful, and carefully documented.

Shahid Rahman discusses in the first part of his essay, “Hugh MacColl and George Boole on Hypotheticals” (pp. 287-310), the views of Aristotle, Boëthius, Hamilton, Boole, and the Booleans on hypothetical and conditional propositions. In the latter part he concentrates on the reformulation of these ideas by Hugh MacColl, who, according to the author, “defined himself as a peacemaker between the Booleans and the traditionalists” (p. 289). The paper is a well-written and important contribution to an understanding of MacColl’s thought.

The last paper is Nicola Vassallo’s “Psychologism in Logic: Some Similarities between Boole and Frege” (pp. 311-325). Vassallo challenges the received view that Boole was a psychologistic philosopher and Frege was an anti-psychologistic philosopher. She opposes the common picture of Frege as an innovative discoverer (*e.g.*, Dummett p. 79, Quine p. 272) and Boole as a thinker who was more closely connected to an older epoch in the history of logic (*e.g.*, Laita p. 53, Van Evra p. 97, Corcoran & Wood p. 102, Nambiar p. 236, Bornet p. 259f.). She proposes that, given a suitable definition, both Boole and Frege appear as psychologistic thinkers (p. 312). Her definition is a version of Susan Haack’s “weak psychologism”, according to which logic is prescriptive of ordinary mental processes [11, p. 238]. Vassallo concludes that “if so-called ‘weak psychologism’ is . . . psychologism, then Frege is no less psychologistic than Boole” (p. 315). However, a well-known loophole in Haack’s definition does not suffice to transform Frege into a psychologistic thinker. As Leila Haaparanta has written: “Haack’s terminology may be somewhat misleading, as many logicians who are regarded as antipsychologists, Frege, for example, would accept what Haack calls weak logical psychologism. However, they would not say that determining the norms of thought would be the only or the basic task of logic” [12, p. 153]. Even though Frege certainly accepted that the laws of logic must prescribe the way in which people ought to think, he also emphasised that the validity of logic is absolutely independent of mental processes or other subjective circumstances. In the spirit

of Haack's "anti-psychologist philosopher" [11, p. 238], Frege believed that pure logic has, in itself, nothing to do with ordinary mental processes. Vassallo's openly provocative essay is motivated by a desire to refute the claim that Boole's philosophy is bad and outdated (p. 320). But who, nowadays, would claim that Boole's ideas are worthless?

All in all, Gasser's *A Boole Anthology* draws us a picture of George Boole, not as the founder of modern logic, but as an important mediator between traditions, and an initiator of an on-going and ever increasing development of the field of logical theory. On the one hand he was a historically sensitive scientist whose work emerges seamlessly from the Scholastic-Aristotelian tradition. On the other his novel ideas in the field of the algebra of logic paved the way for the break-through of a modern, mathematically-oriented logic at the turn of the 20th century. He was not an isolated miracle but an exceptionally gifted mathematician who was greatly influenced by several of his contemporaries. All the contributors of the volume seem to agree with Dummett that "there can be no doubt that Boole deserves great credit for what he achieved, in the sense that in those historical circumstances what he did must have been very difficult to do" (p. 79). Moreover, Gordon Baker and Peter Hacker have said in their book, *Frege: Logical Excavations*, that "if Frege had not made the decisive breakthrough in 1879, others would have made it along the same line within his lifetime (and nobody had been in a position to do so significantly earlier)" [1, p. 16]. I think the same applies, *mutatis mutandis*, also to George Boole.

When it comes to *A Boole Anthology* as an anthology, it is somewhat regrettable, especially with regard to the classical section, that it lacks an introductory chapter that would have provided a critical context for the various contributions. James Gasser's *A Boole Anthology* is an important collection of first-rate Boole-studies. However, its significance is no greater than the sum of its parts.

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