

Gregory J. Chaitin

*Conversations with a Mathematician: Math, Art, Science and
the Limits of Reason*

London: Springer-Verlag, 2002

vi + 158 pp. ISBN 18522335491

REVIEW

GEORGE ENGLEBRETSSEN

In April of 2004 an American mathematician auctioned his services as a co-author on e-Bay. More specifically, as far as I can make out, he offered to help someone research and write a mathematical paper for a scholarly journal. Since he has an Erdős number of 4, he guaranteed that the highest bidder would achieve an Erdős of 5. I laughed. But the situation does raise a number of interesting questions, some of which are: Do mathematicians ever have a sense of humor? How high does an Erdős number have to be before it is meaningless? How many times can I write “Erdős” in a single paragraph? What does research in mathematics amount to? Chaitin answers the first and last of these questions (sometimes simultaneously) in this little collection of his recent lectures and interviews.

Gregory Chaitin, a mathematician at the IBM Watson Research Center, is best known for his formation and exploitation of what he has termed Algorithmic Information Theory (AIT). He has produced a string of important papers and books on AIT, the foundations of mathematics, and randomness. *Conversations with a Mathematician* brings together three of Chaitin’s public lectures and a number of TV and newspaper interviews, all from the last fifteen years. In virtually all of these pieces Chaitin demonstrates a clear affirmative answer to my first question above. He is funny, witty, and delightfully informal and irreverent. He makes it obvious that he believes mathematics is fun and that he is having fun doing it and talking about it. During one of the interviews Chaitin is directly asked the question about humor and mathematicians. In response he laughs and says, “Well, I don’t think we have as much of a sense of humor as physicists do, but I think we **ought** to have a sense of humor” (p. 151). But, in addition to being

fun, mathematics is interesting in its own right and often extremely important. In his lectures and interviews Chaitin is eager to infect his audience with his own enthusiasm for the topics with which he has been most concerned in his research. In answer (at least partially) to my final question above, Chaitin's own research involves historical study, a deep probing into fundamental epistemological questions concerning formal systems, a philosophical approach to the notion of computing, and a deep probing of the limits of mathematics. A common theme in all these areas of Chaitin's work has been the idea of randomness, and his interest in randomness reveals something important about the originality of his ideas. His probing into the foundations and limits of mathematics has been informed by his deep interest in quantum physics. Unlike classical physics (either Newton or Einstein), quantum physics is not deterministic—it is probabilistic. While the equations of classical physics yield certainties, the equations of quantum physics yield only probabilities. So randomness can be said to rule in the quantum world. Chaitin has been among a small handful of researchers who have explored the consequences of recognizing randomness not only in physics but in mathematics as well. His own insight was to see that the incompleteness of mathematically formal systems was a result of randomness and to express that insight in terms of computational complexity. I shall say more about this below.

Conversations with a Mathematician begins with the collection's longest piece, a 1999 lecture, "A Century of Controversy over the Foundations of Mathematics." It is a perfect place to start for anyone unfamiliar with the kinds of problems Chaitin addresses. The lecture is at once both fascinating and breezy, both informative and fun. Here Chaitin lays out the history of work by logicians, mathematician, and computer theorists throughout the 20th century on the nature of and foundations of formal axiomatic systems. Perhaps not so modestly, he sees these developments as leading to his own AIT. As a logician, I have always found the story Chaitin retells here engrossing.

Throughout the early decades of the last century a number of first-rate philosophers and mathematicians turned their attention to the foundations of mathematics to a degree rarely seen before. There were a number of reasons for this. The end of the 19th century saw a general acceptance of the notion that whatever the foundation of mathematics might be it was not what it had traditionally been thought to be—geometry. Developments in analysis, algebra, and the formations of non-Euclidean geometries brought about a so-called crisis in mathematical foundations. This was soon exacerbated by Cantor's discovery of an infinity of transfinite sets. At the same time mathematicians such

as Frege and Peano were formulating formal systems of logic that would be sufficient to secure the foundations of mathematics. This “logician” program was enthusiastically adopted and promoted by Bertrand Russell. However, at the turn of the century Russell discovered a disturbing series of paradoxes in both Cantor’s and Frege’s systems. The crisis had grown. There were a number of responses, but the most important was by Hilbert. Hilbert’s approach was to say that mathematics should be conducted (ideally) in the language of a formal and axiomatic logic. His program was called “formalism”. The advantage of the program was that mathematicians could proceed simply by following very carefully and faithfully the rules of logic. Of course the trick here is that the logic one uses must have some fairly important characteristics; it must be at least consistent and complete. No contradiction can be the result of a proof in a consistent system; in any complete system there is a proof for either a given statement or its negation. So Hilbert’s aim was to formalize mathematics using logic, and it looked as if in doing so he had saved mathematics. But it was not to be. In 1931 Kurt Gödel gave his famous proof that, in effect, no formal system powerful enough to generate arithmetic could ever be complete. Logicians are interested in deduction; they formulate systems of logic that are meant to model how we reason (especially how we reason mathematically). So the story thus far—from Frege to Gödel—is primarily about logic. But, as Chaitin recounts the rest of the story, it turns from deduction to computation. His take on Gödel’s proof is that because it requires defining a large number of functions dealing with lists recursively it can be viewed as making use of a kind of proto-programming language (in particular, LISP). As Chaitin says, “The computer was implicit in Gödel’s paper, but this was really not visible to any ordinary mortal...” (p. 21). The person who spelled out just what a programming language would have to be like in order to check logical deductions (as Hilbert had intended) was, of course, Turing. Now among the many important results of Turing’s work was his formulation of the “halting problem.” He showed that there was an inherent limitation on any computational program: one can never determine in advance whether the program will eventually halt. Moreover, given that all deductions are calculations, it follows that one can never deduce in advance whether the program will eventually halt. It turns out that any formal axiomatic system that could solve the halting problem must generate paradoxes (like Russell’s). Chaitin’s AIT is, in part, the result of his realization that it is randomness in mathematics (as in quantum physics) that accounts for the incompleteness that Gödel discovered and inherent limits on calculation that Turing discovered. Chaitin demonstrated

that one could get an indication of how such limits apply by measuring the complexity of a computing program. In particular, he argued that the size of the program was a matter of how much it could be compressed. An ideal program would be one that could not be compressed further. He connected this idea with the physicist's notion of entropy (so compressibility is a measure of randomness, which is a measure of entropy). The obvious question is: If the degree of complexity of something is a matter of the size of program required to calculate it, then how does one know if he or she has in hand the smallest program? Chaitin showed that, in general, one can never be sure. What all of this ultimately means is that there are inherent limitations on mathematics—but the limitations are “natural and inevitable rather than mysterious and surprising” (p. 34).

It is in the nature of collections such as this one that there is a fair amount of overlap and repetition. The second lecture included in this one, “Algorithmic Information Theory and the Foundations of Mathematics,” retells much of the story of Hilbert, Gödel, Turing, Chaitin's own growing conviction “that mathematics should be pursued more in the spirit of physics” (p. 80), and the formation of AIT. Throughout both his lectures and his interviews Chaitin maintains his relatively light-handed approach to the more technical matters. Chaitin is a thinker who seems genuinely eager to introduce the fundamentals of his ideas in an easy and accessible way to general audiences. More importantly perhaps, he seems to want to share the excitement and pleasure he derives from mathematics with anyone caring to listen. This is especially evident in the interviews gathered here. In his interviews Chaitin is not only informative but also confessional, gossipy, and oddly philosophical. We learn a great deal about his childhood interest in both mathematics and physics, about his attitude concerning the contrasts between intellectual and prosaic life, about how he creates and the joy of encountering new ideas. We are treated to many bits of gossip (mostly light) about people from Gödel and Einstein to Turing and Wheeler, and more. In one interview Chaitin talks about computer software as something like the soul—the soul (whatever that is) as information (p. 150). And he goes on to suggest that perhaps software constitutes a simulation of Plato's world of ideas. Much of what Chaitin says in these interviews is mischievously provocative. A few examples. “I've gotten old enough that I'm not even sure that I believe in mathematics at all any more!” (p. 47). “I think that mathematicians are actually artists. Pure mathematics is really an art form, and I'm acutely sensitive to beauty or to lack of beauty” (p. 108). “It takes tremendous emotion to do good mathematics, it is very difficult”

(p. 71). “To create a new theory of science, you have to be mad” (p. 64).

Anyone, including mathematicians, logicians, and information theorists, can find something worthwhile in this collection. It’s fun for everyone, and the author is probably not so mad.

DEPARTMENT OF PHILOSOPHY, BISHOP’S UNIVERSITY, LENNOXVILLE, QUÉBEC,
CANADA J1M 1Z7

E-mail address: genglebr@ubishops.ca

