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*Alfred Tarski and the Vienna Circle: Austro-Polish Connections
in Logical Empiricism*

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REVIEW

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As the editorial introduction states, the main part of this sixth Yearbook of the Institute “Vienna Circle” collects papers from the international conference “Alfred Tarski and the Vienna Circle”, which was held 12–14 July 1997 at the Science Center of the Polish Academy of Sciences in Vienna. The stated intention of the meeting was “to give an account and interpretation of the relations between the Vienna Circle and the Polish school of philosophers since the beginning of the 20th century.” There are 20 contributed papers grouped in four sections:

- I. The Development of Tarski’s Concept of Truth.
- II. The Interaction between Tarski and the Vienna Circle.
- III. Philosophical Aspects of Tarski’s Concept of Truth and its Application in the Methodology of Science.
- IV. Technical Aspects of Tarski’s Definition of Truth and Model Theory.

There is also a selection of letters to Kurt Gödel from Tarski, 1942–1947 (translated and edited by Tarski’s son, Jan Tarski), and an essay, “Neo-Positivism and Italian Philosophy (1924–1973)” by P. Parrini. These are followed by an unrelated review essay, “Critical Idealism Revisited — Recent Work on Cassirer’s Philosophy of Science” by T. Mormann. There is then a section of book reviews (some, by chance, related to the topic of the conference) and a brief report on the year’s activities of the Institute “Vienna Circle”. The volume ends with an index of names.

Clearly the thesis of the conference is that the young Alfred Tarski was the key link between Poland and the Vienna Circle in the inter-war period, despite the fact that he never had a permanent university

position in Poland. Many Polish philosophers are mentioned in these papers, many visited Vienna, and many Austrians visited Warsaw; but it is certainly true that Tarski had at that time a major influence on later-to-be-famous philosophers such as Carnap, Gödel, Popper, and Quine, as well as on the mathematician, Karl Menger. The Yearbook general editor, Friedrich Stadler, comments:

“Finally, it is shown that Polish philosophers were not so radical as those of the Vienna Circle. This is probably due to the fact that the Polish school was much more influenced by traditional philosophy than was Moritz Schlick and his group. This influence goes back to Franz Brentano, who was the main teacher of Kazimierz Twardowski, the real father of Polish analytic philosophy. This connection, then was generally Austrian. Thus, the relations between the Vienna Circle and the Lvov-Warsaw School are rooted in a more general setting of Austrian philosophy.” (p. ix)

(I.1) The first paper, “Semantic Revolution – Rudolf Carnap, Kurt Gödel, Alfred Tarski” by Jan Woleński, begins with a review of the various meanings of the word “semantic(s)” as used by, among others, M.J.A. Bréal (1897), C.S. Peirce, Edmund Husserl, Karl Bühler, C.K. Ogden and I.A. Richards, Charles Morris, W.V.O. Quine, the early Rudolph Carnap, and Poles such as Tadeusz Kotarbiński, Stanislaw Leśniewski, Kazimierz Ajdukiewicz, and Leon Chwistek. He comments, “However, I did not find in Polish writings before Tarski any explicit statement that the concept of truth belongs to semantics. On the other hand, almost everybody in Polish philosophy accepted the classical (Aristotelian) truth-definition.” (p. 2).

The difficult historical question is the relationship and interaction between Tarski, Carnap, and Gödel. These three had been writing on important developments since 1930, and Carnap was certainly bringing out semantical ideas over that period. Gödel well understood questions of satisfiability and universal validity of logical formulae in the same way as did Hilbert-Ackermann – namely without a formal definition. Menger met Tarski in Warsaw in 1929, and Tarski’s first visit to Vienna was in 1930. However, as the author notes, the big impact of Tarski’s ideas on philosophers came from his paper at the Congress on Unified Science, organized by the Vienna Circle, in Paris in 1935. He says, “I think that Tarski’s paper was a turning point in the philosophical career of our word. Since 1936 the word ‘semantics’, as used in logic and

philosophy of language, denotes considerations about relations holding between expressions and their objectual references.” (p. 3). And the evidence seems to indicate that Carnap did not fully adopt Tarski’s viewpoint until perhaps after 1936.

In this useful and well written paper, the author traces the continuing discussions and objections to various views on “truth” and how it should be defined. And there are many remaining questions on just how Carnap, Gödel, and Tarski influenced each other during the early 1930’s. In particular there are changes between the Polish and German editions of “Der Wahrheitsbegriff”, where in the later edition Tarski refers explicitly to Gödel’s work. In his summary, the author says, “Perhaps it is possible to say the following:

1. the formal definition of truth was mathematically given (Tarski);
2. that truth is undefinable under specified conditions was proved (Tarski, Gödel);
3. arithmetic is the critical point (Gödel);
4. the definition of truth requires non-finitary means (Tarski);
5. the undefinability results require non-finitary reasoning (Tarski formally, Gödel rather informally);
6. incompleteness can be proved by non-finitary means, but it does not lead to concrete undecidable sentences (Gödel informally, Tarski formally);
7. the most important results are present in Carnap’s “Logische Syntax der Sprache”, but without any appeal to transfinite set theory (however, Carnap made strong use of Hilbert’s proof of rule [rule of proof?]).” (p. 11)

That seems a fair overview, but as Woleński concludes, “Judging the general philosophical significance of semantics is still far from being finished.” The good documentation here, however, will be of great assistance to future commentators.

(I.2) The early influences on Tarski that led to his work on truth were both philosophical and mathematical, as I. Niiniluoto points out in “Theories of Truth: Vienna, Berlin, and Warsaw”. On the philosophical side, Tarski had learned a version of the correspondence theory of truth from his teachers Lukasiewicz and Kotarbinski. Mathematically, at the time of his work on truth he was already studying the notions of satisfiability of logical formulas in formal languages. A peculiar feature of Tarski’s early work that was perhaps related to his philosophical orientation was that all interpretations were taken from a fixed, unique domain of “all objects”; there were many languages,

but one world. This was one step beyond what has been called the “universalist” conception of logic that was favored by Frege, Russell, and the early Carnap. According to that conception, there was one language and one world – *i.e.*, a fixed interpretation of it. But Tarski’s view at the time was still not the more modern “logic as calculus” one, according to which there can be many languages and many different interpretations of the basic domain, and which can be investigated from a meta-linguistic standpoint.

Niiniluoto examines Tarski’s work in relation to several theories of truth by his predecessors and contemporaries. Specifically, he considers Schlick’s empiricist theory based on unique coordination between judgments and states of affairs. He also considers the related picture theory of Wittgenstein, as well as Popper’s views. Lukasiewicz and Reichenbach both developed semantic theories based on probability, but these differed greatly from Tarski’s approach. The other wing of the Vienna circle, led by Neurath, developed what has been called a “coherence” view of truth, as an alternative to the correspondence approach. The latter’s reliance on an external world as a realm of semantic values was regarded by Neurath and Hempel as dubious metaphysics. Carnap was quick to adopt Tarski’s semantic approach to truth, but characteristically attempted to shed the metaphysical commitments. The discussions of these points within the Vienna Circle anticipated some familiar contemporary ones, notably by Putnam and Quine.

But the most interesting case of mutual influence was surely that of Carnap. As the author documents, Carnap’s *Logical Syntax of Language* (1934) and his subsequent adoption of semantics in *Introduction to Semantics* (1942) staked out well-considered philosophical positions that were strongly influenced by Tarski’s work, and by conversations between the two. Perhaps the key aspect of this influence was the break from Wittgenstein’s view of the ineffability of language structure, which had been something of a theoretical straight-jacket. The author concludes with the interesting observation that it is in Carnap’s 1942 development of semantics that we find a more modern and satisfactory approach than in Tarski’s own work from the period, even including that of 1944. Only with his return to model theory in the 1950’s does he adopt the fully modern point of view.

(I.3) In “Truth before Tarski,” Hans Sluga presents the situation in semantics, such as it was, at the time that Carnap first encountered Tarski’s work on truth, the formal outlines of which he eventually adopted. Carnap himself recalled his chief prior influences as having been Frege, Russell, and Wittgenstein. Sluga describes the semantic

doctrines of each of these influences, firstly in order to understand Carnap's development, the presumption being that this course was essentially that of contemporary analytic philosophy. He also proposes that such a study may cast light on insights and perspectives that have since been lost, forgotten, or overlooked.

Sluga is surely right that the most important difference between Tarski's point of view and the one that Carnap had inherited was the latter's "universality," in the sense that for Carnap at that point, there was no place outside of logic, as it were, from which to formulate or investigate the kind of semantic questions that Tarski was able to pursue. Following Wittgenstein, one could no more get outside of logic in order to study such notions as logical truth or consistency, than could one get out of one's own skin to measure one's shirt size. With hindsight, Russell's early suggestion that one might escape this predicament by describing the structure of one logical language in another one sounds remarkably Tarskian — and it may indeed have inspired Tarski. But at the time, the proposal need not have been taken as a rejection of the universality of logic. Indeed Carnap's failure to distinguish the logical calculus as a formal system from the interpreted language of logical reasoning was exactly responsible for his disastrously flawed "*Untersuchungen zur allgemeinen Axiomatik*" [5], which never saw the light of day. This abandoned work now provides us with a valuable record of Carnap's attempts at semantic theory immediately before his first encounter with Tarski in Vienna in 1930, and it is a pity that Sluga does not consult it in this connection. He does provide a concise and insightful summary of the relevant background, showing the significance of Frege's context principle, and the central rôle given by him to the notion of *truth*. He briefly describes the special position of the notions of *belief* and *judgment* in Russell's account of propositional meaning. Finally, Wittgenstein's *picture theory of meaning* is recalled, and Frege's objections to it in his late article "*Der Gedanke*" are presented. It is here that Frege emphasizes the logically primitive character of *truth*, and Sluga speculates that the pre-Tarski Carnap was following Frege on this point. Unfortunately, without considering Carnap's *Axiomatik*, the textual evidence is rather slight. In fact, Carnap's "*Metalogik*" position of the late 20's seems more influenced by Russell's hierarchy of languages view than by Frege's one language view.

According to Sluga, what was lost in Carnap's — and analytic philosophy's — shift from truth as an indefinable, primitive logical notion to the Tarskian semantic analysis in terms of satisfaction, reference, and other semantic concepts, was an understanding of the philosophical problem of truth and its relation to meaning. He correctly points

out that Tarski's theory of truth has often been misconstrued by subsequent philosophers as having *solved* a problem that had stumped and befuddled Frege, Russell, and Wittgenstein. But in saddling Carnap with inventing this error, he fails to appreciate the real use to which Carnap put Tarski's work. Under the influence of both Tarski and Gödel, Carnap developed a position — first in his *Logical Syntax* and later elaborated in “Empiricism, semantics and ontology” [6] — in which the “problem of truth” that Sluga is urging us to reconsider is not solved, but rather *dissolved*.

(II.1) In “How the Unity of Science Saves Alfred Tarski” Anita Burdman Feferman recaps Tarski's biography and connections with Vienna in the 1930's (which usefully supplements points made in paper (1)). The key event, then, is the invitation by Quine to Tarski to speak at the Unity of Science meeting at Harvard in 1939, which Tarski only accepted at the last minute. Other Polish invitees were unable to come for a variety of reasons. Arriving in New York on August 21st of 1939 on a temporary visa, Tarski was then persuaded after the conference not to return to Poland after Hitler had invaded on September 1st. This decision meant he would be separated from his wife and two children for the whole length of WWII, but the decision also saved him, as a Jew, from being killed in the war. As his wife, Maria, was not Jewish, she and the children were able to survive, but his parents, brother and sister-in-law were not. The family was not reunited until January of 1946.

In the article, Mrs. Feferman relates the great uncertainty during the war period of finding a job in the U.S. and how Tarski eventually came to the University of California, Berkeley, in 1942, where he remained for the rest of his career. At the Berkeley Mathematics Department, Tarski not only established a flourishing school of logic, but he also instituted the joint Ph.D. in Logic and Methodology of Science in cooperation with the Philosophy Department. As the author comments, “He was aggressive about assembling a great team and by the mid-1950s he had created an atmosphere that equaled or perhaps even surpassed the wonderful ferment that had existed in Warsaw in the years between the wars.” (p. 51). And, as an outgrowth of the ideals of the Unity of Science movement that had been so crucial in Tarski's life, he was the moving force in founding the Division of Logic, Methodology and the Philosophy of Science under the International Union of History and Philosophy of Science, which since 1960 has had such successful large-scale congresses in many world centers. We can hope to read many

more details of Tarski's life and influence in the biography on which the Feferman husband and wife team have been laboring for some years.

(II.2) In "Tarski and Gödel: Between the Lines", Solomon Feferman, a Ph.D. student of Tarski's in the early 1950s, relates how Tarski, five years older than Gödel, came to Vienna in February of 1930 at the invitation of Karl Menger, and how extensive his results in logic, set theory, geometry, and measure theory already were at that time. Gödel had just finished his dissertation under Hans Hahn on the Completeness Theorem, and "Tarski showed great interest in this work" (p. 54).

The author says that at this time "Gödel had essentially been working on logic in isolation and without direct leadership" (p. 54). This seems to be rather unfair to Carnap, for we know Gödel attended his lectures on "Metalogik" in 1928 and 1929. (Feferman later calls this confusingly "Carnap's seminar on foundations of mathematics".) These lectures were based on the unpublished "Allgemeine Axiomatik", which Carnap was then working on, and which we know Gödel studied in manuscript. Note that its content is clearly related to that of Gödel's dissertation. In addition to their frequent (and well-documented) conversations, there is also Carnap's textbook *Abriss der Logistik*, the only modern textbook in German before Hilbert-Ackerman. Unfortunately, Feferman does not mention the *Abriss*, but instead only "a few books and articles by Schröder, Frege, Skolem, Zermelo, and others" (p. 54). Finally, Gödel himself, in a published interview much later, reported his most important early influences in logic to have been: "Hilbert-Ackerman and Carnap's lectures on Metalogic" (see [1], especially p. 163, for a fuller discussion of the relationship between Gödel and Carnap).

After Tarski's visit to Vienna, the two Incompleteness Theorems followed quickly, and Gödel informed Tarski by letter of his fundamental results in January of 1931. Tarski at once understood the significance of these accomplishments and reported on them to the Warsaw Philosophical Society in April of 1931. Feferman then comments on the detail with which Tarski discussed the connections between his and Gödel's ideas in his historical notes for the German translation of "Der Wahrheitsbegriff". Tarski felt that the "method of arithmetizing the metalanguage" was independently considered by him, though more completely worked out by Gödel. Georg Kreisel once told the senior reviewer that Bernays had told him that he and Hilbert had considered arithmetization but had dropped the idea as too complicated. Of course, the idea would have come out of any attempt to axiomatize syntax (say, by concatenization of symbols), which Tarski was capable

of doing and might have done as an outgrowth of his work. Gödel, however, got there first with the most significant connections. Interestingly, Feferman points out that Hermann Weyl in 1910 had already given “a precise explanation of the notion of definability in an arbitrary structure”. If Hilbert and his students had taken in this idea – and Weyl worked for some time in Göttingen, after all – the history of logic would have been different. But as with many results in mathematics, the path to discovery is far from direct.

The author then recounts the connections between Tarski and Gödel after emigration to America, and their first meeting after the war at the Princeton Bicentennial Conference on the Problems of Mathematics in 1946. The notes of their very interesting lectures were not published at the time, and it was not possible therefore to really assess the many interconnections between their ideas and the suggestions they made. Again, as Feferman explains, the history of logic might have been different if there had been greater currency given to their opinions, but the number of working logicians was small at that time. The paper concludes with comments on Gödel’s later ideas in set theory that did not come to fruition, and how Tarski and Gödel had very different views on Platonism. The several volumes of Gödel’s *Collected Works*, for which Feferman was the principal editor, will be extremely valuable in discussing the intellectual development of this period in logic.

(II.3) It has already been mentioned that in the 1930s Carnap’s semantic theory was in some respects already further developed than that of Tarski. This is one of several interesting and well-taken points made by Creath in “Carnap’s Move to Semantics: Gains and Losses,” which provides an accurate, brief survey of the philosophical and logical development leading up to the *Logical Syntax*. He also raises several thought-provoking issues concerning the subsequent shift in Carnap’s position, usually characterized as a move from “syntax” to “semantics”. Creath shows that this description is less accurate than its convenience suggests, and that in fact many notions now considered “semantical” (like that of logical consequence) were already formulated in the *Syntax*, and had important roles to play there. The result is that the later shift to Tarskian “semantics” was actually much less abrupt and momentous than is usually thought. Its main novelty, the adoption of a formal notion of “truth”, actually fit quite smoothly into the rest of Carnap’s *Syntax* framework.

In the same spirit, Creath dispels a number of other common misconceptions and convenient falsehoods, simply by exposing them to the light of fact. We now know, for instance, that Carnap was not laboring

in ignorance of Gödel's results, but rather was among the first to learn of and appreciate them. Indeed, his *Syntax* was an attempt to apply that new-found perspective to the rather unsatisfactory strictures of Wittgenstein's *Tractatus*. In particular, using Gödel's methods, as developed in the *Syntax*, (and aided by Gödel himself, in recently published correspondence [7]) Carnap was able to provide a general definition of validity for logical formulas that not only anticipated Tarski's but was also deemed by Carnap to be adequately "syntactic".

Creath puzzles at some length over a passage from the *Syntax* in which Carnap emphasizes that "truth" is not a properly syntactic notion, a matter of a sentence's "design," *i.e.*, "the kinds and order of its symbols". This comes despite the fact that Carnap has succeeded in defining "validity" for logical formulas. Here the most straightforward reading seems perfectly adequate: while the truth of logical sentences can be determined syntactically, the truth of those containing also descriptive vocabulary will depend on non-syntactic facts about the interpretations of such terms.

The paper begins with an amusing parody of the "standard view" of Carnap's development in the late 20's and early 30's, contrasted by an illuminating time-line of the actual events of that period.

(II.4) The question "What is genuine logic?" is the central concern of "Tarski and Carnap on Logical Truth — or: What is Genuine Logic?" by Gerhard Schurz. In a fanciful "Tarski-Carnap Game" consisting of four rounds, Schurz provides a conceptual analysis of logical truth according to successive attempts by Carnap and Tarski. The development of ideas, if not strictly historical, is nonetheless compelling enough as conceptual history as it might have been, with rounds 1 and 3 going to Tarski, and 2 and 4 to Carnap.

Round 1: Tarski's semantics provide a formal theory of logical truth in response to the lesson of Gödel's incompleteness theorem, that deduction alone cannot suffice for this purpose.

Round 2: As has been argued by Etchemendy, Tarski's definition fails to characterize *logical* truth because it relies on an antecedent, and arbitrary or conventional, distinction between logical and non-logical terms. It thus in effect begs the question "what is logic?". Here Carnap's theory from the *Logical Syntax* has the upper hand, since it includes a characterization of the logical vocabulary in terms of the determinateness of statements consisting only of such terms.

Round 3: There are various, more or less well-known problems with Carnap's approach. Some of these are discussed, and it is concluded

that a different criterion of being a “logical constant” is required. Tarski, of course, proposed one: permutation invariance. The proposal stems from a lecture given by Tarski in 1966 and published only in 1986 [8], so a good deal of authorial license goes into this conceptual history. Briefly, Tarski’s proposal was to define as “logical” those operations on sets that are invariant under all permutations, thus applying the spirit of Felix Klein’s *Erlanger Programm* to Logic instead of Geometry. Rather remarkably, it turns out that all of the usual logical constants, relations and operations—truth functions, identity relation, quantifiers—do indeed have this property of invariance.

Round 4: Unfortunately, however, some apparently *non*-logical properties are also permutation invariant. This includes not only the so-called numerical quantifiers “there are n things x , such that...”, which are arguably logical, but also such “contingent quantifiers” as “there are as many things x as there are planets, such that ...”. Other similar “logical” operations involving contingent, empirical conditions are seen to be permutation invariant, and so the criterion is supposed to fail. It is a necessary condition for a relation, operation, or constant to be logical, but not a sufficient one.

The author concludes with his own proposed solution to the problem of characterizing the genuinely logical, which this reviewer finds less compelling than Tarski’s invariance criterion, even in light of the “critique” that ended Round 4, and tied the match. Be that as it may, the essay is a successful and thought-provoking bit of conceptual historical fiction.

(III) Part III of the book is entitled “Philosophical Aspects of Tarski’s Conception of Truth and its Application in the Methodology of Science”. It consists of the following eight papers:

1. Jan Tarski: “Interplay between philosophy and mathematics in classical theory of truth”
2. Andrzej Grzegorzczak: “Is antipsychologism still tenable?”
3. Arthur Rojszczak: “Why should a physical object take on the role of truth-bearer?”
4. Maria Luisa Dalla and Roberto Giuntini: “Lukasiewicz’s theory of truth, from a quantum logical point of view”
5. Göran Sundholm: “Intuitionism and logical tolerance”
6. Wilhelm K. Essler: “Tarski on language and truth”
7. Thomas Mormann: “Neurath’s opposition to Tarskian semantics”

8. Ladislav Kvasz: “Tarski and Wittgenstein on semantics of geometrical figures”

These well-written and thought-provoking papers include many novel insights into Tarskian semantic theory and related matters, both from contemporary philosophical and historical viewpoints. We here only briefly indicate their contents:

Jan Tarski provides a summary and assessment of the background and the influence, both in mathematics and philosophy, of (Alfred) Tarski’s theory of truth. Special consideration is given to the reception of the theory in philosophy, given its essentially mathematical aspects.

Grzegorzczak applies a modern-day psychologistic approach, rooted in semantics, to the so-called “semantic paradoxes” of Grelling and Nelson. The conclusion is reached that, from this point of view, these paradoxes are no longer contradictions in the strict sense.

Rojszczak asks, specifically, “Why had Tarski chosen physical objects as truth-bearers in his original work from 1933 about truth in formalized languages?” and “Are there any general grounds for the choice of physical objects as the bearers of truth?”. The answer to the first question bears on the second one, and the answer to the second one bears on contemporary discussions of “physicalism” by authors such as Hartry Field and John McDowell.

The interesting paper by Dalla and Giuntini compares Lukasiewicz’s three-valued logic with subsequent developments in multi-valued logic, focusing particularly on the quantum logic of von Neumann. Interestingly, the possibility of using his logic for this purpose in physics was apparently considered, but not pursued, by Lukasiewicz himself.

Sundholm considers a wide range of “intuitionistic provocations”, in discussing the works and thought of various authors from the point of view of Brouwerian intuitionism. Cantor, Zermelo, Bolzano, Kronecker, Frege, Wittgenstein, Menger, Carnap, and Gödel are all considered, while touching on the “*Grundlagenstreit*” and several other topics along the way.

The difficult paper by Essler is concerned with the success and failure of axiomatic descriptions of mathematical concepts, such as the real numbers or the theory of sets. Conditions of adequacy for metalanguages are considered, and the fact that elementary (*i.e.*, first-order) axiomatizations of such concepts are not categorical is discussed. A non-standard model of first-order set theory is compared to Lobachevsky’s non-standard geometry.

Mormann reconsiders the dispute between Carnap and Neurath on the use of Tarskian semantics in the Vienna Circle’s empiricist program.

The former whole-heartedly embraced semantic theory, and regarded the latter's reluctance to do the same as based on "misunderstanding". Mormann argues to the contrary that Neurath indeed had legitimate theoretical grounds for rejecting Tarskian semantics in the context of his radical empiricist theory of knowledge.

In the provocative paper by Kvasz, the "picture theory" of Wittgenstein is regarded as an alternative approach to semantics, and compared to the semantical approach of Tarski. The two theories are compared with respect to an unusual object theory, namely Geometry, in which geometrical pictures play an important and distinguished rôle.

(IV) Part IV is entitled "Technical Aspects of Tarski's Definition of Truth and Model Theory" and consists of the following five papers:

1. Paul Weingartner: "Tarski's truth condition revisited"
2. Roman Murawski: "Undefinability vs. definability of satisfaction and truth"
3. Jaakko Hintikka and Gabriel Sandu: "Tarski's guilty secret: compositionality"
4. Ryszard Wójcicki: "Should Tarski's idea of consequence operation be revisited?"
5. Georg Gottlob: "Remarks on a Carnapian extension of S5"

Here, again, we only summarize their respective contents.

Weingartner considers a revision of Tarski's truth condition intended to accommodate two purported objections. After a brief historical review of precursors of Tarski's theory, these objections are formulated as presuppositions, which are then taken into account in a revised theory. Finally, it is shown how the revision solves the paradox of the liar.

Murawski provides a very clear review of Tarski's theorem on the indefinability of truth and considers its consequences with respect to Peano arithmetic in particular. An explicit specification of the notion of satisfaction, and thus also of truth, is given for Peano arithmetic in a weak fragment of second-order arithmetic.

The thought-provoking contribution by Hintikka and Sandu is concerned with the dependence of Tarskian semantics on "compositionality": roughly, the meaning of a complex symbol is a function of the meanings of its parts. The authors show that Tarski's own well-known doubts about applications of his work to natural languages probably stemmed from the reliance of formal semantics on compositionality, which is generally agreed to fail in natural languages. They consider a non-compositional formal language, independence-friendly first-order

logic, which admits no natural, Tarskian compositional semantics, but does have a compelling game-theoretic semantics.

Wójcicki begins with the question of whether the notion of truth is an indispensable element of Tarski's conception of logical consequence. The author considers the relevance of ideas derived from cognitive science involving belief revision, which lead to a non-monotonic logic, to Tarski's way of approaching the notion of logical consequence. The conclusion is reached that standard Tarskian methods are indeed adequate for non-monotonic reasoning.

Gottlob provides a survey of a particular system of modal propositional logic \mathbf{C} that is related to the systems studied by Carnap in his late investigations in modal logic. Like Carnap's systems, \mathbf{C} is motivated by the idea that necessity $\Box p$ means " p is logically true" and possibility $\Diamond p$ means " p is not logically false". While \mathbf{C} is strictly stronger than S5, its valid formulas are not closed under (uniform) substitution. Indeed S5 can be characterized as precisely the tautologies of \mathbf{C} that are preserved under such substitutions. In addition to surveying its logical properties, the paper provides a useful summary of the literature on the system, including its relation to Carnap's own work.

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