

**PHILOSOPHY IN HAO WANG'S CONVERSATIONS
WITH GÖDEL**
Review of
**HAO WANG, *A LOGICAL JOURNEY. FROM GÖDEL
TO PHILOSOPHY***

Cambridge and London: The MIT Press, 1996
xiv + 391 pp. ISBN 0-262-23189-1

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This is a long-awaited book. It contains Hao Wang's records of his conversations with Kurt Gödel (1906 -1978) from 1971 to 1976, ordered by subjects, and some reflections by Wang about Gödel's life, personality and ideas. The contents of the conversations are mainly philosophical in nature, which is very understandable; although Gödel almost never dared to make his philosophical ideas public, he sometimes felt the need of some philosophical interchange. Consequently, I will consider only the philosophical side of the book, though some other interesting things about Gödel can be found here.

The review is divided into seven sections. The first and second of these are devoted to the origins of the book, as it was actually published, and to the origins of the conversations themselves. The third tries to present a global scheme of Gödel's more general philosophical conceptions. Sections 4-6 will sum up the main ideas of what he thought about philosophy of mind, of mathematics and of logic and set theory. Finally, I will attempt some assessment of the book itself, and also of some of the philosophical ideas appearing here for the first time, in the context of Gödel's well-known ideas available in his previous publications, including the recent ones containing his philosophical unpublished manuscripts [1, 2]¹.

1. THE ORIGINS OF THE BOOK.

For many years Wang did not find a satisfactory way to publish the records of his conversations with Gödel. He told me in the early

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¹The first of Gödel's philosophical manuscripts to be published appeared in my "Un inédito de Gödel contra el convencionalismo: historia, análisis, contexto y traducción," *Arbor* **CXLII**, 558-559-560, 1992: pp. 323-348.

nineties that after many different attempts, he had arrived at a general presentation where the different fragments appeared in chronological order. Yet he was not satisfied, mainly because the global result was too sketchy and many fragments were rather loose in the general context. After that, in correspondence, he told me that he was trying to find someone to assist him for this matter, and even sent me a complete typed copy, from which I quoted some fragments concerning some of Gödel's philosophical views (see [2, pp. 27, 33]).

Hao Wang finally proposed Michael Detlefsen to assist him in completing the projected book, but after taking some time to consider the offer, Detlefsen declined and suggested a former student of him in Notre Dame, Anthony Everett, for the job, who was accepted by Wang. Nevertheless, Detlefsen continued to be in touch with Wang, both in correspondence and over the phone, about his writings, some of which he read and made comments about. Yet Wang must have been somehow annoyed with Detlefsen's former negative decision, for he is not mentioned in the published book.

Wang does mention Everett in the Preface (p. xiii), to credit him for his help during two years in making an index and an outline of the notes, an index of the book, and for his comments on a previous draft of the book. Everett is at present happy with that acknowledgement, but it seems to me that it is not as generous as it should have been, according to Everett's own description of his actual work on the book, which I now quote because of its importance in understanding the whole history:

Wang had several hundred pages of fragmentary notes of his conversations with Godel [*sic*], interspersed with some of Wang's own observations. For many years he had been trying to work these into a book of some form, a final tribute to Godel [*sic*]. I don't think he was at all sure how best to do this. I sorted through these notes and indexed them. I also tried to order them into various topics, order the notes within each topic in a philosophically coherent manner, and order the topics. So the final order in which the notes appear in the book is probably largely my responsibility.

After talking with Wang I understood my task to be this: to try to *re-construct* or *re-write* Godel's [*sic*] observations into a more coherent and flowing body of text that could stand on its own. I was not completely sure how much leeway I was allowed. I am pretty sure

that Wang thought this was my task too, but one can never be completely certain. As I saw it the result would not have quite been Godel [*sic*], but in Godel's [*sic*] spirit, perhaps in much the same way that people try to complete the novels and symphonies of dead authors and composers. It would not have been presented as "Godel's word" [*sic*] but as an explicit reconstruction of it.

Anyway I spent much time trying to do this. Obviously this was a difficult way of proceeding, and maybe the result would have been of only limited interest. Quite rightly, I think, Wang disliked the result of my work and resolved to include the fragments as they stood. He wrote connecting text and comments quite quickly I believe. I had read and commented upon drafts of most, but not all, of the chapters before Wang died. To some extent the form of the book may still be partly a compromise, since Wang was very ill and knew that he was dying, and wanted to see the thing finished while he lived.

The original idea, as suggested by Mic [Detlefsen], and as I understood it, was that the result should have been co-edited by Wang and me. However since Wang ended up writing most of the book it is really his own child and I cannot take much credit. I see my own most important contribution to the book as so horrifying Wang that they allowed him to better understand what he wanted the book to be and what he didn't want it to be. Perhaps they also helped motivate him to finish it (out of fear that my "textual reconstruction" would otherwise be published?).

Let us finish this section with a comparison between the final presentation of the notes with the original one in the original manuscript of the book which I — and also Everett — mentioned above. In what seems to be its last stage, that manuscript was entitled *For perspicuous objectivity. Discussions with Gödel and Wittgenstein*. It was divided into two parts. The first one constitutes a book in itself, containing five chapters, respectively entitled: 1. "To philosophy, from the views of Wittgenstein and Gödel on mathematics"; 2. "Five islands of agreement with Gödel"; 3. "Creation and discovery: on mathematical realism"; 4. "Dialectic of intuition and idealization: on conceptual

realism”; 5. “Objective certainty and the nature of mathematics”. Besides, there are a preface, a section of abbreviations and reference, an introduction, and several pages of notes after each chapter. As a whole, the book — which is dedicated “To the memory of Kurt Gödel” — extends to xiv +178 typed pages and seems to be ready for publication. I think it should be published as it is.

The second part of the manuscript, entitled “The conversations”, starts with an introduction (17 pp.), and is divided into four chapters: 1. “October 1971 to February 1972” (77 pp.); 2. “March to December 1972” (46 pp.); 3. “October 1975 to January 1976 ” (79 pp.); 4. “February to June 1976 ” (40 pp.) . Probably Wang thought at one time of publishing an entire book including all this material, and then found the idea rather unconvincing.

Some of the things Wang says in the manuscripts do appear in the book under review, here and there. Unfortunately, the numbering system of the Gödel passages has been heavily modified. In the published book they appear simply with three numbers, the first being the number of the chapter, the second the number of the section, and the third the number of the particular passage (*e.g.*, 5.3.22); also, the remaining quotations, whether by Gödel — in published or unpublished writings — or by other authors, follow the same pattern. On the contrary, the system of the manuscript was described by Wang this way: “I have split the conversations into about 520 segments, which are numbered according to their place in the year and the month. For instance, 602.1 refers to the first segment (1) in February (02) of 1976 (6)”. Obviously enough, the original system would have been preferable, as it would give the reader and the scholar additional information about the particular time in which the idea was originally formulated, which is necessary to understand Gödel’s preferences according to particular stages of the evolution of his ideas. In the actual book the fragments appear divided according to the subject matter dealt with in each of them; yet this could have been done in the same way, but preserving the old numbering system.

2. THE ORIGINS AND CONTEXT OF THE CONVERSATIONS.

After some more or less casual contacts, the relation between Wang and Gödel began in 1967, when Wang sent Gödel a draft of a manuscript on Skolem for comments, and Gödel provided some. A few paragraphs of Gödel’s letters were finally published in Wang’s *From Mathematics to Philosophy* [3] and are well-known. As it happened, Gödel invited Wang in January 1970 to visit him, and then Wang asked

Gödel to comment on a draft of his projected book, which Gödel must have read carefully, especially the parts where his name occurred. As a matter of fact, the conversations began by making extensive comments on those parts, then they extended to other parts.

From October 1971 to December 1972 Wang and Gödel met regularly in Gödel's office in Princeton, from 11 am to 1 or 1:30 pm. They had twenty sessions where Wang was allowed to take notes. Unfortunately, he did not bring a tape recorder ("at that time I did not find the idea congenial and did not propose it", p. 136). Usually Gödel brought slips of paper with the ideas he wished to talk about and Wang took notes during the sessions. Yet he recognized that the final notes remained in a very unsatisfactory state, despite his efforts to reorganize them after each session.

They usually started talking about philosophy and Wang's manuscript. After it was sent to the printer in June 1972, Gödel's philosophical views constituted the nutshell of the conversations. Several times Gödel wanted to show Wang some of his unpublished manuscripts — probably the Gibbs lecture, given in 1951, in the first instance — but he always wound up by finding the manuscript "not yet in a form fit to be shown" (p. 136). Also, Gödel declined to expose his philosophy in a systematic way, arguing that it was not been yet developed to the point to lecture on it.

In 1975 Wang became a visitor at the Institute for Advanced Study for more than one year, and they resumed the contacts. Yet Gödel had stopped going to his office — by then he had health problems, so most of the conversations were over the phone, and based upon written versions of former discussions which Wang had previously sent him.

Wang's whole conception of his reports is important. He says that some of them were fully approved by Gödel, while others were not. Some were never seen by Gödel. As a result, Wang tells us that Gödel "would not have wished to publish much of the material in the form I "quote", and it is quite possible that there are places where I am mistaken about what he actually says" (p. 137). Also, Wang adds that he made no serious attempt to look to Gödel's *Nachlass*, whose materials might prove some of his reports to be mistaken. The reader is warned: "what I report in this book must be understood *cum grano salis*" (p. 137).

3. GÖDEL'S GLOBAL PHILOSOPHICAL VIEWPOINT.

Gödel's own philosophy took a clear side in spiritualism, rationalism, idealism and theology, against skepticism, materialism and positivism.

His philosophical global viewpoint was summarized by him in a manuscript from 1960 this way:

The world is rational. Human reason can, in principle, be developed more highly. There are systematic methods for the solution of all problems. There are other worlds and rational beings of a different and higher kind. The world in which we live is not the only one in which we shall live or have lived. There is incomparably more knowable *a priori* than is currently known. The development of human thought since the Renaissance is thoroughly intelligible. Reason in mankind will be developed in every direction. Formal rights comprise a real science. Materialism is false. The higher beings are connected to the others by analogy, not by composition. Concepts have an objective existence. There is a scientific philosophy and theology, which deals with concepts of the highest abstractness; and this is also most highly fruitful for science. Religions are, for the most part, bad — but religion is not (p. 316).

Accordingly, Gödel defended a dualistic ontology based on a sort of Leibnizian monadology, an epistemology based on a probable, rationalistic knowledge, which should be always guided by observation — including intuition — and a series of opinions about the philosophers — either from the past or contemporaries — which were inspired by them.

His conception of the monads is highly speculative. Monads, which in the end should compose matter, are neither material nor spatial, although they act in space. Also, they have a consciousness and maintain relationships with other particles, while having “something inside”. Their spiritual nature will be correctly understood just when “the true theory of physics is found” (p. 292). Among the different kinds of being, concepts and objects seem to be the essential ones, although the relations between them and monads are by no means clear. Concepts are wholes, composed of primitive concepts (“such as negation, existence, conjunction, universality, object, concept, whole, meaning, and so on”, p. 295), and are wholes of an organic nature, in a way that sets are not. As for objects, including monads and sets, they can be of different kinds, including physical and mathematical ones. The following quotation may perhaps clarify this a little:

A set is a special kind of whole. Sets are unities which are just the multitude; but generally wholes are more

than multitudes which are also unities. That is why sets are a limiting case of wholes. A whole must have parts. A monad is a unity but not a whole because it is indivisible: it is only an *uneigentlich* [improper] whole. Primitive concepts, like monads, are unities which are not wholes, because they are not composed of parts (p. 296).

Be that as it may, primitive concepts are to be looked for by us, and then systematically exposed to give an account of the world; thus we come to knowledge.

All knowledge depends upon observation of the different kinds of beings; it is not infallible, but every mistake is due to the empirical component. It can be corrected through our whole experience, properly guided by pure reason, which “does not commit mistakes” by itself and seems to include some “probable intuition” (p. 291). Thus skepticism is to be rejected. Yet “experience” should not be understood in the empirical manner, for the sort of observation presupposed here includes *Wesenschau*, which is explained by Gödel as “essential intuition, grasp of essence, categorial intuition, perception of concepts” (p. 292). Therefore, what is usually called experience is irrelevant. On the contrary, everything is inside ourselves: “Don’t collect data. If you know everything about yourself, you know everything. There is no use in burdening yourself with a lot of data. Once you understand yourself, you understand human nature and then the rest follows.” The results of the investigations ruled by the correct use of our knowledge should be expressed as a sort of grand axiomatic, theoretical metaphysics, apparently based on some use of mathematical logic, somehow conceived as the proper science of general concepts.

The relation between axiomatics and ontology is clearly expressed when Gödel writes: “The axioms correspond to the concepts, and the models which satisfy them correspond to the objects. The representations give the relation between concepts and objects” (p. 141). Also, the relation between ontology and epistemology can be better understood through this: “Sets are objects but concepts are not objects. We perceive objects and understand concepts. Understanding is a different kind of perception: it is a step in the direction of reduction to the last cause” (p. 235).

Upon these bases it is not very difficult to figure out Gödel’s philosophical friends and also his enemies, which constitutes the kernel of chapter 5 of the book. Among the first Plato, Kant and Husserl have to be mentioned. We can try to summarize his views about them. Plato

can be regarded as the founder of philosophy because he studied the definition of concepts. Kant is to be acknowledged because, although his philosophy somehow gives the pre-eminence to architectonic over an absolute and systematic starting-point, however he looked for a trascendental — yet rather subjective — philosophy where “all categories should be reduced to something more fundamental” (p. 171). Husserl should be central in what Gödel thought to be the true method of philosophy, because his phenomenology tries to go back to the absolute foundations of our knowledge, “to the process of how we form the knowledge, and to uncovering what is given to us from inside” (p. 167). Yet there are numerous shades in the fragments which have to be carefully analyzed to obtain a more articulate viewpoint about them.

As for Gödel’s philosophical enemies, most of his critical passages are written against the logical positivists, and other “linguistic philosophers”, Carnap and Wittgenstein being the names appearing more often. However, as the kernel of his criticisms are now well-known, after other publications by Wang, and also through the recent appearance of some of Gödel’s unpublished manuscripts, I will leave those passages without further comment.

4. PHILOSOPHY OF MIND: SPIRITUALISM.

Gödel’s main views in the field of the philosophy of mind seem to be a consequence of his struggle against materialism and positivism, and can be summarized by saying that for him the mind cannot work like a computer, and that the brain is not enough to explain mental phenomena, so even if the brain were shown to be like a computer, very interesting mental abilities have yet to be explained. From the publication of Wang’s *From mathematics to philosophy* in 1974 we know Gödel’s main arguments against any possible attempt to identify minds and computers (in the context of his comments about Turing machines). They were: that the use of the mind is in constant development, while computers are static; that the states of every computer are necessarily finite, while mental states might converge to infinity because of their development, and that there may exist mental procedures which are not mechanical in nature (he was obviously thinking of mathematical intuition, which for Gödel cannot be reduced to any mechanical procedure).

Also, from his Gibbs lecture, given in 1951 but published for the first time in 1994 (in Spanish) and 1995 (in English) (see the works cited in [2]), we know that for Gödel his celebrated incompleteness results

show that mathematics is inexhaustible, so that the human mind is unable to mechanize all its mathematical intuitions. This was expressed by him as an interesting logical disjunction: either the system of all demonstrable propositions (subjective mathematics) surpasses all machines, or the system of all true mathematical propositions (objective mathematics) surpasses subjective mathematics. If the first, the human mind cannot be reduced to the brain; if the second, mathematical objects and facts cannot be our creation, so they are independent of our mental acts. Obviously enough, both alternatives were to be unacceptable for materialists, so they together may be indirectly argued in favour of spiritualism.

Most of the passages published in Wang's new book concerning Gödel's philosophy of mind express very similar ideas, or contain comments on them, as for instance it is the case with these:

The incompleteness results do not rule out the possibility that there is a theorem-proving computer which is in fact equivalent to mathematical intuition. But they imply that, in such a — highly unlikely for other reasons — case, either we do not know the exact specification of the computer or we do not know that it works correctly. My incompleteness theorem makes it likely that mind is not mechanical, or else mind cannot understand its own mechanism. If my result is taken together with the rationalistic attitude which Hilbert had and which was not refuted by my results, then [we can infer] the sharp result that the mind is not mechanical. This is so, because, if the mind were a machine, there should, contrary to this rationalistic attitude, exist number-theoretic questions undecidable for the human mind (pp. 186-187).

But there also are other passages in which spiritualism is openly defended: “Even if the finite brain cannot store an infinite amount of information, the spirit may be able to. The brain is a computing machine connected with a spirit” (p. 193). As usual then, Gödel was much more cautious in his publishable writings than in his private reflections.

For similar general reasons, Gödel was also against psychophysical parallelism: it is not compatible with his strong conviction that the mind cannot be reduced to the brain. But in this case a new argument appears here, which connects the issue with empirical matters: parallelism can be disproved scientifically, as “there aren't enough nerve cells to perform the observable operations of the mind” (p. 190). Similarly,

Gödel wrote that the brain cannot be explained in the usual Darwinian way, as “life force” works according to laws which are neither simple, nor mechanical. Yet the most specific argument in the same direction seems to be just that the formation of a human body by the laws of physics “is as unlikely as the separation by chance of the atmosphere into its components” (p. 192).

5. PHILOSOPHY OF MATHEMATICS: OBJECTIVISM.

Objectivism, or Platonism, was the kernel of Gödel’s philosophy of mathematics, and consisted of the belief that mathematical objects and concepts (the entities referred to by mathematical symbols) and facts (those expressed by mathematical propositions) are not our creation; yet they do objectively exist with total independence from the existence and working of our minds. This philosophical position was fully developed in the Gibbs lecture and in Gödel’s essay on Carnap.

The Gibbs lecture studied the philosophical implications of the incompleteness theorems. According to it, mathematics is inexhaustible, so we cannot do mathematics without intuition, which cannot be replaced by algorithmical methods. Mathematical concepts and facts are objective, and the human mind can perceive them in a way which cannot be done by any finite machine. As for the Carnap essay, mathematics cannot be reduced to a formal system of logical syntax of language, for — after the incompleteness results — no similar system could do the work unless it makes use of concepts similarly powerful to the ones to be reduced, so any attempt on these lines would be vacuous. However, these writings by Gödel are already published, so I will limit myself here to identifying some of the ideas which are not appearing in them.

In the conversations with Wang, Gödel gave similar arguments, but they were more simply stated, and emphasized the importance of number theory. He clearly said that the “real argument” for objectivism is simply that, as we know that many propositions about natural numbers are true, and as we believe that many related conjectures (*e.g.*, Goldbach’s) make sense, then “there must be objective facts about natural numbers” and these facts “must refer to objects” which “are unchangeable in time” (p. 211). As for the argument against conventionalism (*i.e.*, against Carnap), Gödel added that logic and mathematics must have a real content which can be seen by studying number theory, where “we encounter facts that are independent of arbitrary conventions. These facts must have a content because the consistency of number theory cannot be based on trivial facts, since it is not even known in the strong sense of knowing” (p. 212).

However, Gödel was aware that as long as we go up to higher mathematics, the solid ground seems somehow to vanish. In the published writings he used to talk about the help of intuition for that case, but here he used the rather loose notion of “idealization”:

Strictly speaking we only have clear propositions about physically given sets and then only about simple examples of them. If you give up idealization, then mathematics disappears. Consequently it is subjective matter where you want to stop on the ladder of idealization ... Without idealizations nothing remains: there would be no mathematics at all, except the part about small numbers. It is arbitrary to stop anywhere along the path of more and more idealizations. We move from intuitionistic to classical mathematics and then to set theory, with decreasing certainty. The increasing degree of uncertainty begins [at the region] between classical mathematics and set theory. Only as mathematics is developed more and more, the overall certainty goes up. The relative degrees remain the same (p. 217).

It seems that for Gödel idealization was not only a stronger notion than intuition, but something somehow related to his holistic overall conception of mathematics and general science; also, it might be connected to Kant’s “regulative” use of some ideal concepts in order to obtain progress in our research, which is always inspired by the principle of getting more and more general concepts to be used in explaining the less general ones.

Finally, there seems to be nothing significantly new concerning two of Gödel’s most important features characterizing the objectivity of mathematics: its analytical, but not tautological, nature, and its deep analogy with physics, in that both sciences have to postulate entities from the viewpoint of their fruitfulness. For Gödel, objectivism is clearly a fruitful position for research, and it cannot be adopted just because of its fruitfulness — through some sort of *as if* attitude — for in that case the position is adopted precisely because it has been shown as fruitful (p. 239).

6. PHILOSOPHY OF LOGIC AND SET THEORY: CONCEPTIVISM.

Both in his published and unpublished essays, Gödel spoke rather obscurely about concepts; he seemed to think that concepts are indispensable for philosophy, foundations of mathematics, and logic, and that only through the right concepts the usual paradoxes and other

problems of set theory were able to be overcome. Yet not even a minimal theory of concepts and its relations with sets, in the field of a philosophy of logic and mathematics, can be found in those writings. One of the novelties of the publication of these records is that here we can finally find some pieces of such a theory. That is why this section will contain more quotations.

The crucial difference between logic and mathematics is that while logic deals with concepts, mathematics deals with sets:

The subject matter of logic is intensions (concepts); that of mathematics is extensions (sets). Predicate logic can be taken either as logic or as mathematics: it is usually taken as logic. The general concepts of logic occur in every subject. A formal science applies to every concept and every object. There are extensional and intensional formal theories ... Mathematicians form and use concepts, but they do not investigate generally how concepts are formed, as is to be done in logic (p. 274)

The subject matter of mathematics is sets:

A set is a unity of which its elements are the constituents. It is a fundamental property of the mind to comprehend multitudes into unities. Sets are multitudes which are also unities ... : this is the main fact of mathematics ... Mathematical objects are not so directly given as physical objects. They are something between the ideal world and the empirical world, a limiting case and abstract. Objects are in space or close to space ... Sets are quasi-spatial (p. 254). Mathematicians are primarily interested in extensions and we have a systematic study of extensions in set theory, which remains a mathematical subject except in its foundations (p. 274).

On the contrary, logic deals with concepts, although it also deals with sets from the viewpoint of its foundations, that is, from the formal side:

Set is a formal concept. If we replace the concept of *set* by the concept of *concept*, we get logic. The concept of concept is certainly formal and, therefore, a logical concept. But no intuition of *this* concept, in contrast to that of set, has been developed (p. 267). Logic is the theory of the formal. It consists of set theory and the theory of concepts. The distinction between elementary (or predicate) logic, non elementary logic, and set theory is a subjective distinction ... Elementary logic is the

logic for finite minds. If you have an infinite mind, you have set theory (p. 268).

In this connection, we find more about the relation between sets and concepts than what was offered in the ontology (see section 3 above). Sets seem to be the extensions of concepts, very like in Frege's way:

It is not in the ideas (of *set* and *concept*) themselves that every set is the extension of a concept. Sets might exist which correspond to no concepts. The proposition "for every set, there is a [defining] concept" requires a proof. But I conjecture that it is true. If so, everything (in logic and mathematics) is a concept: a set, if extensional; and a concept (only) otherwise (p. 274). It is not evident that every set is the extension of some concept. But such a conclusion may be provable once we have a developed theory of concepts and a more complete set theory. While it is an incorrect assumption to take it as a property of the concept of *concept* to say that every concept defines a set, it is not a confusion to say that sets can be defined by concepts or that set is a certain way of speaking about concept (p. 276).

We can try to go beyond the concept of concept, but to do that we need first a theory of concepts (the one we also need for another important reason: to overcome the paradoxes involved in an incorrect use of them, as it is already suggested in Gödel's publications):

If you introduce the concept of *concept*, the result is still logic. But going "higher" would be too abstract and no longer logic. The concept of *concept* calls for only the lowest level of abstract intuition ... The older search for a satisfactory set theory gives way to a similar search for a satisfactory theory of concepts that will, among other things, resolve the intensional paradoxes. Quine's idea of stratification is arbitrary, and Church's idea along the line of limited ranges of significance is inconsistent in its original form and has not been worked out (p. 268).

Such a theory of concepts is even suggested along very general lines:

Even though we do not have a developed theory of concepts, we know enough about concepts to know that we can have also something like a hierarchy of concepts (or also of classes) which resembles the hierarchy of sets

and contains it as a segment (p. 278). Just as set theory is formulated in the predicate calculus by adding the membership relation, concept theory can similarly be formulated by adding the relation of application: a concept A applies to something B (which may also be a concept), or B participated in the idea A . Logic studies only what a concept applies to. *Application* is the only primitive concept apart from the familiar concepts of predicate logic with which we define other concepts (p. 277).

However, a fundamental difference does exist between set theory and a theory of concepts: “the concept of clearly defined concept is not a clearly defined concept. A concept, unlike a set, can apply to itself. Certainly the concept of *concept* is a concept (p. 278).

To avoid this problem, we could perhaps reach very general, abstract concepts, going higher and higher well beyond the concept of concept itself, which is already rather close to Kantian well-known ideas. There are a few records on this point as well:

The general concept of *concept* is an *Idea* [in the Kantian sense]. The intensional paradoxes are related to questions about Ideas. Ideas are more fundamental than concepts ... Absolute demonstrability and definability are not concepts but inexhaustible [Kantian] Ideas. We can never describe an Idea in words exhaustively or completely clearly. But we also perceive it, more and more clearly ... Ideas cannot be used in precise inferences: they lead to the theory of types. It is a kind of defeatism to think that we have this vague idea which is the very basis of our precise idea. We understand the special concept only because we previously had the general idea. We restrict the general idea to individuals to get the concept of the first type. The general idea of concept is just generality (pp. 268-9).

So we arrive at the same point we arrived at above, when talking about more and more general concepts from the viewpoint of their “regulative”, Kantian use.

7. SOME ASSESSMENT.

The book itself has to be highly recommended, in spite of its numerous defects, for a very simple reason: there is no other place where Gödel’s philosophical thoughts are available in so frank and open a way.

Even a superficial comparison between most of the things Gödel said to Wang and most of the things he wrote in the essays he meant for publication (whether actually published or not), shows that in the first case his extreme caution and pathological fear to controversy disappeared to a very high degree. Also, in these fragments Gödel made several attempts to insert some of his well-known philosophical doctrines into a global framework, both ontological and epistemological.

However, the book is not well-organized, and contains an enormous amount of repetitions, while Wang's comments are very uneven in value and perspicuity. It seems that he tried to put into the book all the things he actually had thought about Gödel, even though many of them had already appeared in his former books ([3, 4]), instead of trying to concentrate just on the new fragments.

Also, the system of numbering, as I said above, is not very informative; even worse, the same system is applied to Gödel's new fragments, to quotations from other materials already published, and even to quotations from other authors. Perhaps it would have been more useful simply to offer the reader the whole list of fragments as they were held by Gödel, together with brief comments by the editor, and then a global attempt to reconstruct the main lines of Gödel's philosophical thought, somehow in the way Everett seems to have attempted first.

As for Gödel's ideas in themselves, I think that many of them are not clearly expressed, so it is very difficult to try to actually insert them into what we already know about his philosophy. For instance, take the concept of concept. He says that the concept of concept is a concept, which is not true with sets, so a theory of concepts similar to set theory cannot be built up. So, it is not clear how a theory of concepts could be built up as a logical, formal foundation of the concept of set, then of set theory, as Gödel seemed to believe, unless we accept explanations of the *obscurum per obscurius* type.

Besides, although Gödel's theory of concepts is based just upon one primitive relation, that of application, this is difficult to relate to Frege's theory (from which it could have been inspired). For one thing, Frege's concept of concept is not a concept, but an object: this is Frege's celebrated paradox of concepts. As we write "the concept of...", we put a concept in an object position, for we use a singular term to name it. Also, for Frege, although the theory of concepts could perhaps be reduced to one fundamental relation, this relation seems to be different from that of application. Frege, in his posthumous writings, wrote about this relation to be that of "subsumption", and this relation is highly problematic, as it may give way to another paradox: the well-known paradox of the relations between the related terms and

the relations actually relating them. That is why Frege invented an ontology of saturated and unsaturated entities (*i.e.*, objects and concepts), which need no further entities to connect them. Yet no similar attempt can be found in the Gödel fragments we now have available.

Finally, Gödel says that concepts could rest on ideas. However, as Gödel himself had to recognize, Kant's distinction between ideas and concepts, although useful in trying to define concepts more precisely, "is not clear" (p. 269). So, at the very end, it seems very problematic that a theory of concepts, based on primitive concepts and finally on regulative ideas, could be built up in order to throw light on the most obscure zones of the foundations of logic and set theory when, as it happens, concepts and ideas themselves are very far from being clear entities.

Although this book should be studied by scholars interested in Gödel's ideas, one thing is already clear: we know now why Gödel was so reluctant to make his most deep philosophical ideas public: they are simply rather obscure and rather unconvincing.

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