

Review of
PIERGIORGIO ODIFREDDI, EDITOR,
KREISELIANA:
ABOUT AND AROUND GEORG KREISEL

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There are many good reasons for regarding the task of writing a review of a volume about and dedicated to Georg Kreisel (G.K.) as a challenging, and even rather intimidating task. To mention just two obvious ones: Firstly, very few people, and the present author is certainly not among them, have the necessary scientific and philosophical competence to deal with the wide range of Kreisel's interests and publications. (Perhaps G.K. himself, the author of some memorable reviews, would be the natural choice.) Secondly, it is hard to resist the temptation to write about Georg Kreisel the man rather than this book about him.

Kreisel is difficult. His papers are difficult, most of his ideas, insights, appraisals, criticisms and suggestions are anything but simple. This is by no means an isolated personal opinion. Similar remarks will be found in many places in the book under review. And, as the reader of this volume will surely agree, especially after having read its first part, the man himself is difficult.

This volume, *Kreiseliana: About and around Georg Kreisel*, is a heterogeneous collection of twenty-five papers. The list of authors is a most impressive one. Most of them have played, and many continue to play, a fundamental role in contemporary logic. The book is divided into four parts. The first, "Reminiscences," as the title indicates, consists of personal recollections of various kinds. The second and third parts are devoted to Kreisel's work in mathematics and in philosophy, respectively. The final part, entitled "Technical Tribute," contains two original technical papers. Clearly, a volume of this kind cannot be assessed according to normal standards.

The first part contains thirteen personal recollections, some of them very vivid, which will undoubtedly contribute to the aura of mystery

and enchantment (or disenchantment) which surrounds the *legend* of Georg Kreisel. (I would be willing to bet that anyone who has had any kind of relationship, direct or indirect, with G.K., has a story to tell.) But this first part is much more than just that: some of the texts in it contain privileged firsthand accounts of the otherwise hardly accessible history of (stories about) the flow of ideas in some fundamental areas of contemporary logic. This is particularly true of the contributions by Barwise, Barendregt, Crossley, Nerode, Parikh and Sacks. Much interesting information concerning the emergence and development of whole areas, such as higher recursion theory, or of some trends in proof theory and model theory can be found in these contributions. Behind these personal recollections, the interested reader will find elegant expositions of the history of many important (logical-mathematical-philosophical) concepts.

The part dedicated to Kreisel's mathematics includes three very interesting papers (C.H. Delzell, S. Feferman and H. Luckhardt) whose main topic is Kreisel's *Unwinding Program*. The tone of Feferman's paper ("Kreisel's Unwinding Program") is mainly critical. Feferman presents a thought-provoking analysis of the main proposed successes of the unwinding program (Littlewood's Theorem, Roth's Theorem and van der Waerden's Theorem) and has a clearly negative perspective concerning the nature of the purported applications of proof-theoretical techniques to the unwinding of non-constructive proofs. A more report-like account of Kreisel's results on the extraction of bounds from (*prima facie*) non-constructive proofs is given by H. Luckhardt paper "Bounds Extracted by Kreisel from Ineffective Proofs." The relevant examples reappear (Littlewood's Theorem, Roth's Theorem, L-series, Hilbert's 17th Problem) with some variations of details and additional information. A short and elegant section on *Instructive Examples* briefly describes important features of proof analysis such as, *e.g.*, what sort of parameters one should take into account when submitting a proof to analysis. C.N. Delzell's monograph-paper ("Kreisel's Unwinding of Artin's Proof") is entirely concerned with an analysis of the unwinding of Artin's proof.

Some fundamental ideas, intuitions and results in proof theory in the sixties parade by in W. Howard's short, and sometimes almost cryptic, but exceedingly elegant and interesting paper ("Some Proof Theory in the 1960's"). It contains a clear description of the aims and goals of proof theory in the sixties (the so-called reductive character of proof theory) as well as an informative intuitive account of (Kreisel's ideas on) autonomous progressions (not to mention the interesting suggestions

for further work the attentive reader will find in the paper). Howard's paper serves not only as "an indication of the nature and importance of Kreisel's work and influence in proof theory," but is also one of the best examples in the volume of what I referred to at the beginning of this review, as a "privileged firsthand account of a flow of ideas" (the discussion with Schütte is a high point).

G.K.'s contribution to the problem of intuitionistically acceptable proofs of the completeness theorem for first-order intuitionistic logic is discussed in D. McCarty's paper "Completeness for Intuitionistic Logic." McCarty analyses the main reasons (most of them suggested by Kreisel) for a negative attitude with respect to intuitionistic completeness results. In particular, he examines in some detail the relation between, on the one hand, intuitionistic completeness proofs and, on the other, Church's Thesis, (different versions of) the Markov Principle and Lawless Sequences.

Although related to results obtained by G.K. on continuous functionals of finite types in the early sixties, H. Schwichtenberg's essay ("Density and Choice for Total Continuous Functionals") can be considered as an independent technical tribute. The main purpose of the paper is to present complete proofs of (a) the density theorem and of (b) the choice principle for total continuous functionals (the author says in the introduction of the paper that the main results are located in section 6 when in fact they appear in section 5).

The part devoted to Kreisel's Philosophy is by no means homogeneous. The three papers it contains are very different in nature. C. Cellucci's polemical paper "Mathematical Logic: What Has It Done for the Philosophy of Mathematics?" submits Kreisel's position concerning the nature and aims of Mathematical Logic to an overall criticism. According to Cellucci, we can find strong evidence of Kreisel's commitment to the traditional and, still according to Cellucci, untenable view that the method of mathematics is the axiomatic method and that the main goal of mathematical logic is precisely to study the nature of the axiomatic method.

P. Odifreddi's paper "Kreisel's Church" is a very interesting (Kreisel-style) reading-guide to Kreisel's views on the subject of *Church's Thesis*. As in the case of Howard, Odifreddi also hints at several points to suggestive further work.

The final paper in the philosophical part is P. Weingartner's "Some Critical Remarks on Definitions and on Philosophical and Logical Ideals." The paper could certainly appear as a philosophical tribute to Kreisel. Although it contains several references to G.K., the paper is not exactly a report on (or an exposition of, discussion of, *etc.*)

G.K.'s views. The paper is mainly concerned with two topics, which are, to a certain extent, independent: (1) certain views on the nature of definitions and (2) two traditional attempts to construct universal languages (heroic ideals in Weingartner's own words).

The final part contains two technical tributes, one in model theory (A. Macintyre and A.J. Wilkie) and one in proof theory (G. Mints). In the paper "Normal Forms for Sequent Derivations," Mints takes up the old problem (in proof theory) of uniqueness of normal forms in sequent calculus. It is well known that cut-free forms are not unique in different sequent calculi. The main purpose of the paper is to define a concept of normal form for which uniqueness holds in a sequent calculus formulation of intuitionistic first-order logic. In the final part of the paper, Mints considers an extension of his main result to the fragment of intuitionistic multiplicative linear logic that contains linear implication and tensor product (in this context, Mints refers to an example 2, after theorem 5, but it is not given in the text). One thing is clear: Mints's paper would have benefited from a thorough revision.

Macintyre and Wilkie ("On the Decidability of the Real Exponential Field") take up model-completeness and decidability questions for the theories T_e and T_{exp} of the structures $\langle \bar{R}, e \rangle$ and $\langle \bar{R}, \text{exp} \rangle$, where $\bar{R} = \langle \mathbf{R}, +, \cdot, -, 0, 1, < \rangle$.

Some final remarks:

A volume about and dedicated to G.K. naturally raises some further questions:

- (1) One could always question the scope and structure of the volume: why not introduce a paper on Kreisel and the proof theory of the 70s and 80s? (why not the 90s?). Kreisel's influence on this area is anything but negligible. His discussions with other proof theorists on the nature and aims of proof theory during this period are very important (proof theory versus theory of proofs, identity criteria for proofs, *etc.*).
- (2) The parts devoted to Kreisel's mathematics and to Kreisel's philosophy are unbalanced (at least in number).
- (3) Given that some of the contributions are mainly critical in character, it would be interesting to see how Kreisel would have replied to what was said on what he said and thought about different subjects. This is particularly true of the papers by C. Cellucci and by S. Feferman. Why not add some comments by G.K.?

- (4) Although most of Kreisel's publications appear (scattered in reference lists) in the volume, it would have been appropriate (and very useful) to compile a Kreisel bibliography.
- (5) I cannot end without observing that this volume should have undergone a thorough technical revision. As it is, there are many misprints, and references to sections and examples which do not exist.

We could hardly expect unanimous agreement when the topic is G.K. It is true that very few people in academic circles would justify a volume like this. But Georg Kreisel, both because of his scientific reputation and his idiosyncracies is without doubt one of them. At the same time, it seems also to be true that in many academic circles the relevance of this publication might be questioned. I do not have the authority or competence to judge (as Nerode did), but, in my opinion, the volume is not only a well deserved homage to this complex man of many important and influential ideas and results, but, and perhaps this is even more important, it is also required reading for all those interested in the unwritten history of contemporary ideas, concepts and results in logic, philosophy and mathematics.

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