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Review of
THEODORE HAILPERIN, *SENTENTIAL PROBABILITY LOGIC*

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The present book can be seen both as a sequel and as a companion book to Hailperin former *Boole's Logic and Probability*, but with some substantial enhancements. Nonetheless, before analyzing the book itself it would be useful to make clear in what sense the author uses the term "probability logic", for its use has been somewhat varied and bewildering among the several specialists working in the field. In the present context, probability logic should be understood as a logic of which the semantics is given by probability values (as such, standard two-valued classical logic is a probability logic according to this description).

The first three chapters (and the bulk) of the book is composed of material previously published in *History and Philosophy of Logic* and offer a historical overview of the subject. It starts with a short mention Leibniz's idea of developing a doctrine of degrees of probability for deciding between contrary claims (his main concern was with legal disputes) but as was the case with many of his seminal ideas, this one also was not developed into a coherent theory. For this reason, Hailperin quickly passes to Jakob Bernoulli, perusing his method for providing numerical assignments to the degrees of probability of an argument — at this point, probabilities were not thought of as applied to propositions, but as measures to the likelihood of an argument from its (contingent) premises to its conclusions. There is here a nice presentation of Bernoulli's ideas, particularly of his often difficult to understand combination of pure and mixed arguments for the obtaining of a single conclusion. Next comes, J. H. Lambert, who was one the first authors to consider probabilities in connection to propositions themselves, although Hailperin makes clear that Lambert did so using traditional syllogistic arguments within an intensional context. We are then presented with a brief mention of T. Bayes for his celebrated theorem that contains the first statement of a result in probability logic involving conditional probabilities (likewise an astronomical paper by J. Mitchell deserves a passing mention for its discussion of the relation between the conditional probabilities A given B and not-B given not-A). Bolzano is the next author to be examined and his much-neglected work on probability is finally given here its due value. Bolzano's role in the development of modern semantics has recently been acutely pointed out by J. Alberto Coffa [3], especially his concept of *validity*, which adumbrates Tarski's model-theoretic definition of truth. But his analogous definition of *relative validity* and its use in probability have been almost ignored up to now, and at this point Hailperin gives a careful discussion of Bolzano's several insights on probability (including their pitfalls and shortcomings). He stresses that Bolzano was perhaps the first to note that the probability of a necessary conclusion may not be equal to the product of the probabilities of the premises (a common mistake then).

After that, we come to the authors who have already been analyzed elsewhere by Hailperin as well by other scholars: first, G. Boole with his "interval" probability logic (this material was taken mainly from Hailperin's earlier book cited above). Then follows a very readable (but not novel) discussion of Peirce's ideas on conditional probabilities, including his introduction of a new logical symbol for this very concept. When this historical account enters the twentieth century (certainly the golden century of probability logic) we encounter more standard material: Keynes, Carnap, Nicod and Reichenbach are given the usual expounding of their works, but the interesting point is the mention of Janina Hossiasson's (later Mrs. Adolf Lindenbaum) axiomatization of the notion of "confirmation". Since her contributions to probability are not usually acknowledged in Western references and Hailperin does justice to her work here, it is really a pity that he did not spot her sharp criticisms of some aspects of Keynes' *Treatise on Probability* that appeared in the proceedings of the *First Congress of Mathematicians of Slavic Countries* (the same congress in which Presburger presented his now famous arithmetic; see [3]). Finally this survey ends with probabilistic inference (highlighting works by P. Suppes), the use of linear programming in probability logic (here the authors studied are E. W. Adams, H. P. Levine and N. Nilsson) and with a sentential version of D. Scott and P. Krauss' probability logic. In particular, the works by Adams, Levine and Nilsson are strongly related to some of Hailperin's own original contributions to probability logic, *i.e.* his use of probability intervals to give semantic values for an absolutely free algebra of propositions and his recourse to linear programming for explicitly calculating the bounds for such values. In fact, a very neat summary of such contributions are given in Chapter 4, where the subject is worked in such a way that its reading is not marred by unnecessary technical details. It is fair to say that anyone acquainted with the first chapter of Chang-Keisler's *Model Theory* should have no difficulty in reading through it.

Next, in Chapter 5 we proceed to the construction of a conditional probability logic that is an extension of a logical system called suppositional logic by Hailperin. The latter is obtained in the following way: we introduce a new symbol (a vertical bar "—", not to be confused with Sheffer's stroke) in the underlying language of classical logic and provide this enlarged syntactical set with a particular three-valued semantic. From the point of view of a logician, this is one of the main contributions of this book and Hailperin shows that this suppositional logic can be extended to the above mentioned conditional probability logic in the same way (and with the same methods) that classical logic can be extended to the probability logic of Chapter 4. Furthermore, it can be argued that this strengthening of the suppositional logic provides a formalization of Peirce's conditional probabilities. Finally, in Chapter 6 some technical applications of probability logic to problems of circuit fault testing and network reliability are shown in order to suggest the practical usefulness of the logical systems developed in the previous chapters.

After this summary of Hailperin's book, we now make some observations about its content:

1) At the end of Chapter 3, Hailperin maintains that the Scott-Krauss paper of 1996 contains the earliest instance of a probability logic. We find this claim difficult to accept: as early as 1913 J. Łukasiewicz published a book with a fully formalized probability logic — for this he used an algebra of propositional functions with values in finite models. The resulting probability logic was astonishingly simple and beautiful. In addition, it also presented (as far as the reviewers know) the first occurrence of relevant-type logical implication (thus, three years before C. Lewis; see [4] or the English translation in [5]).
2) In relation to probabilistic inference, we must note that already in 1958 the Polish philosopher K. Ajdukiewicz was developing models of probabilistic inference such that the variables of his functions had probability values which lay in intervals of natural numbers (see [1]).

3) Linear programming is a well-known tool for economists; then, a future and possible application for Hailperin's ideas would be in the domain of decision theory under uncertainty. The important point to stress is that in the standard decision theories the underlying logic is the classical one. In principle, nothing prevents the substitution of Hailperin's probability logic for the classical logic in order to construct a modified and extended decision theory for economics. The greatest advantage of Hailperin's system is that it furnishes an effective means for calculating the bounds for the probability values of a proposition using machinery that is not alien to economic practice. This is not the case with the use of other non-classical logics, which either provide only qualitative estimates for their semantic values or employ technical calculations incomprehensible to most economists.
4) A few corrections on misprints: on page 44, the first row of the matrix A the element 'a_{im}' should be 'a_{1m}'; on page 141, 'Jeffreys' should be replaced by 'Jeffrey'; on page 188, '$P(A_1) = p$' should be '$P(A_2) = p$'. Finally, 'Rudolph Carnap' should be written 'Rudolf Carnap' on page 290 and Łukasiewicz's *Selected Works* [5] should be added to the references.

The above remarks are by no means intended to decrease the overall value of the book under review. On the contrary, we believe that anyone interested in probability logic should read it. Its author, one of the greatest researchers working in this field, produced here both a much-needed historical guide to the subject and an invaluable source of inspiration for future developments in this philosophically ever-fascinating field of logic.

References

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