Modern Logic
Volume 8, Number 3 & 4 (May 2000-October 2001), pp. 31–52.

THE LANGUAGE OF TERNARY DESCRIPTION AND ITS FOUNDER

(To the 70th anniversary of Professor Avenir Uyemov.)

LEONID LEONENKO

1. The man of marked individuality

Avenir Uyemov is, undoubtedly, an outstanding philosopher, who has made his original contribution in different domains of philosophy. His most important achievements are in the field of logic, so his logical system is the main point of the discussion below. However, even those people who could not look into the details of Uyemov's logical works but had a chance to communicate with him or at least watch him, were impressed by his extraordinary personality.

I first saw Avenir Uyemov at a lecture on philosophy. In September 1968 he was the forty-year-old head of the Department of Philosophy at Odessa University, when I was a second-year student in the Faculty of Mathematics. The subject of the lecture was "Dialectical Materialism." We had already studied the beginnings of that subject at school, so our new professor didn't tell us any surprising new facts at this first lecture. But he impressed us by his uncommon appearance and especially by his manner of expounding material. We all felt that this professor was somehow unlike his colleagues lecturing on human sciences and, on the contrary, somehow ressembled our mathematics instructors. Perhaps at that time I did not clarify for myself the essence of those differences and similarities. Now I think that it may be expressed briefly as "professionalism." Both the mathematical disciplines which we studied as well as our mathematics lecturers were supported by the age-long cultural tradition worked out by generations of professionals. This tradition demands a wideness of prospect as well as an accuracy in details. Uyemov represents another tradition, but of the same antiquity and depth, and which requires almost the same demands of its adherents.

I continued my acquaintance with A. Uyemov at the seminars of the so-called “Studio of philosophized mathematicians” which was organized by a group of students in our faculty who felt some kind of “hunger for the humanities”. However, at that time there were many different seminars, study groups and occasional meetings that were grouped around A. Uyemov. It may have been of a scientific or semi-scientific nature, and many students, lecturers and other people of different interests willingly visited it. A variety of problems were discussed there, but because of the mixed character of the audience, one tried to formulate these problems in a form clear to the majority. I do not remember the subjects of most of these topics, but I still remember my impression of professor Uyemov during those discussions. His analysis of a problem was always the most thorough one, and (in spite of his utmost delicate behavior) his intellectual advantages over his opponents and collocutors were obvious.

Now I think that these advantages were also a result of his genuine professionalism. He simply knew far more about things being discussed. Behind him there was a cultural tradition, which his opponents (as well as I together with my companion-listeners) lacked.

Avenir Ivanovich Uyemov was born on April 4, 1928, in the Shuya district of the Ivanovo region in Russia. In 1935, after his parents divorced, he moved to Vladivostok with his mother. Here, at the age of 15, Avenir became a student at The Polytechnic Institute. He was interested in mathematics, but didn’t attend his lectures regularly. His preferable method of studying mathematics was reading books. As time went by, the number of attended lectures was decreasing to zero, but the number of books read was increasing. He read, naturally, not only mathematical books, and one of those — “On Man” by Helvetius — made a great impression on him. Comparing the subject of this book (and other books of the same kind) with the subjects of the institute curriculum, the young student gradually came to the idea that all that he studied was not the matter of the most importance. Undoubtedly, the notion of “integral” possesses both beauty and profit. It is useful to know what “limit” is and how to reach it. Nevertheless, the most important notions in human life are different. It will be more profitable to know what “bliss” is and how can we reach it, but mathematics does not teach these things.

Such and similar ideas finally led Avenir Uyemov to leave the Polytechnic Institute (in his second year of studies). He went to Moscow and after several adventures became a student of the philosophical faculty of Moscow State University (MSU).

He met good companions and good teachers there. Almost all of his colleague-students were former front-line soldiers who had seen a lot and looked into the essence of current affairs. Stalin was not their idol.

As for the teachers, they came (not all, but many of them) from hereditary Russian intelligentsia and maintained traditions with Moscow University. Their demands were to learn philosophy not from current textbooks but from the original works of professionals, such as Aristotle, Hegel, Marx. Comparison of these works with the works of comrade Stalin and other Party comrades led the students to very definite conclusions.

Some of the faculty professors were not only specialists but also persons of encyclopedic brains, belonging to the cultural elite of the country. Professor Valentine Asmus was a close friend of Boris Pasternak and Korney Chukovsky. Professor Pavel Popov wrote the first biography of Mikhail Bulgakov. They were both professors of logic, and perhaps partially under their influence student Uyemov had chosen logic as his specialty.

At that time the students of the philosophical faculty of MSU were taught only the "traditional" branches of logic, the kernel of which was Aristotelian syllogistics. It depended partially on historical and partially on political reasons. Though the classics of Marxism did not leave behind works on formal logic, in Stalin's time a Marxist philosopher was required to be Marxist in everything, *ergo* in formal logic too. Studying logical works of non-Marxist scientists — especially contemporary — may be therefore estimated as a flirtation with the enemy ideology. On the other hand the logicians who taught A. Uyemov themselves belonged to the traditional school of logic, which had arisen in Russia before the revolution and then was "conserved" after the Bolsheviks' victory.

That's why the logical tradition, in which A. Uyemov was reared by his teachers, can be depicted by the names of Aristotle, Leibniz, De Morgan, Mill, Jevons, Minto, Poretsky.

On the other hand, during those years A. Uyemov got acquainted also with the tradition that was begun by Frege, Russell and Hilbert. He studied it while attending lectures of professor Sophia Yanovskaya, who conducted "united" seminars on logic for philosophers and mathematicians. After finishing his post-graduate study at the university (1952), A. Uyemov worked for a long time at the Pedagogical Institute in Ivanovo. At that time the scientific school of mathematical logic, headed by the outstanding mathematician Anatoly Maltsev, was being practiced in that Institute. The participation in school seminars helped A. Uyemov to become familiar with mathematical methods of construction and investigation of symbolic logical systems.

One can characterize the scientific interests of A. Uyemov at the beginning of his career as belonging mainly to "traditional" logic. In his candidate (1952) and doctor (1964) dissertations, as well as in many later works he investigated inferences by analogy. In the seventies he published two large monographs on analogy, in which he summarized his previous work. The investigation scheme in these works may be depicted by the present-day term "practical logic". Formal logic analysis is also very detailed and successful — a number of formal types of analogy are picked out, and for some of these types certain rules of correctness are formulated. But the application of the mathematical apparatus (predicate logic) in the considered works is on the level of descriptions, not inferences.

Significant results were obtained by A. Uyemov in the field of philosophical analysis of the main categories of logic. His great success was a book *Things, Properties and Relations* (1963) [1]. As far as I know, today this book remains the best one on this subject, in spite of the passage of 35 years. The width and the depth of the analysis was combined with the brilliant style of exposition that reminds one of Russell's style in his *Human Knowledge*. But the application of mathematics here is again minimal. It was scarcely the author's intention — more probably the matter had offered a resistance (namely, the resistance to the application of predicate logic). For example, in his book, A. Uyemov had substantiated the so-called "principle of mutual transformations". According to this principle, one and the same object can act in different contexts as either thing, property, or relation. But if we regard a property as a one-place predicate and relation as a many-placed one, then the considered principle leads to the ability of changing the arity of one and the same predicate. If we assume it, we must then make changes to the notion of propositional function.

Speaking figuratively, we can say that the Aristotelian tradition in logic votes for the principle of mutual transformations, while the Frege-Russell tradition votes against that principle. I think that the originality of A. Uyemov's formal logical system (which is stated below) is connected with the verdict he makes in this and the similar cases: the first tradition is right in essence, but the second is right formally. In other words, we need an accurate mathematical apparatus that allows identity of predicates with different arities.

Among the scientific achievements of A. Uyemov one can't omit his contribution to the elaboration of general systems theory. He is the author of one original variant of that theory (based on categories "thing", "property" and "relation"). This variant is called "Parametrical general systems theory" because in it the "system" is treated as an object characterized by special kinds of properties — the so-called "system parameters". Perhaps the main part of A. Uyemov's reputation is connected with his works on systemology, and most of his disciples are also working in that field. I can't present here any detailed characteristics of Uyemov's system concept. One who is interested can take up the basic monograph *System approach and general systems theory* (1978). The last theoretical and practical results in that field were connected with economical and ecological systems' researches conducted while A. Uyemov was the chief of the research group at the Economics Institute of the Ukrainian Academy of Sciences (Odessa, 1974-1996).

A. Uyemov has been living in the Ukraine since 1966, when he left Ivanovo Pedagogical Institute for Odessa State University.

There were (and, I hope, will be) many other significant and interesting events in A. Uyemov's life. He published a dozen scientific monographs (some were republished outside the former Soviet Union), became a professor of renown in the scientific community, and founded his personal scientific school. But his main achievement is the creation of a specific formal logical system and the construction of a mathematical calculus based on its principles. This calculus is called "The Language of Ternary Description" (LTD). The rest of this article is devoted to an exposition of its foundations.

2. The Language of Ternary Description

2.1. Philosophical premises and syntactic conventions. Now one may find different mathematical formulations of the language of ternary description (*e.g.*, Uyemov [12, 26, 47, 49]; Leonenko [29]), but they are all based on one specific system of logical analysis. This system has many features similar to that of Frege-Russell, but differs from the latter in a set of principles. Here I will give short descriptions of the main (but not all) of those principles, not trying — for obvious reasons — to present their complete justifications. At the same time, main syntactical constructions will be described, also with the omission of details.

1. The principle of mutual transformations.
Main predicate formulas of LTD.

Weak form of the principle: One and the same object may act in different contexts as either a thing, a property, or a relation. *Strong* *form*: For any given object it is possible to point out a context where it acts as a thing (and, respectively, a property or a relation).

I shall adduce an example for the weak form of the principle only. In the proposition "A game gives pleasure" the term "a game" denotes a thing. But in "Hockey is a game" the same term denotes a property, and in "A game took place between Russia and Canada" it denotes a relation.

Omitting details, we can say that the weak form of the considered principle is admitted by many philosophers and logicians (it is not so for the strong form, but A. Uyemov admits it too). If we have a purpose to explicate this principle in a formal calculus then such a calculus must allow predicates of predicates (to make properties and relations able to act as things). But more unusual is the requirement to transform a property into a relation and vice versa. It means that a predicate may change its arity.

Let me adduce another example. The card game "Preference" we may consider to be a specific relation between the gamblers. But this game allows three persons to play as well as four. The key question is: do both these variants mean the same game? If we answer "yes" then we will be forced to assume the identity of predicates with different arities (even if we do not recognize the principle of mutual transformations).

Certainly, one can speak not about the identity, but about the equivalence of predicates in different forms (that approach was assumed by Pavel Materna, who had examined the same problem of predicate arity). But A. Uyemov's view is that one and the same predicate may be applied to different number of correlates.

We shall soon speak about the notion introduced in LTD to replace the notion of "n-ary sequence of correlates" and realize the above principle. For the present it is essential to mention that if the identity of predicates with different arities is assumed, then we can't syntactically differentiate property from relation using the distinguishing feature "one correlate / two or more correlates". Instead of the latter, the following positional distinguishing principle is introduced in LTD. If a term denotes a property, then it is placed to the *right* of the parentheses that contain a thing being its correlate. But if a term denotes a relation, then it is placed to the *left* of the parentheses. Thus in each of the following formula schemes

$$(\mathfrak{A})\mathfrak{B}; \qquad \mathfrak{C}(\mathfrak{A}); \qquad \mathfrak{C}([(\mathfrak{A})\mathfrak{B}])$$

$\mathfrak{A}$ denotes a thing, $\mathfrak{B}$ denotes a property, and $\mathfrak{C}$ denotes a relation.

The third of above schemes contains not only parentheses, but also square brackets. Brackets of that kind mark a specific operation of LTD, that corresponds the transformation of a sentence to a noun group in natural languages (so as: "The king is bald" — "The bald king").

Finally, it is necessary to remark that together with formula types $(\mathfrak{A})\mathfrak{B}$ and $\mathfrak{B}(\mathfrak{A})$ "dual" formulas of types $(\mathfrak{A}^*)\mathfrak{B}$ and $\mathfrak{B}({}^*\mathfrak{A})$ are introduced. These formulas express inversions: "Socrates is wise" — $(\mathfrak{A})\mathfrak{B}$; "Wisdom is attributed to Socrates" — $(\mathfrak{A}^*)\mathfrak{B}$; "The wise Socrates" — $[(\mathfrak{A})\mathfrak{B}]$; "The wisdom of Socrates" — $[(\mathfrak{A}^*)\mathfrak{B}]$.

2. "Definiteness", "indefiniteness", "arbitrariness" — three main categories of names in LTD.

There are three primitive names that serve as terms in constructing all other names in LTD. They are marked by symbols t, a and A. The notion being linked with symbol t is "definite object" ("given", "fixed" object); correspondingly symbol a is linked with the notion "indefinite object" ("some object", "something"); and symbol A — "arbitrary object" ("any object", "anything").

Thus it is clear that when comparing LTD with predicate logic one must relate symbols a and A to quantifiers and relate t to individual constants or definite descriptions. There is no novelty in quantifier-free predicate calculus. W. Quine, A. Church, J. Slupecki and other logicians have built a number of systems that do not take quantifiers as primary constructions. But motivations for quantifier elimination and, certainly, technical facility of its use in those systems and in LTD are very different.

As a "first approximation" we may regard quantifiers to be substituted in the LTD by the subdivision of language terms in categories of definiteness. One can watch something similar in natural languages, where the sense of a clause like "Man is trustful" becomes quite clear only after clause terms ("Man" in particular) are characterized by one of three categories: "any", "some" or "concrete, definite". Such a characterization is realized in many languages by means of articles. In a number of contemporary researches, logicians and linguists (Z. Vendler, S. Kuno, E. Barth and others) have shown that articles very frequently act just as operators that relate language names to the three categories mentioned above.

Consideration of the analogy to articles leads to the idea of introducing three formal operators that can be applied to names of some symbolic language. In the case where such a language does not include constants (*i.e.*, proper names), there arises a possibility of introducing three different *types of names* instead of operators. That is the situation that takes place in LTD.

Using names t, a and A of LTD, we may express the sentence "Something possesses property P" by the formula scheme $(a)P$. Analogously, the sentence "Everything possesses property P" is expressed as $(A)P$, and the sentence "The given object has property P" — as $(t)P$. If we substitute symbol P in these schemes by some formula of the LTD then we get LTD-explications of the three considered types of clauses. For example, substitution of P for t gives explications of sentences:

"Some object possesses the fixed property": $(a)t$
"Any object possesses the fixed property": $(A)t$
"The fixed object possesses the fixed property": $(t)t$

If we substitute for P the formula $[(t*)A]$, that denotes, as it was explained above, "any property of the fixed object", then we get respectively:

"Some object possesses any property of the fixed object": $(a)[(t*)A]$

and so on.

An appropriate "substitution rule for the arbitrary object A" is introduced in the LTD. Using that rule, it can be proved that if $\mathfrak{A}_A$ is an arbitrary formula with an occurrence of the symbol A (this occurrence must fulfill certain conditions), $\mathfrak{B}$ is also an arbitrary formula, and $\mathfrak{A}_{\mathfrak{B}}$ is the result of substituting the mentioned occurrence of A in $\mathfrak{A}_A$ with $\mathfrak{B}$, then the following inferences are valid:

$$\mathfrak{A}_A \vdash \mathfrak{A}_{\mathfrak{B}} \qquad \text{and} \qquad \mathfrak{A}_{\mathfrak{B}} \vdash \mathfrak{A}_a$$

It is an obvious analogy for the corresponding relations of quantifiers.

Perhaps, the most similar to LTD in what concerns the explication of notions "arbitrary" and "something" is Hilbert's and Bernays' ε-calculus. The indefinite description $\varepsilon_x\mathfrak{A}(x)$ of that calculus reflects the notion "some individual x that possesses a property $\mathfrak{A}$". It makes it possible to associate the term $\varepsilon_x\mathfrak{A}(x)$ and the term $[(a)\mathfrak{A}]$ of LTD. Like in LTD, each formula in the ε-calculus has subject and predicate parts in its structure, since quantifier expressions $\exists x\mathfrak{A}(x)$ and $\forall x\mathfrak{A}(x)$ are substituted respectively for $\mathfrak{A}(\varepsilon_x\mathfrak{A}(x))$ and $\mathfrak{A}(\varepsilon_x\neg\mathfrak{A}(x))$. Nevertheless the differences between the two considered systems are more essential than their similarity. Not to mention that there are not any "arbitrary individuals" in the ε-calculus: even the construction of an "indefinite individual" $\varepsilon_x\mathfrak{A}(x)$ presupposes the fixed set of predicate constants of the language (this set is a value domain for symbol $\mathfrak{A}$). But constants of that kind are *not present* in LTD. Moreover, the "indefiniteness" of the description $\varepsilon_x\mathfrak{A}(x)$ consists in the fact that there is no explicit assignment of any individual constant to this term. At the same time, the term $\varepsilon_x\mathfrak{A}(x)$ denotes *one and the same object* in any of its occurrences in any formula. The last is not true for the term a in LTD (we shall discuss the details below). And finally, the elimination of quantifiers in the ε-calculus is realized by means of binding the variable x in $\mathfrak{A}(x)$ by the ε-operator. An analogous operation is impracticable in LTD, because there are *no variables* (in the usual sense of that notion) in it. In spite of the fact that the relation between terms a and t reminds one of that between a variable and a constant, neither a nor t possesses the characteristic feature of a variable, *i.e.*, its ability to take values.

By the way, the applicability of the last feature to quantified variables of predicate logic depends on the chosen interpretation of quantifiers (and in most cases it also doesn't take place, though in others — like in Hintikka's game interpretation — it does). The conception used in LTD can be simplified as follows. If a variable is bound by a quantifier, then objects that form the value domain of the variable are somehow "united" into a new object — a or A — and a predicate in quantifier scope is applied to this new object. L. Sumarokova had substantiated this conception in Sumarokova [3]. Her analysis is based on watching various rôles of quantifier words in natural languages.

3. The identity operator. Specific character of the "object domain" in LTD.

I have pointed out that names a and A mark in LTD the *types* of objects' indefiniteness. Therefore, for each of those names, its different occurrences in any formula may denote *different* elements of the language's "object domain" — when these elements have the same type of indefiniteness. The same is true for non-elementary terms of the language. In other words, the [usual for most of the formal languages (but not for natural ones)] identification principle, "any occurrence of the same sub-formula in a given formula denotes the same object", is not adopted in LTD.

To express the identity of objects being denoted by given occurrences of formulas $\mathfrak{B}$ and $\mathfrak{C}$ in some formula $\mathfrak{A}$, one has to use in LTD the special functor which is called *$\imath$-operator*: the mentioned occurrences of $\mathfrak{B}$ and $\mathfrak{C}$ are prefixed in $\mathfrak{A}$ by the same symbol $\imath$. For instance, the formula $(\imath a*)\imath a$ means that some object is attributed as a property to itself. Such an explication of identity is analogous to its being expressed in natural languages by pronouns. To fix the identity in more than one group of objects, one can subscribe indices to the symbol $\imath$ or duplicate it (if the latter way is chosen, then combinations like $\imath\imath$, $\imath\imath\imath$, *etc.*, are treated not as a repeated $\imath$-operator, but as indivisible symbols).

The explication of identity inside any given logical system is strongly related to the conception of object domain — “universe of discourse” — presupposed by that system (such a relation had been examined for predicate logic in works of Hilbert and Bernays, Kleene, Hintikka.) Let **U** designate the universe of discourse of LTD. The main distinctive feature of **U** (when comparing it with the domain of individuals in predicate logic) is that elements of **U** are treated as “qualitatively interpreted things”, in the sense that is substantiated in A. Uyemov’s book *Things, properties and relations*. Such a treatment implies that:

a) each object of the domain **U** is characterized by one of the types of indefiniteness: t, a or A.
b) one and the same object of **U** may act as either a thing or a property or a relation in different contexts (the principle of mutual transformations).
c) the identity of elements of **U** is treated as congruence of its “essential” attributes.

The last item c) reflects the adoption of the identity principle different from what is usually called “Leibniz’s principle”. It can be explicitly stated as follows:

Two things x and y can be identical
while some of their properties are different.

I shall call this “the principle of Aristotle”. A. Uyemov is being quite consistent in adopting it just because he assumes the identity of relations with a different number of correlates. But the latter is not the main reason. The partition of an object’s attributes into essential ones and nonessential ones (coming from the traditional logic) is a part of the foundations of the whole of A. Uyemov’s system. To determine essential attributes, one must examine whether the given object becomes not identical to itself when it loses these attributes.

There are well known critical arguments against the assuming of the above partition (one can recall a number of critics of “essentialism” from Locke to Russell and Quine.) But these arguments may be taken into account using one important addendum to the principle of Aristotle: *an attribute’s essentiality is not absolute* but can arise and vanish when the context of an object’s inspection is changed. Thus objects identical in one relation may be not identical in the other. For example, under different circumstances we may consider different copies or issues of the same book as the same or not the same object.

So we can state that the concept of identity in the language of ternary description differs from the classical one. It resembles the identity that is expressed in predicate logic not by the binary functor " $=$ ", but by the uniformity of terms' occurrences. In fact, it is rather weaker than the last. Using usual mathematical terminology it is perhaps more relevant to speak about objects of the domain **U** not being "identical" but "equivalent in some sense." According to different senses of equivalence there appears a partition of the universe of discourse **U**, which consists of several classes of equivalence. Each of these classes is represented by the same $\imath$-operator that prefixed a given language term if it denotes an element of the class. Thus one $\imath$-operator may be used to mark a term that denotes one and the same *book* ; another, to mark the same *issue* of that book; and the third, to mark a given *copy* of that issue.

In his last works, A. Uyemov has made an attempt to express stronger conditions of object identity, which reflect Leibniz's principle (though the principle of Aristotle is still assumed in these works). Omitting details, it may be noted that the basis for such an attempt is the absence of notions other than "thing", "property" and "arbitrariness" in the formulation of Leibniz's identity principle (see Uyemov [49, 53]).

4. Operators for individual and propositional negation.

Any logic system that tries to express the difference between "essential" and "contingent" clashes with the necessity of passing over the law of contradiction. Conjunctions of sentences that describe different contexts where one and the same attribute acts as either essential or contingent may violate this law. There are different ways of overcoming that difficulty — *e.g.*, in the semantics for modal logic using the notion of possible world, the law of contradiction is valid in a fixed world but may be not valid when passing from one world to another. In paraconsistent logic one may find special forms of negation that do not satisfy the law of contradiction.

As for LTD, different modes of introducing negation may be observed in different formulations of the language. In Uyemov [12] a special operation of "individuals' distinction" was introduced. It allows the construction of various things to be different from the given object. Using that operation it is possible to express propositional negation, and for the latter, the law of contradiction will be valid only under certain conditions. The statement "It is false that object $\mathfrak{A}$ possesses property $\mathfrak{B}$" is considered as equivalent to "Any property of object $\mathfrak{A}$ differs from $\mathfrak{B}$". Let / be a sign for "individuals' distinction" functor, then we may express the last statement as $[(\mathfrak{A}*)A] \;/\; \mathfrak{B}$.

On the other hand, in his latest works A. Uyemov tried to express various types of negation in his system without any additional functors. He based this upon the general assumption that notions "true" and "false" can be sufficiently explicated inside the system of concepts "thing", "property", "relation", "definite", "indefinite" and "arbitrary" — that is the basic concept system of LTD. His arguments for the possibility of avoiding additional concepts are substantiated in Uyemov [42, 49].

5. Other operations and relations in the LTD.

I shall describe these operations with maximum brevity. First, let me note that in the latest of A. Uyemov's papers, the system of LTD operations is regularly constructed using a set of principles that we can't concern ourselves with here. Nevertheless, there are a few most important operations in all formulations of LTD, and I will discuss them below.

5.1. The relation of "attributive implication": $\Rightarrow$.

This is a relation between two individuals (*i.e.*, two things). It expresses one of the meanings of the connective "is" in natural language. (Uyemov's interpretation of that meaning has varied over time. Therefore he may object to some details of the interpretation given below. Here, as well as in other cases, I present examples that reflect not only Uyemov's, but also my own understanding of LTD concepts.)

It is assumed that in the following sentences

(1) Socrates is a man (2) Courage is a virtue

"is" denotes attributive implications; while in

(3) Socrates is courageous (4) Socrates is virtuous

"is" denotes some other relation (namely, "pure" predication). Attributive implication may be considered to be a special type of predication. In fact, "a man" in (1) denotes a property as well as "courageous" in (3). Both of these properties are attributed to Socrates. But the relation (1) of Socrates to a man does not reduce to predication, unlike his relation (3) to courage.

Attributive implications in (1) and (2) possess the following peculiarities that distinguish them from predication in (3) and (4):

a) Every reference to Socrates is (or "means", or "implies") a reference to a man; every reference to courage is a reference to a virtue. In that sense the relation denoted by "is" in (1) and (2) reminds one of plain implication.

b) As a *concrete example* of a man one may take Socrates; as a *concrete* virtue one can point to courage (but not Socrates, in spite of the fact he is virtuous). That's why in some of his works A. Uyemov had used the term "concrete implication" instead of "attributive".

Formal syntax properties of the functor ⇒ reflect the above peculiarities a) and b). Namely, *modus ponens* for ⇒ is adopted in accordance with a), and the so-called "axioms of restriction" correspond to b).

The adoption of *modus ponens* for the functor ⇒ which can connect *non-propositional* terms of the language means that in LTD one may conduct inferences based on notions (individuals) only. It makes LTD resemble nominalistic logic systems (other tools for inferences based on individuals are also present in LTD).

In Leonenko [29] some relations of A. Uyemov's interpretation of the connective "is" to other interpretations that took place in the history of logic (Leibniz, Hobbes, Jevons, Hoffding, Frege, Russell, Kotarbinski, Lesniewski, Hintikka) were considered.

5.2. "Mereological implication" ⊃.

It is a functor included in LTD in order to express the relation "the whole — its part". The underlying philosophical interpretation of that relation presupposes that its partial examples may be simultaneously such kinds of relations as "the set — its element", "the set — its subset", "the object — its attribute", "the sum — its summand", "the system — its subsystem", *etc.*

Nevertheless, this does not mean that A. Uyemov assumes the requirement (being typical to nominalistic conceptions represented by Lesniewski, Slupecki, Goodman, Quine) that the relation "the whole — its part" should possess common, generic properties of all partial relations mentioned above. There are well-known difficulties and "oddities" of formal explications which take "the whole — its part" as a generic relation of "the set — its element" and "the set — its subset". (For example, this generic approach in Lesniewski's mereology forces one to regard two elements as always forming a new one; thus the existence of a whole that contains a number of parts other than one of the numbers in the sequence $2^2 - 1, 2^3 - 1, ..., 2^n - 1, ...$ is impossible).

But another approach can be used. In all of the above examples of partial relations, the second correlate is — by intuition —- a part of the first. Nevertheless, it shouldn't mean that every property of the relation "the whole — its part" must characterize also each partial relation in the above group. It will be sufficient to recognize only that any relation in the group somehow *implies* the existence of the relation "whole — part" between its correlates. Thus "the whole — its part" may be considered as "more fundamental" than any relation in the referred group *not* for the reason that the latter is a *kind* of the first, but because the latter implies (and in this sense includes or presupposes) it. In Leonenko [29] examples and comparisons of this approach with nominalistic interpretations of "the whole — its part" were discussed.

Among formal properties of the functor $\supset$, let me mention its transitivity. It should be pointed out that transitivity is not necessary for every "partial" relation of the group considered above. For example, if x is an element of y, and y is an element of z, then x is a part of y, and y is a part of z. Therefore (due to transitivity of $\supset$) x will be *a part* of z, but it may *not be an element* of z in the same case.

The next formal property of $\supset$ is its implicative character: if x is a part of y then any reference to x "means" or "implies" (maybe only tacitly) a reference to y. Therefore *modus ponens* is assumed for the mereological implication $\supset$.

5.3. More other functors of LTD.

It will be not surprising for the reader to see usual propositional connectives among those "more other" functors. In fact, these connectives are present in some formulations of LTD (see Leonenko [29, 37]), though negation — as it had been emphasized above — is expressed through a specific operation of individuals' distinction.

But one may find it very bizarre that in his own formulations of LTD A. Uyemov has *not* introduced any classical propositional connectives. His arguments on behalf of this approach may be simplified by the following. Both propositions and notions can be regarded as "objects", *i.e.*, things. Therefore operations applied to both first and second may be interpreted as operations over the domain of things. The intersection $A \cdot B$ of notions is usually expressed by the conjunction $a \& b$ of propositions; but we may treat both first and second as partial cases of some "synthesis of objects" — $\mathfrak{A} \times \mathfrak{B}$. The characteristic feature of that synthesis is "the presence of object $\mathfrak{A}$ as well as $\mathfrak{B}$ under the condition of the presence of object $\mathfrak{A} \times \mathfrak{B}$". Analogously the implication "if a then b" and two relations of individuals A and B: "A is B" and "A contains B as its part" — may all be treated as three partial cases of the relation $\mathfrak{A} \rightarrow \mathfrak{B}$ of things $\mathfrak{A}$ and $\mathfrak{B}$ (things of arbitrary category type) that means "$\mathfrak{B}$ is present in any case when $\mathfrak{A}$ is present".

Different senses of the above "presence" of objects $\mathfrak{A}$ and $\mathfrak{B}$ can be explicated inside LTD by means of some system of postulates that regularize usage of connectives $\times$ and $\rightarrow$. And then one can obtain the "propositional part" of LTD (that corresponds to a fragment of classical calculus determined by conjunction and implication) by considering all formulas of forms $\mathfrak{A} \times \mathfrak{B}$ and $\mathfrak{A} \rightarrow \mathfrak{B}$ and distinguishing among them those where $\mathfrak{A}$ and $\mathfrak{B}$ denote propositions.

Such a program of "deriving" a propositional part of LTD seems interesting enough, though its realizations in some publications are, as I think, not free from slips. I will not analyze this program, but only say that if one rejects it then it is still possible to express in LTD specific properties of the implication and other functors in the cases when these functors are applied not to individuals but to propositions. In other words, using certain precautions we can include fragments of the "usual" propositional calculus in LTD.

2.2. **Formulations and properties of LTD calculi.** A. Uyemov's first publication that presented some basic principles and a fragment of the formal calculus of LTD appeared in 1968 (Uyemov [4]). The evolution of ideas and formalism was reflected in some following articles (Uyemov [8, 12, 13, 16, 17, 25]). The formulation presented in Uyemov [26] may be regarded as a rather complete description of the conception that had been developed previously. The characteristics of LTD that I have discussed above reflect mainly that conception (though many important moments remained behind this discussion).

From the middle of the 1980's A. Uyemov was working on the problem of the expressibility of LTD's operations through basic categories "thing", "property", "relation", "definiteness", "indefiniteness", "arbitrariness". His results were summed up in a series of articles starting with Uyemov [47, 49]. The next article of this series will contain a new formulation of the calculus.

Formal logical properties of LTD were analyzed in Leonenko [29, 33, 34, 37]. Because of the absence of propositional negation one should examine not simple consistency of LTD but some other forms of its consistency. In particular, for the LTD calculus examined in Leonenko [29, 37], I proved that it is absolutely consistent (*i.e.*, it includes some unprovable formulas).

The possibility of inferences that are based on non-propositional formulas implies some specific properties of LTD calculi. In particular, it was shown that the adoption of *modus ponens* rules for the attributive and mereological implications implies the falsity of the deduction theorem (but only for those inferences where the indicated rules are used).

As for imperfections, a considerable awkwardness of all existing formulations of the language may be pointed out. The problem of constructing a sequence of calculi, each "embedded" in the other, and also problems of axioms' independence and operations' separability are not solved. The question of a formal semantics for LTD is still open.

2.3. **Applications of LTD.** Main efforts in the field of LTD applications were directed at the explication of basic concepts of parametrical general system theory. In Uyemov [13] the meanings of different system parameters were expressed. Formal definitions for general notions of "system parameter" and "system regularity" were given in Leonenko and Sarayeva [28]. I showed in Leonenko [29] how to construct an applied calculus of LTD in order to make provable some basic assertions of parametrical system theory (in particular, the principle which states that any object can be represented as a system). In the bibliography at the end of this article I cite some publications where LTD was applied to economic, ecological and geo-mechanical systems' modeling.

It should be mentioned also that the language of ternary description has interesting applications to some philosophical problems. For example, it was used to clarify the problem of formal expression of ontological assertions; in formal explications of explanation and understanding processes; in sciences' and researches' classification (see bibliography below).

I think that perhaps more interesting may be the future applications of LTD in the field of philosophical logic. Having no possibility to discuss it in detail, let me note that, as I believe, LTD can successfully advance the following problems that have been already set up:

— formal explication of the concept of a proposition's "theme" (that was suggested for consideration by Strawson);

— clarifying of the "improper" syllogistic inferences based on specific interpretations of the connector "is" (previously examined by Hintikka in the framework of game semantics);

— explanation of the failure of the law of the excluded middle for certain assertions of abstract objects (J. Slupecki had investigated that problem by application of some nominalistic ideas).

3. The conclusion.

It may be stated that logical problems that served as a source for the Language of Ternary Description are still the subject of intensive research in contemporary logic. Problems such as: the approximation of formal inferences' structure to the one that takes place in natural language; including into the scope of logic some "calculi of individuals" that reflect non-propositional operations applied to objects of reasoning; explication of the concept of "essential" attribute, *etc.*, — had caused the rise of various specific logical calculuses. The same or similar problems induced A. Uyemov to apply in the LTD his own original concepts.

In addition, A. Uyemov aspires to solve with the help of LTD some problems that belong to the scope of general systems theory (especially to his own "parametrical" kind of it). The results obtained in that field may be an object of attention for logicians if they take an interest in general systems theory.

But independently of that, the system of logic analysis being a foundation of LTD seems to be very original and interesting. As for me, I consider my acquaintance with A. Uyemov and his ideas to be a very felicitous fortune. These ideas are a significant contribution to logic.

Main publications on the LTD.

Below I cite only the most important works that concern the essential problems of LTD and its applications. A lot of articles that consider philosophical problems and especially concepts of parametrical general system theory remain beyond the bibliography given below. (The total number of publications written by A. Uyemov personally, is over 300).

I compile the following references indicating for each the language of publication and its general area of discourse. The ordering of the references is by year of publication.

References

[1] A. Uyemov, *Things, Properties and Relations*, Moscow: USSR Academy of Sciences, 1963.
Russian. Philosophical foundations of LTD

[2] A. Uyemov, *Dinge, Eigenschaften und Relationen*, Berlin, Akademie-Verlag, 1965.
German. Reprint of [1]

[3] L. Sumarokova, *Logical Problems of Linguistic Systems' Simplicity*, Dissertation for Ph.D., Odessa, Odessa State University, 1966.
Russian. Certain philosophical-linguistic prerequisites of LTD

[4] A. Uyemov, "Concerning a Variant of the Logic-Mathematical Apparatus for Systems Research," in *The problems of Formal Analysis of Systems*, Moscow, 1968, p.35-69.
Russian. First variant of LTD calculus

[5] P. Kovaljev, "Concerning Problem of the Axiomatic Construction of the System Description Language," in *Philosophic Problems of the Modern Natural Science*, Issue 27. Kiev, 1972, p.128-131.
Ukrainian. Mathematical aspects of LTD

[6] A. Tsofnas and A. Uyemov, “Formal Expressions of the Ontological Statements,” in *Philosophical sciences*, 1973, No. 3, p.51-59.
Russian. Application of the LTD to some philosophical problems

[7] P. Valentchyk and Uyemov, “Formal Typology of the Scientific Knowledge and the Problem of its Unity,” in *Philosophy and Natural Science*, Moscow, Nauka, 1974, p.133-151.
Russian. Applications of LTD

[8] A. Uyemov, “Logical Apparatus of the System Research,” in *The 5th Intern. Congress of LMPhS. Contributed Papers*, London, Ontario, Canada, 1975, p.129-130.
English. Abstract of the Language of Ternary Description

[9] E. Prichodko, I. Sarayeva, and A. Uyemov, “Problems of Identificators in the Structure of the Formal Language of the Systems Description,” in *The Methodological Problems of the Language Analysis*, Erevan, 1976, p.230-244.
Russian. Formal calculus concepts

[10] A. Uyemov, “Theoretic-System Approach to the Formalization Problem of Basic Features of the World Ocean as Ecosystem,” in *The Problems of Sea Economics*, Odessa, 1977. Issue No. 6., p.34-40.
Russian. Application of LTD to some economical and ecological problems

[11] A. Uyemov, “An Application of the Language of Ternary Description to the Logic of Estimations Construction,” in *Filozofia*, XX, Szeged, 1977, p.21-32.
Hungarian. Applications of LTD in the scope of ethics

[12] A. Uyemov, “Formal Aspects of Systematization of Scientific Knowledge and Procedures of its Development,” in *Systems Analysis and Scientific Knowledge*, Moscow: Nauka, 1978, p.95-141.
Russian. New formulation of a calculus and its applications

[13] A. Uyemov, *System Approach and the General Systems Theory*, Moscow: Musl, 1978.
Russian. The foundations of LTD, its basic concepts and applications to parametric general systems theory

[14] A. Uyemov, “Kinds of Uncertainty and the Problem of the Foundation of the Inductive Inference Validity,” in *Philosophical Problems of the Modern Natural Science*, Kiev, 1978, No.44, p.79-84.
Russian. Applications of LTD in the scope of logic

[15] I. Sarayeva and A. Kozdoba, “On the Method of System Parameters’ Values Construction in Terms of the Language of Ternary Description,” in *System Method and Modern Science*, Novosibirsk, NSU, 1979, p.50-60.
Russian. Application of LTD to parametrical general systems theory

[16] A. Uyemov, “The Language of the Ternary Description as an Alternative of the Predicate Calculus,” in *The 6th Intern. Congress of LMPhS*, Hannover, 1979, Section 5, p.194-198.
English. Abstract of LTD’s version previously proposed in [12] and in [13]

[17] A. Uyemov, “Concerning the Logic-Mathematical Apparatus of the General Systems Theory,” in *Urgent Problems of Logic and Methodology of Science*, Kiev, Naukova dumka, 1979, p.299-309.
Russian. Description of a calculus proposed in [12]

[18] N. Savusin N., “System-Parametric Model for the ‘Expenditure - Output’ Leontyev’s System,” in *System Method and Modern Science*, Novosibirsk, NSU,

1980, p.94-105.
Russian. Applications of LTD

[19] L. Terentjeva, A. Uyemov and A. Tsofnas, "An Understanding and Explanation as Scientific Procedures and their Formalization in the Language of Ternary Description," in *System Method and Modern Science*, Issue 6, Novosibirsk, NSU, 1980, p.45-51.
Russian. Philosophical applications of the LTD

[20] L. Leonenko and G. Shtakser, "The Operation of Distinction in the Language of System Description," in *System Approach and Modern Science*, Novosibirsk, NSU, 1981, p.80-91.
Russian. Formal mathematical properties of LTD

[21] I. Sarayeva and Uyemov, "About Algorithmic Approach to the Problem of Exposing some System Regularities," in *System Approach and Modern Science*, Novosibirsk, NSU, 1981, p.92-103.
Russian. Application of LTD to parametrical general systems theory

[22] A. Uyemov, V. Barabash and E. Kovriga, "Classification and Coding of the Scientific Researches of the World Ocean," in *Econology of the World Ocean*, Kiev, 1981, p.43-51.
Russian. Applications of LTD

[23] A. Uyemov and A. Tsofnas, "Concerning Problem of the Explication of the Structure of Understanding and Explanation in the Framework of the United Language," in *Philosophical Sciences*, 1981, No.2, p.142-144.
Russian. Philosophical applications of the LTD

[24] A. Uyemov, "Methodological Foundations of the Formalization of the Conceptual Simplicity-Complexity of Systems in the Language of Ternary Description," in *System approach in Modern Science*, Novosibirsk, NSU, 1982, p.57-64.
Russian. Applications of LTD

[25] A. Uyemov, "Self-Applicability as the property of the Formal Language of the Parametrical General Systems Theory," in *Logic, Methodology and Philosophy of Science. VII Intern. Congress (Salzburg)*, Moscow, 1983. Abstracts Sections 6, 8-13. p.68-70.
English. Abstract of some logical properties of the LTD

[26] A. Uyemov, "A Formalism for Parametrical General Systems Theory," in *System Researches. Methodological Problems*, Yearbook 1984, Moscow, Nauka, 1984, p.152-180.
Russian. Detailed formulation of a new variant of the calculus.

[27] A. Uyemov and L. Leonenko, "On the S-representability of formulas in the Language of Ternary Description," in *The 7th All-Union Conference on the Mathematical Logic*, Novosibirsk, 1984, p.182.
Russian. Formal mathematical properties of the LTD that reflect certain concepts of general systems theory

[28] L. Leonenko and I. Sarayeva, "On the Application of the Language of Ternary Description to the Modeling of System Parameters' Values and to the Detection of System Regularities," in *System Researches. Methodological Problems*, Yearbook 1984, Moscow, Nauka, 1984, p.181-193.
Russian. Application of LTD calculus formulated in [Uyemov,1984] to parametrical general systems theory

[29] L. Leonenko, *Logic-Philosophical Analysis of System Representation of Objects (Calculuses of the Language of Ternary Description*, Dissertation for Ph.D. Kiev, Institute of Philosophy, 1985.
Russian. Philosophical foundations, formal logical properties and applications of LTD

[30] I. Sarayeva, "About the Types of Proofs in the Language of Ternary Description," in *Problems of System Analysis*, Novosibirsk, NSU, 1985, p.82-89.
Russian. Philosophical and logical premises of the LTD

[31] N. Savusin, "The "System" Definition Formalization using a new version of the Language of Ternary Description," in *Problems of System Analysis*, Novosibirsk, NSU, 1985, p.89-95.
Russian. Application of LTD to parametrical general systems theory

[32] A. Uyemov, "Formalization of the Elementary Modes of the Cognitive Activity in the Language of Ternary Description," in *System-Cybernetic Aspects of Knowledge*, Riga, Zinatne, 1985, p.41-80.
Russian. Attempt to express a set of LTD operations through six basic categories

[33] L. Leonenko, "Rules of 'Modus ponens'-type and the Deduction Theorem in the Language of Ternary Description Calculuses," in *The 8th All-Union Conf. on the Math. Logic*, Moscow, 1986, p.102.
Russian. Formal logical properties of the LTD

[34] L. Leonenko, "Quantification in the Language of Ternary Description," in *Logic and system methods of scientific knowledge analysis*, Abstracts to 9-th All-Union Conf. on LMPhS. Moscow, 1986, p.139-141.
Russian. Formal logical properties of the LTD

[35] D. Glazov and S. Andronov, "Prerequisites of Artificial Intelligence for Geomechanical and Technological Systems," in *Soviet Mining Journal*, Oxford & IBH Publ.Co., Rotterdam, Vol.1, No.3, September 1987, p.12-21.
English. Applications of LTD

[36] D. Glazov and S. Andronov, "Formal Achieving of New Technological Solutions on the Basis of Knowledge-base in the Language of Ternary Structure," in *Soviet Mining Journal*, Oxford & IBH Publ.Co., Rotterdam, Vol.1, No.3, September 1987, p.144-155.
English. Applications of LTD

[37] L. Leonenko, "The Logical Properties of some Calculuses of the Language of Ternary Description," in *The 8th Congress of LMPhS*, Abstracts. Moscow, 1987. Vol. 1, p.278-280.
English. Formal properties of LTD calculuses

[38] A. Uyemov, "Fundamental Features of Language of the Ternary Description as a Logical Formalism in the Systems Analysis," in *The 8th Congress of LMPhS*, Moscow, 1987. Abstracts.Vol. 5. Part 1. Section II. p.337-339.
English. Abstract of LTD basic concepts

[39] I. Sarayeva, *The Logic-Methodological Analysis of the Problem of General Systems Laws establishing with the help of Deductive Methods*, Dissertation for Ph.D. Kiev, Institute of Philosophy, 1988.
Russian. Philosophical and logical premises, methods of reasoning and applications of LTD

[40] A. Uyemov, Y. Veselov, V. Glushkov et al., *Complex Goal Programs of the Economic Exploitation of the World Ocean's Researches*, Kiev, Naukova dumka, 1988.
Russian. Applications of LTD

[41] A. Uyemov, "Formalization of Semi-defined System Models in the Language of Ternary Description," in *Prakseologia*, 1989, No.4, p.165-174.
Polish. Applications of LTD

[42] A. Uyemov, "Analysis of Operations as Means of the Study of the Systems Dynamics," in *Philosophy: Questions of the Methodology and Logic.* Transactions of Latvian University, Issue 551, Riga, 1990, p.143-170.
Russian. Construction the set of LTD operations through six categories: thing, property, relation, definiteness, indefinite-ness, arbitrariness

[43] D. Glazov and A. Orishin, *The Flexible Technology for the Complex-mechanized Coal's Excavation*, Moscow, Nedra, 1992.
Russian. Applications of LTD

[44] A. Uyemov and G. Shtakser, "The Aristotle's Types of Essences and Fregean Discerning between Sense and Meaning," in *Frege's and Hilbert's Heritage in the XXth Century. Logic, Philosophy and Mathematics: Abstracts of All-Russian Conference*, Kaliningrad, 1992, p.22.
English. Philosophical foundations of the LTD

[45] A. Uyemov and I. Sarayeva, *The Method of Analogies as the base of Forecasting of the Social-Economic Systems Development*, Odessa, 1992.
Russian. Application of LTD to the analysis of the structure of conclusions by analogy

[46] A. Uyemov, "Parametric General Systems Theory and the Estimation of Proposals for Mankind's Survival," in *Proceedings of the 38 annual meeting of ISSS. Pacific Grove, California, 1994*, Vol.1, p.247-259.
English. Application of LTD to social-economic development processes

[47] A. Uyemov, "The Language of Ternary Description as a Deviant Logic," in *Boletim da Sociedade Paranaense de Matematica*, 1995, Vol.15, No.1-2, p.25-35.
English. First part of new language formulation, expounds philosophical and logical premises

[48] I. Sarayeva and A. Uyemov, "On the Problem of Interrelation and Integration of Systems Theories," in *System Researches. Methodological Problems*, Yearbook 1992-1994, Moscow, Editorial URSS, 1996, p.79-96.
Russian. Applications of LTD

[49] A. Uyemov, "The Language of Ternary Description as a Deviant Logic, II," in *Boletim da Sociedade Paranaense de Matematica*, 1997, Vol.17, No.1-2, p.71-81.
English. 2nd part of the work started in [47], expounds the notions of identity, implication and valent values

[50] A. Uyemov, *Foundations of Practical Logic with Problems and Exercises*, Odessa, OSU, 1997.
Russian. This text-book for students contains description of the LTD and exercises on it

[51] A. Uyemov, "System Approach to the Problem of the Classification of Sciences and Scientific Researches," in *Issues and Images in the Philosophy of Science*,

Kluwer Acad.Publ., 1997, p.377-390.
English. Application of not namely LTD but its basic concepts to sciences' classification

[52] A. Uyemov, "The Lvov-Warsaw School and the Problem of a Logical Formalism for General Systems Theory," in *The Lvov-Warsaw School and Contemporary Philosophy*, Kluwer Acad. Publ., 1998, p.355-363.
English. The philosophical foundations of LTD

[53] A. Uyemov, "The Ternary Description Language as a Formalism for the Parametrical General Systems Theory: Part 1," in *Int. J. General Systems*, Vol.28(4-5), p.351-366.
English. Main ideas for the applying of LTD to Parametrical Systems Theory

[54] A. Tsofnas, *Systems Theory and Theory of Knowledge*, Odessa, AstroPrint, 1999.
Russian. Applications of LTD to various epistemological problems

Kinna Street, 22, apt. 6, Odessa, Ukraine, 65029
E-mail address: `domos@eurocom.od.ua, domos@te.net.ua`