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Corrigenda: On the product theory of singular integrals

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We wish to acknowledge and correct an error[1] in a proof in our paper *On the product theory of singular integrals*, which appeared in Revista Matemática Iberoamericana, volume 20, number 2, 2004, pages 531-561. In Lemma 2.3.2, part (a), we wish to show that for $\lambda > 0$, the operator $R(\lambda, \mathcal{L}) = (\lambda I + \mathcal{L})^{-1}$ is bounded on $L^\infty(M)$ with a norm that may depend on λ. We write the operator as

$$R(\lambda, \mathcal{L})[f](x) = \int_M f(y)\, r_\lambda(x, y)\, dy.$$

It then follows that

$$r_\lambda(x, y) = \int_0^\infty e^{-\lambda s}\, H(s, x, y)\, ds$$

where $H(s, x, y)$ is the heat kernel for the operator $\mathcal{L}$. We assert in equation (2.12) that there is a constant C so that for all $\lambda > 0$

$$|r_\lambda(x, y)| \leq C\, \frac{d^2(x, y)}{V(x, y)}. \tag{2.12}$$

This is not correct when M is a compact manifold. It should be replaced by the statement that when M is compact and of dimension at least 3, there is a constant C so that for all $\lambda > 0$

$$|r_\lambda(x, y)| \leq C\, \frac{d^2(x, y)}{V(x, y)} + \frac{C}{\lambda}. \tag{2.12a}$$

This estimate still shows that the operator $R(\lambda, \mathcal{L})$ is bounded on $L^\infty(M)$ if M is compact.

[1]We wish to thank Professor Dachun Yang of Beijing Normal University for bringing this error to our attention.

The error in our proof occurs on page 539 in the estimation of the integral for $r_\lambda(x,y)$. We do not use the decay given by $e^{-\lambda s}$ and assert that one part

$$\int_{d(x,y)^2}^{\infty} e^{-\lambda s}\,|H(s,x,y)|\,ds \lesssim \int_{d(x,y)^2}^{\infty} V_{\sqrt{s}}(x)^{-1}\,ds$$

arises only in the case of noncompact M. This is not true, and one must deal with this integral in all cases. However we can keep the decay term and write

$$\int_{d(x,y)^2}^{\infty} e^{-\lambda s}\,|H(s,x,y)|\,ds \lesssim \int_{d(x,y)^2}^{1} V_{\sqrt{s}}(x)^{-1}\,ds + \int_{1}^{\infty} e^{-\lambda s}\,ds\,.$$

The first term on the right is handled as in the non-compact case, using the estimate

$$V_{d(x,y)\,t}(x) \geq t^3 V(x,y)\,,$$

which is valid when $t > 1$ and $t\,d(x,y) \leq 1$. The second term on the right gives $C\,\lambda^{-1}$.

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