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ADDENDUM TO: "SOME NEW TYPES OF FILTER LIMIT THEOREMS FOR TOPOLOGICAL GROUP-VALUED MEASURES"

Abstract

The purpose of this note is to point out some corrections to the paper: A. Boccuto and X. Dimitriou, "Some new types of filter limit theorems for topological group-valued measures," *Real Anal. Exchange* **39** (1) (2014), 139-174.

We use the notation and terminology developed in [1].

On page 145, the definitions of $m^{\mathcal{L}}$ and m^+ should be formulated as follows:

$$m^{\mathcal{L}}(A) := \{m(B) : B \in \mathcal{L}, B \subset A\}, \quad A \in \mathcal{L},$$

$$m^+(A) := m^{\Sigma}(A) = \{m(B) : B \in \Sigma, B \subset A\}, \quad A \in \Sigma.$$

On page 148, formula (7), the definition of "positive" measure defined in a σ-algebra Σ and with values in a topological group R should be stated as follows:

> A finitely additive measure $m : \Sigma \to R$ is said to be *positive* iff every neighborhood W of 0 contains a neighborhood U_0 of 0 such that, for every $A \in \Sigma$ with $m(A) \in U_0$ and for each $B \in \Sigma$ with $B \subset A$, we also get $m(B) \in U_0$.

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On page 148, Proposition 2.10 should be formulated as follows.

Proposition 2.10 *Let $m : \Sigma \to R$ be a σ-additive measure. Then*

$$\lim_k m^+(H_k) = 0$$

for each decreasing sequence $(H_k)_k$ in Σ, satisfying

$$m\Big(B \cap \Big(\bigcap_{k=1}^{\infty} H_k\Big)\Big) = 0 \quad \text{for every } B \in \Sigma.$$

On page 150, Theorem 2.13 should be stated as follows.

Theorem 2.13 *Let $m : \Sigma \to R$ be an (s)-bounded measure. Then for each disjoint sequence $(C_k)_k$ in Σ there exists an infinite subset $P_0 \subset \mathbb{N}$, with*

$$\lim_r \Big\{ m\Big(\bigcup_{k \in Y, k \geq r} C_k \Big) : Y \subset P_0 \Big\} = 0, \tag{1}$$

and m is σ-additive on the σ-algebra generated by the sets C_k, $k \in P_0$.

On page 150, Theorem 2.14 should be formulated as follows.

Theorem 2.14 *Let $m_j : \Sigma \to R$, $j \in \mathbb{N}$, be a sequence of finitely additive (s)-bounded measures. Then for any disjoint sequence $(C_k)_k$ in Σ there exists an infinite subset $P \subset \mathbb{N}$, with*

$$\lim_h \Big\{ m_j\Big(\bigcup_{k \in Y, k \geq h} C_k \Big) : Y \subset P \Big\} = 0$$

for every $j \in \mathbb{N}$, and each m_j is σ-additive on the σ-algebra generated by the sets C_k, $k \in P$.

The following result, which is more general than Theorem 2.17 on page 152, was proved in [2, Corollary 6.2].

Let $(R, +)$ be a group, G be a locally compact Hausdorff topological space and m be a finitely additive R-valued set function, defined on the δ-ring of the relatively compact Baire subsets of G. Then m is regular if and only if m is σ-additive.

On page 156, formula (17), instead of

$$m^+_{j_{2h-1}}([l(n_{2h}, +\infty[) \subset U_{2h} \subset U_2$$

there should be

$$m^+_{j_{2h-1}}(]l(n_{2h}, +\infty[) \subset U_{2h} \subset U_2.$$

On page 159, formula (26) should be written as follows:

$$\begin{aligned} m^+_j(H_k) &= \{m_j(B) : B \in \Sigma, B \subset H_k\} = \\ &= \{m_j(B \setminus H_\infty) : B \in \Sigma, B \subset H_k\} = \\ &= \{m_j(C) : C \in \Sigma, C \subset H_k \setminus H_\infty\} = \\ &= m^+_j(H_k \setminus H_\infty) = m^+_j\Big(\bigcup_{l=k}^{\infty} C_l\Big) \end{aligned}$$

for every j, $k \in \mathbb{N}$.

On page 159, two lines above formula (28), instead of

$$\nu^+_j([k, +\infty[) := \bigcup \{\nu_j(D) : D \subset [k, +\infty[\} \subset m^+_j\Big(\bigcup_{l=k}^{\infty} C_l\Big)$$

there should be

$$\nu^+_j([k, +\infty[) := \{\nu_j(D) : D \subset [k, +\infty[\} \subset m^+_j\Big(\bigcup_{l=k}^{\infty} C_l\Big).$$

On pages 160-161, Theorem 3.5 should be formulated as follows.

Theorem 3.5 *Let G be any infinite set, $\Sigma \subset \mathcal{P}(G)$ be a σ-algebra, $m_j : \Sigma \to R$, $j \in \mathbb{N}$, be a sequence of positive (s)-bounded measures, $\mathcal{F}$ be a diagonal filter of $\mathbb{N}$. Assume that $m_0(E) = (\mathcal{F}) \lim_j m_j(E)$ exists in R for every $E \in \Sigma$, and that m_0 is σ-additive and positive on Σ.*

Then for every disjoint sequence $(C_k)_k$ in Σ and $I \in \mathcal{F}^*$ there exists $J \in \mathcal{F}^*$, $J \subset I$, with

$$\lim_k \Big(\bigcup_{j \in J} m^+_j(C_k)\Big) = \lim_k \{m_j(C_k) : j \in J\} = 0.$$

On page 161-162, after formula (30), the proof of the equality

$$\lim_{j \in J} m_j(B) = m_0(B) \text{ for all } B \in \mathcal{K}$$

in Theorem 3.5 should be as follows:

Arbitrarily choose $U \in \mathcal{J}(0)$, and let $W \in \mathcal{I}(0)$ be such that $5\,W \subset U$. In correspondence with W, let $U_0 \in \mathcal{J}(0)$, $U_0 \subset W$, satisfy the condition of positivity, that is $m(B) \in U_0$ whenever $A \in \Sigma$, $m(A) \in U_0$ and $B \in \Sigma$, $B \subset A$. In correspondence with U_0 there exists $k_0 \in \mathbb{N}$ with $m_0\Big(\bigcup_{k>k_0} C_k\Big) \in U_0$ and therefore, by positivity of m_0,

$$m_0\Big(\bigcup_{k>k_0, k \in P} C_k\Big) \in U_0.$$

Moreover, there is $j_0 \in J$, $j_0 = j_0(U, k_0)$ such that for every $j \in J$ with $j \geq j_0$ we have:

$$m_j\Big(\bigcup_{k \leq k_0, k \in P} C_k\Big) - m_0\Big(\bigcup_{k \leq k_0, k \in P} C_k\Big) \in U_0,$$

$$m_j\Big(\bigcup_{k \leq k_0} C_k\Big) - m_0\Big(\bigcup_{k \leq k_0} C_k\Big) \in U_0,$$

$$m_j\Big(\bigcup_{k=1}^{\infty} C_k\Big) - m_0\Big(\bigcup_{k=1}^{\infty} C_k\Big) \in U_0,$$

$$m_j\Big(\bigcup_{k>k_0} C_k\Big) - m_0\Big(\bigcup_{k>k_0} C_k\Big) \in 2\,U_0,$$

and hence,

$$\begin{aligned} m_j\Big(\bigcup_{k>k_0} C_k\Big) &= m_j\Big(\bigcup_{k>k_0} C_k\Big) - m_0\Big(\bigcup_{k>k_0} C_k\Big) + m_0\Big(\bigcup_{k>k_0} C_k\Big) \\ &\in 3\,U_0. \end{aligned}$$

By positivity of m_j, we have also $m_j\Big(\bigcup_{k>k_0, k \in P} C_k\Big) \in U_0$. Thus, for every $B \in \mathcal{K}$, $B = \bigcup_{k \in P} C_k$, we get

$$\begin{aligned} m_j(B) - m_0(B) &= m_j\Big(\bigcup_{k \in P} C_k\Big) - m_0\Big(\bigcup_{k \in P} C_k\Big) \\ &= m_j\Big(\bigcup_{k \le k_0, k \in P} C_k\Big) - m_0\Big(\bigcup_{k \le k_0, k \in P} C_k\Big) + \\ &\quad + m_j\Big(\bigcup_{k > k_0, k \in P} C_k\Big) - m_0\Big(\bigcup_{k > k_0, k \in P} C_k\Big) \\ &\in U_0 + 3\,U_0 + U_0 = 5\,U_0 \subset 5\,W \subset U. \end{aligned}$$

Thus, $\lim_{j \in J} m_j(B) = m_0(B)$ for all $B \in \mathcal{K}$.

On pages 162-163, Theorem 3.6 should be formulated as follows.

Theorem 3.6 *Let Σ, $\mathcal{F}$ be as in Theorem 3.5, τ be a Fréchet-Nikodým topology on Σ, $m_j : \Sigma \to R$, $j \in \mathbb{N}$, be a sequence of positive finitely additive (s)-bounded and τ-continuous measures. Assume that $m_0(E) := (\mathcal{F}) \lim_j m_j(E)$ exists in R for each $E \in \Sigma$, and that m_0 is σ-additive and positive on Σ.*

Then for every set $I \in \mathcal{F}^*$ and for each decreasing sequence $(H_k)_k$ in Σ with τ-$\lim_k H_k = \emptyset$ there exists a set $J \in \mathcal{F}^*$, $J \subset I$, with

$$\lim_k \Big(\bigcup_{j \in J} m_j^+(H_k)\Big) = \lim_k \{m_j(H_k) : j \in J\} = 0.$$

On pages 163, Theorem 3.7 should be formulated as follows.

Theorem 3.7 *Let Σ, $\mathcal{F}$ be as in Theorem 3.6, $m_j : \Sigma \to R$, $j \in \mathbb{N}$, be a sequence of positive σ-additive measures. If*

$$m_0(A) := (\mathcal{F}) \lim_j m_j(A)$$

exists in R for each $A \in \Sigma$, and m_0 is σ-additive and positive on Σ, then for each $I \in \mathcal{F}^*$ and for every decreasing sequence $(H_k)_k$ in Σ with $\bigcap_{k=1}^{\infty} H_k = \emptyset$ there exists $J \in \mathcal{F}^*$, $J \subset I$, with

$$\lim_k \Big(\bigcup_{j \in J} m_j^+(H_k)\Big) = \lim_k \{m_j(H_k) : j \in J\} = 0.$$

On page 168, formula (34), instead of

$$\lim_s \Big(\bigcup_{j \in M^*} m_j(C_{k_{r_s}}) \Big) = 0$$

there should be

$$\lim_s \Big(\bigcup_{j \in M^*} m_j^+(C_{k_{r_s}}) \Big) = 0.$$

On page 169, formula (35), instead of

$$\lim_k \Big(\bigcup_{j \in M^*} m_j(C_k) \Big) = 0,$$

there should be

$$\lim_k \Big(\bigcup_{j \in M^*} m_j^+(C_k) \Big) = 0.$$

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References

[1] A. Boccuto and X. Dimitriou, *Some new types of filter limit theorems for topological group-valued measures,* Real Anal. Exchange, **39** (1) (2014), 139-174.

[2] H. Weber, *Fortsetzung von Massen mit Werten in uniformen Halbgruppen,* Arch. Math. (Basel), **27** (4) (1976), 412-423.