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ERRATA: MB-REPRESENTATIONS AND TOPOLOGICAL ALGEBRAS

Theorem 5(b) from [BC] says the following.

> *Let $|X| = \kappa \geq \omega$, $\mathcal{F}_0 \subset [X]^\kappa$ be an almost disjoint family, and $\mathcal{F} = \{F \triangle A \colon F \in \mathcal{F}_0 \ \&\ A \in [X]^{<\kappa}\}$. If $|\mathcal{F}_0| > \kappa$ then the algebra $S(\mathcal{F}) = \{A \subset X \colon (\forall P \in \mathcal{F})(\exists Q \in \mathcal{F})(Q \subset A \cap P \text{ or } Q \subset P \setminus A)\}$ is not topological, that is, $\mathcal{A} \neq S(\tau \setminus \{\emptyset\})$ for any topology τ on X.*

Unfortunately, the printed proof has a gap. (It shows only that the pair $\langle S(\mathcal{F}), S_0(\mathcal{F})\rangle$ is not topological.) A correct proof for this result follows.

Proof. By way of contradiction suppose that there exists a topology τ on X such that $S(\mathcal{F}) = S(\tau_0)$, where $\tau_0 = \tau \setminus \{\emptyset\}$. Notice that for every $F \in \mathcal{F}$ we have $F \in S(\mathcal{F}) = S(\tau_0)$ and

$$U \notin S(\mathcal{F}) \text{ for every } U \in [F]^\kappa \text{ with } |F \setminus U| = \kappa. \tag{1}$$

In particular, $\mathcal{P}(F) \not\subset S(\tau_0)$; so F does not belong to $S_0(\tau_0)$, which is defined as $\{A \subset X \colon (\forall P \in \mathcal{F})(\exists Q \in \mathcal{F})(Q \subset P \setminus A)\}$. Thus,

$$\operatorname{int}_\tau(F) \neq \emptyset \ \text{ for every } F \in \mathcal{F}. \tag{2}$$

For every $F \in \mathcal{F}_0$ let $\mathcal{V}_F$ be a maximal pairwise disjoint subfamily of $\tau \cap [F]^{<\kappa}$ and notice that $|\bigcup \mathcal{V}_F| < \kappa$. Indeed, otherwise we could find a subfamily $\mathcal{V}$ of $\mathcal{V}_F$ with $|\bigcup \mathcal{V}| = |F \setminus \bigcup \mathcal{V}| = \kappa$. But then $U = \bigcup \mathcal{V} \in \tau \subset S(\tau_0) = S(\mathcal{F})$ would contradict (1). So, $F \setminus \bigcup \mathcal{V}_F \in \mathcal{F}$ and $V_F = \operatorname{int}_\tau(F \setminus \bigcup \mathcal{V}_F)$ is nonempty by (2). To finish the proof it is enough to notice that $\{V_F \colon F \in \mathcal{F}_0\}$ is a family of nonempty pairwise disjoint subsets of X, contradicting the fact that $|\mathcal{F}_0| > \kappa = |X|$. □

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