STABLE SAMPLING AND FOURIER MULTIPLIERS

Basarab Matei, Yves Meyer, and Joaquim Ortega-Cerdà

Abstract: We study the relationship between stable sampling sequences for band-limited functions in $L^p(\mathbb{R}^n)$ and the Fourier multipliers in L^p. In the case that the sequence is a lattice and the spectrum is a fundamental domain for the lattice the connection is complete. In the case of irregular sequences there is still a partial relationship.

1. Introduction

When $\omega > 0$, $1 < p < \infty$, and $f \in L^p(\mathbb{R})$ we write $f \in E_\omega^p$ if the Fourier transform $\hat{f}$ of f vanishes outside $[-\omega, \omega]$. If $0 < h \leq \pi/\omega$ the Shannon theorem states that any $f \in E_\omega^p$ can be completely recovered from its samples $f(kh)$, $k \in \mathbb{Z}$. When $h = \pi/\omega$, the map $S_\omega \colon f \mapsto \sqrt{h} f(kh)$, $k \in \mathbb{Z}$, is an isometry between E_ω^2 and $\ell^2(\mathbb{Z})$. This map S_ω is an isomorphism between E_ω^p and $\ell^p(\mathbb{Z})$ when $h = \pi/\omega$ and $1 < p < \infty$. This fails if $p = 1$ or $p = \infty$.

The problem addressed in this paper is to extend the Shannon theorem to functions $f \in L^p(\mathbb{R}^n)$, $n > 1$, $1 < p < \infty$. We are given an integrable compact set $K \subset \mathbb{R}^n$ and we want to know if every $f \in L^p(\mathbb{R}^n)$ whose Fourier transform is supported by K can be recovered from its samples on the grid $\Gamma = \mathbb{Z}^n$. The general case where $\mathbb{Z}^n$ is replaced by an arbitrary lattice Γ will follow by a linear change of variables.

2. Stable sampling and stable interpolation

Let $K \subset \mathbb{R}^n$ be a compact set and $p \in (1, \infty)$. Let E_K^p be the closed subspace of $L^p(\mathbb{R}^n)$ consisting of all $f \in L^p(\mathbb{R}^n)$ whose Fourier transform $\hat{f}$ is supported by K,

$$(2.1) \qquad E_K^p = \{ f \in L^p(\mathbb{R}^n) : \operatorname{supp} \hat{f} \subset K \}.$$

The Fourier transform $\hat{f}$ of f is defined by

$$(2.2) \qquad \hat{f}(\xi) = \int_{\mathbb{R}^n} \exp(-ix \cdot \xi) f(x) \, dx, \quad \xi \in \mathbb{R}^n,$$

where $x \cdot \xi = x_1 \xi_1 + \cdots + x_n \xi_n$. When $1 \leq p \leq 2$ the Fourier transform $\hat{f}$ belongs to L^q with $1/p + 1/q = 1$. If $p > 2$ this Fourier transform is defined in the distributional sense. The following lemmata are well known.

Lemma 2.1. *If $K \subset \mathbb{R}^n$ is a compact set and $1 \leq p \leq \infty$, then the restriction operator S^p which is defined by*

$$(2.3) \qquad f \mapsto (f(k))_{k \in \mathbb{Z}^n}$$

is continuous from E_K^p to $\ell^p(\mathbb{Z}^n)$.

The restriction of f to any uniformly separated sequence belongs to ℓ^p. This the well-known Plancherel–Polya inequality.

Lemma 2.2 (Poisson formula)**.** *If F is a compactly supported distribution and if $G(x) = \sum_{k \in \mathbb{Z}^n} F(x - 2\pi k)$ then the Fourier coefficients $c(k)$, $k \in \mathbb{Z}^n$, of G and the Fourier transform $\hat{F}$ of F are related by*

$$(2.4) \qquad c(k) = (2\pi)^{-n} \hat{F}(k), \quad k \in \mathbb{Z}^n.$$

If $1 < p < \infty$ we denote by $\mathcal{F}L^p$ the Banach space of all Fourier transforms of functions in $L^p(\mathbb{R}^n)$. The norm of $\hat{f}$ in $\mathcal{F}L^p$ is the norm of f in L^p. The space of restrictions of $\mathcal{F}L^p$ to an open set Ω will be denoted by $\mathcal{F}L^p(\Omega)$. Similarly $\mathcal{F}\ell^p$ will denote the Banach space consisting of all $2\pi\mathbb{Z}^n$ periodic functions (or distributions) whose Fourier coefficients belong to $\ell^p(\mathbb{Z}^n)$ and its norm is the ℓ^p norm of its coefficients.

Lemma 2.3. *For $F \in \mathcal{F}\ell^p$ and for ϕ in the Schwartz class $\mathcal{S}$ the product ϕF belongs to $\mathcal{F}L^p$.*

Indeed let $F(x) = \sum_{k \in \mathbb{Z}^n} c(k) \exp(ik \cdot x)$, where $c(k) \in \ell^p$ and let θ be the Fourier transform of ϕ. Then the Fourier transform of ϕF is $u(x) = \sum_{k \in \mathbb{Z}^n} c(k)\theta(x - k)$. By the Hölder inequality

$$|u(x)|^p \leq \left(\sum_{k \in \mathbb{Z}^n} |c(k)|^p |\theta(x - k)| \right) \left(\sum_k |\theta(x - k)| \right)^{p-1}.$$

Thus it follows that $\|u\|_p^p \leq C \sum_{k \in \mathbb{Z}^n} |c(k)|^p$.

From now on the compact set K is assumed to be *regular*.

Definition 2.1. A compact set $K \subset \mathbb{R}^n$ is *regular* if the following two conditions hold:

(a) K is connected,
(b) for each x_0 belonging to the boundary $\Gamma = \partial K$ of K there exist a neighborhood V of x_0, a suitable coordinate system $\mathcal{R}_{x_0}$, and a continuous function $A_{x_0} \colon \mathbb{R}^{n-1} \mapsto \mathbb{R}$ such that Γ coincides on V with the graph of A_{x_0} in $\mathcal{R}_{x_0}$ and K coincides on V with $\{x_n \geq A_{x_0}(x'),\ x' = (x_1, \ldots, x_{n-1})\}$.

A Lipschitz domain is regular. Definition 2.1 is required in this note since the counter example which is given below is not a Lipschitz domain.

We denote by $\mathcal{S} = \mathcal{S}(\mathbb{R}^n)$ the Schwartz class. We then have:

Lemma 2.4. *If $1 < p < \infty$ and if K is regular, then $\mathcal{S} \cap E_K^p$ is dense in E_K^p.*

Proof: Let $f \in E_K^p$. Then the compact support L of $F = \hat{f}$ is contained in K. Using a smooth partition of the identity we split F into a finite sum of pieces for which the local description of K can be used. Then the interior Ω of K is locally defined by $x_n > A(x_1, \ldots, x_{n-1})$ while K is defined by $x_n \geq A(x_1, \ldots, x_{n-1})$ where A is a continuous function. Let $F_\varepsilon = F(x_1, \ldots, x_{n-1}, x_n - \varepsilon)$ where $\varepsilon > 0$ is a small positive number. The support L_ε of F_ε is $L_\varepsilon = L + \varepsilon e_n$ where $e_n = (0, \ldots, 0, 1)$. Therefore L_ε is contained in Ω. Moreover $\|F_\varepsilon - F\|_{\mathcal{F}L^p}$ tends to 0 with ε. Indeed for every $f \in L^p$ Lebesgue dominated convergence theorem implies that $\|[\exp(i\varepsilon x_n) - 1]f\|_p \to 0$ as ε tends to 0. To conclude the proof it suffices to approximate F by a test function whose compact support is contained in Ω. To reach this goal we replace F_ε by the convolution product $F_\varepsilon \star \theta$ where θ is a smooth bump function supported by a sufficiently small ball $|x| \leq \eta$ with $\int \theta = 1$. $\qquad\square$

Lemma 2.4 does not hold for a Riemann integrable compact set K. Here is a counter example in two dimensions. Let K be the circle centered at 0 with radius 1. The compact set K is Riemann integrable. We consider the arc length measure $d\sigma$ on K and its inverse Fourier transform f, that is a Bessel function that decays as $|x|^{-1/2}$ when $x \to \infty$. Thus f belongs to E_K^p for $p > 4$ but f is not the limit in L^p of a sequence of test functions in E_K^p because there are no test functions in E_K^p. Any test function g will belong to $L^2(\mathbb{R}^2)$ and $\hat{g}$ will be supported in K that has measure 0, thus it will vanish.

We now follow the seminal work of H. J. Landau [**2**].

Definition 2.2. A uniformly separated sequence $\Gamma \subset \mathbb{R}^n$ is a set of stable sampling for E_K^p if there exists a constant C such that

$$(2.5) \qquad f \in E_K^p \Rightarrow \|f\|_p^p \le C \sum_{\gamma \in \Gamma} |f(\gamma)|^p.$$

In other words $\Gamma \subset \mathbb{R}^n$ is a set of stable sampling for E_K^p if $S^p \colon E_K^p \mapsto \ell^p(\Gamma)$ is an isomorphism between E_K^p and its image in $\ell^p(\Gamma)$. If this condition is not satisfied two problems may occur. The first one is named aliasing. Aliasing means that there exists a function $f \in E_K^p$, $f \ne 0$, such that $f(\gamma) = 0$, $\gamma \in \Gamma$. Even if aliasing does not occur the reconstruction of f from its samples $f(\gamma)$, $\gamma \in \Gamma$, is not stable when (2.5) is not satisfied.

When $\Gamma = \mathbb{Z}^n$, by Lemma 2.2 we can "code" the samples $f(\gamma)$ in a periodic function $G \in \mathcal{F}(\ell^p)$ as follows. Let F be the Fourier transform of f. It has compact support and we can periodize it:

$$G(x) = \sum_{k \in \mathbb{Z}^n} F(x - 2\pi k).$$

By Lemma 2.2, the Fourier coefficients $c(k)$ of G satisfy $c(k) = f(-k)$. Hence, the sampling inequality (2.5), on the Fourier side, amounts to say that all $F \in \mathcal{F}(L^p)$ supported in K are controlled by its periodized $G \in \mathcal{F}(\ell^p)$. Let $|E|$ denote the Lebesgue measure of a set E.

Lemma 2.5. *Let $K \subset \mathbb{R}^n$ be a Riemann integrable compact set.*

(a) *If $\mathbb{Z}^n$ is a set of stable sampling for E_K^p then*

$$(2.6) \qquad \forall\, k \in \mathbb{Z}^n, \quad k \ne 0 \Rightarrow |K \cap (K + 2\pi k)| = 0.$$

(b) *Conversely if the sets $K + 2\pi k$, $k \in \mathbb{Z}^n$, are pairwise disjoint, then $\mathbb{Z}^n$ is a set of stable sampling for E_K^p.*

Observe that the sufficient condition (b) is more demanding than (a) and Lemma 2.5 does not fully answer the problem.

Proof: If (2.6) is not satisfied there exists a $k_0 \in \mathbb{Z}^n$ such that the Riemann integrable set $K \cap (K - 2\pi k_0)$ has a positive measure. Therefore this set contains a small ball B. Then $B \subset K$ and $B + 2\pi k_0 \subset K$. Let f be any test function whose Fourier transform is supported by B. Then $g(x) = (\exp(2\pi i k_0 \cdot x) - 1) f(x)$ belongs to E_K^p and vanishes on $\mathbb{Z}^n$. Aliasing occurs.

We now prove (b). Let F be the Fourier transform of the function $f \in E_K^p$ and G its periodized version as above. If $\varepsilon > 0$ is small enough and if $B(0, \varepsilon)$ is the ball centered at 0 with radius ε, the compact set $K' = K + B(0, \varepsilon)$ still satisfies (b). Let ϕ be a test function supported by K' and such that $\phi(x) = 1$ on K. This implies $F = \phi G$ and we can apply Lemma 2.3. □

Here is an example illustrating Lemma 2.5. Let us assume that K is the disc $|x| \leq r$. If $0 < r < \pi$, condition (b) is satisfied and if $r > \pi$, condition (a) does not hold. However Lemma 2.5 does not give any answer if $r = \pi$. To treat this case, we consider the cube $Q = [-\pi, \pi]^n$ to which Theorem 3.1 below can be applied. Therefore $\mathbb{Z}^n$ is a set of stable sampling for E_Q^p. Since $K \subset Q$, $\mathbb{Z}^n$ is a set of stable sampling for E_K^p.

When $p = 2$, stable sampling is equivalent to (2.6). The goal of this note is to show that this property does not suffice when $p \neq 2$.

Definition 2.3. We say that $\Gamma \subset \mathbb{R}^n$ is a set of stable interpolation if every sequence $a(\gamma) \in \ell^p(\Gamma)$ can be interpolated by a function f in E_K^p.

It means that there exists $f \in E_K^p$ such that $f(\gamma) = a(\gamma)$, $\gamma \in \Gamma$.

Lemma 2.6. *Let $K \subset \mathbb{R}^n$ be a Riemann integrable compact set.*

(a) *If $\mathbb{Z}^n$ is a set of stable interpolation for E_K^p then*

$$(2.7) \qquad \bigcup_{k \in \mathbb{Z}^n} (K + 2\pi k) = \mathbb{R}^n.$$

(b) *Conversely if Ω is the interior of K and if*

$$(2.8) \qquad \bigcup_{k \in \mathbb{Z}^n} (\Omega + 2\pi k) = \mathbb{R}^n$$

then $\mathbb{Z}^n$ is a set of stable interpolation for E_K^p.

Here also the sufficient condition (b) is more demanding than (a). The proof is similar to the one used in Lemma 2.5. It is left to the reader.

We want to know when $S^p \colon E_K^p \to \ell^p(\mathbb{Z}^n)$ is an isomorphism. If it is the case, Lemmata 2.5 and 2.6 imply that the translated sets $K + 2\pi k$, $k \in \mathbb{Z}^n$, are a partition of $\mathbb{R}^n$ up to sets of measure 0. We then say that K is a *fundamental domain* for $\mathbb{Z}^n$. But the converse implication is not true. The fact that K is a *fundamental domain* for $\mathbb{Z}^n$ does not imply that the operator S^p is an isomorphism between E_K^p and $\ell^p(\mathbb{Z}^n)$ when $p \neq 2$. This will be proved in the next section.

3. Our result

A Borel function $m(x)$ is a multiplier of $\mathcal{F}L^p$ if we have

(3.1) $F(x) \in \mathcal{F}L^p \Rightarrow m(x)F(x) \in \mathcal{F}L^p$

and if a constant C exists such that

(3.2) $\|m(x)F(x)\|_{\mathcal{F}L^p} \leq C\|F(x)\|_{\mathcal{F}L^p}.$

This does not make any sense if $2 < p$ since $F(x)$ may be a distribution. This issue is settled by the following remarks. It suffices to prove (3.2) when $f \in L^2 \cap L^p$ and a density argument yields the general case. Moreover if m is a multiplier of $\mathcal{F}L^p$ then m is also a multiplier of $\mathcal{F}L^q$ when $1/p + 1/q = 1$. This reduces the case $p > 2$ to $1 < q < 2$.

The following lemma will be seminal in the proof of Theorem 3.1.

Lemma 3.1. *Let $m \in L^\infty(\mathbb{R}^n)$ be a compactly supported function. Then the following two properties are equivalent:*

(a) *m is a multiplier of $\mathcal{F}L^p$,*
(b) *m maps $\mathcal{F}\ell^p$ into $\mathcal{F}L^p$.*

Proof: For proving (a) $\Rightarrow$ (b) let us denote by ϕ a smooth and compactly supported function such that $\phi = 1$ on a neighborhood of the compact support of m. If $G \in \mathcal{F}\ell^p$ we have $\phi G \in \mathcal{F}L^p$ by Lemma 2.3 and $mG = m\phi G \in \mathcal{F}L^p$ since m is a multiplier of $\mathcal{F}L^p$. We now prove (b) $\Rightarrow$ (a). If $F \in \mathcal{F}L^p$ we have $mF = m\phi F$ as above. One uses a smooth partition of the identity to decompose ϕF into a finite sum $F = \sum_1^m F_j$ where $F_j \in \mathcal{F}L^p$ and where the support of each F_j is contained in a cube Q_j centered at x_j with side length 1. It suffices to show that $mF_j \in \mathcal{F}L^p$ to conclude. We consider $G_j(x) = \sum_{k \in \mathbb{Z}^n} F_j(x - 2\pi k)$ and Lemmata 2.1 and 2.2 yield $G_j \in \mathcal{F}\ell^p$. Since m maps $\mathcal{F}\ell^p$ into $\mathcal{F}L^p$ we have $mG_j \in \mathcal{F}L^p$. Let χ_j be a test function such that $\chi_j(x) = 1$ on Q_j and $\chi_j(x) = 0$ on every $Q_j + 2k\pi$, $k \neq 0$. Then $mF_j = \chi_j mG_j \in \mathcal{F}L^p$. $\square$

Theorem 3.1. *Let K be a regular compact set. Assuming that K is fundamental domain for $2\pi\mathbb{Z}^n$ the following four properties are equivalent ones:*

(a) *The indicator function of K is a multiplier of $\mathcal{F}L^p$.*
(b) *The lattice $\mathbb{Z}^n$ is a set of stable sampling for E_K^p.*
(c) *The lattice $\mathbb{Z}^n$ is a set of stable interpolation for E_K^p.*
(d) *The operator $S^p\colon E_K^p \mapsto \ell^p(\mathbb{Z}^n)$ is an isomorphism.*

Proof: Let us show that (a) implies (b). For $f \in E_k^p$ we consider F and G as above. If K is fundamental domain for $\mathbb{Z}^n$, then formally $F = \chi_K G$. A priori G could be a distribution if $p > 2$, so we need to assume that $f \in \mathcal{S} \cap E_k$ which we may by Lemma 2.4. The property of stable sampling is first proved if $f \in \mathcal{S} \cap E_K^p$ and extended by continuity to the general case. Hence the sampling inequality (2.5) amounts to

$$(3.3) \qquad \|\chi_K G\|_{\mathcal{F}L^p} \leq \|G\|_{\mathcal{F}\ell^p}, \quad f \in E_K^p.$$

This immediately shows, by Lemma 3.1, that (a) implies (b). The converse is also true because we can start from $G \in \mathcal{F}(\ell^p)$ and take $F = \chi_K G$ and f the function that has F as its Fourier transform. The assumption (b) means

$$(3.4) \qquad \|F\|_{\mathcal{F}L^p} \leq C\|f(k)\|_{\ell^p}$$

which is the same as

$$(3.5) \qquad \|F\|_{\mathcal{F}L^p} \leq C\|G\|_{\mathcal{F}\ell^p}.$$

The equivalence between (a) and (c) follows just by looking to the situation in terms of the sequence of Fourier coefficients of G, which is an arbitrary sequence in ℓ^p.

The equivalence between (a) and (d) obviously follows from the preceding steps. $\qquad\square$

4. An example

4.1. Fefferman's theorem and stable sampling. Fefferman's theorem, see [**1**], says that in any dimension $n \geq 2$ the indicator function χ_B of a ball B is not a multiplier of $\mathcal{F}L^p$ when $p \neq 2$. Moreover the proof of Fefferman's theorem or an elementary reasoning shows that the result is local. If ϕ is any function which does not vanish identically on ∂B then $\phi(x)\chi_B(x)$ is not a multiplier of $\mathcal{F}L^p$ if $p \neq 2$. This paves the way to our example.

The compact set $K \subset \mathbb{R}^2$ is defined as follows. We start with the square $Q = \{0 \leq x_1, x_2 \leq 2\pi\}$ and we call D_1 (resp. D_2) the closed discs centered at $x_1 = (\pi, 0)$ (resp. $x_2 = (\pi, 2\pi)$) with radius π. Let

$K_1 = Q \cup D_1$ and $K = K_1 \setminus D_2$. It is trivial to prove that K is a fundamental domain for $2\pi\mathbb{Z}^2$ as in the picture:
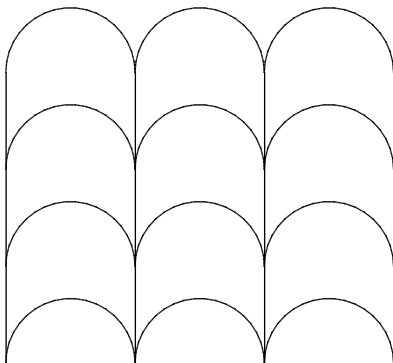
But we know from Fefferman's theorem that the indicator function of K is not a Fourier multiplier of $\mathcal{F}(L^p)$ if $p \neq 2$. Therefore the functions $f \in E_K^p$ cannot be sampled on $\mathbb{Z}^2$ if $p \neq 2$.

5. Irregular sequences and multipliers

A sequence $\Lambda \subset \mathbb{R}^n$ is a complete interpolating sequence for E_K^p when for any sequence of values $v_\lambda \in \ell^p(\Lambda)$ there is a *unique* function $f \in E_K^p$ such that $f(\lambda) = v_\lambda$. It can be seen that complete interpolating sequences are simultaneously stable sampling and stable interpolating sequences.

We are not assuming now that Λ has any structure. In such generality very little is known. In the case of dimension one, and K being an interval there is a complete description of the complete interpolating sequences in E_I^p due to Lyubarskii and Seip [**3**] that generalizes to any $p > 1$ the description of Pavlov, [**4**] when $p = 2$.

Nevertheless it is still possible to see that the existence of complete interpolating sequences is related to the boundedness of the multiplier at least in one direction.

Theorem 5.1. *Given any compact K, and a complete interpolating sequence Λ for the space E_K^p, the function χ_K is a multiplier for $\mathcal{F}L^p$.*

Proof: Take a smooth compactly supported function ϕ such that $\phi \equiv 1$ in a neighborhood of K. If $\hat{F} = \phi$, then for any $f \in E_K^p$ we have the reproducing formula:

$$(5.1) \qquad f(z) = f \star F(z) = \int_{\mathbb{R}^n} f(x)F(z - x)\,dx.$$

Consider the following two operators:

$$\text{(5.2)} \qquad \begin{aligned} T_1 &: L^p(\mathbb{R}^n) \to \ell^p(\Lambda), \\ f &\to \{f \star F(\lambda)\}_{\lambda \in \Lambda}. \end{aligned}$$

Clearly the operator T_1 is linear and bounded by the Plancherel–Polya inequality since the sequence Λ is separated and $f \star F \in E_B^p$, where B is a ball containing the support of ϕ.

Now consider the operator:

$$\text{(5.3)} \qquad T_2 : \ell^p(\Lambda) \to E_K^p$$

that to any sequence of values $\{v_\lambda\} \in \ell^p(\Lambda)$ associates the unique function $f \in E_K^p$ such that $f(\lambda) = v_\lambda$. This function exists because we assume that Λ is a complete interpolating sequence. Moreover it defines a bounded linear operator. Thus the composition operator $T = T_2 \circ T_1$ maps $L^p(\mathbb{R}^n)$ to E_K^p linearly and it is bounded. Because of the reproducing property, and the fact that Λ is a uniqueness set for E_K^p it follows that T is a projection, i.e. $T \circ T = T$.

We are going to produce now another bounded projection invariant under translations. Denote by τ_x the translation operator by x. Then $T_x = \tau_{-x} T \tau_x$ is another projection with the same norm as T. We average them over a big ball and denote

$$\text{(5.4)} \qquad T_R = \frac{1}{|B(0,R)|} \int_{B(0,R)} T_x \, dx.$$

The operator T_R is again a projection with norm bounded by the norm of T. Since all the operators $T_R \colon L^p \to L^p$ are bounded uniformly, by the Banach–Alouglou theorem we can extract a sequence $R_n \to \infty$ such that T_{R_n} converges to a bounded operator $\widetilde{T}$ in the weak operator topology, i.e. $\langle f, T_{R_n}(g) \rangle \to \langle f, \widetilde{T}(g) \rangle$ for all $f \in L^q$ and $g \in L^p$. The linear operator $\widetilde{T}$ is a projection and has bounded norm (all these properties are inherited from the T_{R_n}). We will see now that it commutes with the translations. Indeed if we fix $y \in \mathbb{R}^n$,

$$\text{(5.5)} \qquad \begin{aligned} \tau_y T_R &= \frac{1}{|B(0,R)|} \int_{B(0,R)} \tau_{y-x} T \tau_x \, dx \\ &= \frac{1}{|B(0,R)|} \int_{B(y,R)} \tau_{-x} T \tau_{x+y} \, dx \\ &= \frac{1}{|B(0,R)|} \int_{B(0,R)} \tau_{-x} T \tau_{x+y} \, dx + G_R, \end{aligned}$$

where $\|G_R\| = O(1/R)$.

Thus $\|T_{R_n}\tau - \tau T_{R_n}\| \to 0$ for any translate τ, but since $T_{R_n}\tau - \tau T_{R_n} \to \widetilde{T}\tau - \tau\widetilde{T}$ in the weak operator topology, then $\widetilde{T}\tau = \tau\widetilde{T}$.

Since $\widetilde{T}$ is a bounded linear operator that commutes with the translations then it is a convolution operator, i.e. it is given by a Fourier multiplier against a bounded function.

The fact that it is a projection onto the functions with spectra lying in K implies that $\widetilde{T}$ is the multiplier given by χ_K. $\qquad\square$

Corollary 5.1. *For any smooth compact $K \subset \mathbb{R}^n$ there are no complete interpolating sequences for the space E_K^p for any $p \neq 2$, $1 < p < \infty$.*

Proof: If such Λ existed, χ_K will be a multiplier in $\mathcal{F}(L^p)$ but Fefferman theorem states that it cannot be bounded because K has points with positive curvature. $\qquad\square$

References

[1] C. Fefferman, The multiplier problem for the ball, *Ann. of Math. (2)* **94** (1971), 330–336. DOI: 10.2307/1970864.

[2] H. J. Landau, Necessary density conditions for sampling and interpolation of certain entire functions, *Acta Math.* **117** (1967), 37–52. DOI: 10.1007/BF02395039.

[3] Y. I. Lyubarskii and K. Seip, Complete interpolating sequences for Paley–Wiener spaces and Muckenhoupt's (A_p) condition, *Rev. Mat. Iberoamericana* **13(2)** (1997), 361–376. DOI: 10.4171/RMI/224.

[4] B. S. Pavlov, The basis property of a system of exponentials and the condition of Muckenhoupt, (Russian), *Dokl. Akad. Nauk SSSR* **247(1)** (1979), 37–40; English translation in: *Soviet Math. Dokl.* **20(4)** (1979), 655–659.

Basarab Matei:
LAGA
Université Paris Nord
99, avenue Jean-Baptiste Clément
93430 Villetaneuse
France
E-mail address: matei@math.univ-paris13.fr

Yves Meyer:
CMLA
ENS-Cachan
61, avenue du Président-Wilson
94235 Cachan cedex
France
E-mail address: yves.meyer@cmla.ens-cachan.fr

Joaquim Ortega-Cerdà:
Departament MAIA
Universitat de Barcelona
Gran Via 585
08007 Barcelona
Spain
E-mail address: `jortega@ub.edu`