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NOTE ON A PAPER BY UPPULURI

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By using the Battacharya bounds for the variance of an unbiased estimator V. R. Rao Uppuluri [1] claims to have established the result

$$\frac{\Gamma(m+1)}{\Gamma(m+\frac{1}{2})} > \left\{m + \tfrac{1}{4} + \frac{9}{48m+32}\right\}^{\frac{1}{2}} \quad \text{for } m = 1, 2, 3, \cdots,$$

but a numerical calculation easily shows this to be incorrect, e.g. for $m = 1$. In fact Watson [2] has shown, by using Gauss' theorem

$$F(a, b; c; 1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \quad \text{for } c > a + b,$$

that

$$\begin{aligned}\frac{\Gamma^2(x+1)}{x\Gamma^2(x+\frac{1}{2})} &= \frac{\Gamma(x)\Gamma(x+1)}{\Gamma^2(x+\frac{1}{2})} \\ &= F(-\tfrac{1}{2}, -\tfrac{1}{2}; x; 1) \qquad \text{for } x > -1 \\ &= 1 + \frac{1}{4x} + \frac{1}{32x(x+1)} \\ &\quad + \sum_{r=3}^{\infty} \frac{\{-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdots(r-\frac{3}{2})\}^2}{r!\,x(x+1)\cdots(x+r-1)}.\end{aligned}$$

It follows that

$$\frac{\Gamma^2(x+1)}{\Gamma^2(x+\frac{1}{2})} = x + \tfrac{1}{4} + \frac{1}{32x+32} + O(x^{-2}) \text{ as } x \to \infty,$$

so that it is not possible to have

$$\frac{\Gamma(m+1)}{\Gamma(m+\frac{1}{2})} > \left\{m + \tfrac{1}{4} + \frac{1}{am+b}\right\}^{\frac{1}{2}}$$

for all positive integers m if $a < 32$.

It also follows that, for $m = 1, 2, 3, \cdots$,

$$\begin{aligned}\left\{m + \tfrac{1}{4} + \frac{1}{32m+32}\right\}^{\frac{1}{2}} &< \frac{\Gamma(m+1)}{\Gamma(m+\frac{1}{2})} \\ &= (m+\tfrac{1}{2})\Big/\frac{\Gamma(m+\frac{3}{2})}{\Gamma(m+1)} \\ &< \left\{\frac{(m+\frac{1}{2})^2}{m + \frac{3}{4} + \frac{1}{32m+48}}\right\}^{\frac{1}{2}}.\end{aligned}$$

REFERENCES

1. V. R. Rao Uppuluri, *On a stronger version of Wallis' formula,* Pacific J. Math. **19** (1966), 183-187.
2. G. N. Watson, *A note on gamma functions,* Proc. Edinburgh Math. Soc. **11** (1959), Notes, 7-9.

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