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HOMOMORPHISMS OF ANNIHILATOR BANACH ALGEBRAS, II

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Let A be a semi-simple annihilator Banach algebra, and let ν be a homomorphism of A into a Banach algebra. In this paper it is shown that there exists a constant K and dense two-sided ideals containing the socle, I_L and I_R, such that $\|\nu(xy)\| \leqq K\|x\| \cdot \|y\|$ whenever $x \in I_L$ or $y \in I_R$. If A has a bounded left or right approximate identity, then ν is continuous on the socle. Thus if $A = L_1(G)$, where G is a compact topological group, then any homomorphism of A into a Banach algebra is continuous on the trigonometric polynomials.

In [1] we considered the problem of deducing continuity properties of a homomorphism ν from a semi-simple annihilator Banach algebra A into an arbitrary Banach algebra. The main theorem there (Theorem 5.1) had a conclusion more restrictive than the one stated above and required the additional hypothesis that $I \oplus \Re(I) = A$, for all closed two-sided ideals I, where $\Re(I) = \{x \mid Ix = (0)\}$. The main theorem of this paper applies when $A = L_p(G)$, $1 \leqq p < \infty$ or $C(G)$, where G is a compact topological group and multiplication is convolution, and when A is topologically-simple, whereas the earlier theorem did not.

Any terms not defined in this paper are those of Rickart's book [10]. For facts about annihilator algebras, the reader is referred to [4] or [10].

Given the left-right symmetry in the definition of annihilator algebras, it follows that, given any theorem about left (right) ideals, the corresponding theorem for right (left) ideals also holds. Specifically, this is the case for the theorems in [4, § 4] and [1, § 4]. We will make tacit use of this fact throughout this paper.

2. Structural lemmas.

In this section several lemmas are established which will be used later in proving the main result. Throughout this section, we assume that A is a semi-simple annihilator Banach algebra.

LEMMA 2.1. *If $\{x_1, \cdots, x_n\}$ is contained in the socle of A, then there exist idempotents e and f such that $x_i \in eAf$, $1 \leqq i \leqq n$.*

Proof. By [1, Corollary 4.9], for each i there exist idempotents e_i and f_i such that $x_i \in e_iA \cap Af_i \subset e_iAf_i$. By [1, Th. 4.8], there exist idempotents e and f such that $e_1A + \cdots + e_nA = eA$ and $Af_1 + \cdots + Af_n = Af$. Thus $x_i \in e_iAf_i = ee_iAf_if \subset eAf$, $1 \leqq i \leqq n$.

LEMMA 2.2. *Suppose A is topologically-simple, and e is a minimal idempotent in A. Then there exists a constant L such that:*

Given $f = f^2 \in A$ and $x \in eA$, there exists $g = g^2 \in A$ such that:

(1) $x(1 - f)g = x(1 - f) = xg$

(2) $fg = gf = 0$

(3) $\|g\| \leqq (1 + \|f\|)L$.

The corresponding statement holds for $x \in Ae$.

Proof. Let F_0 denote the bounded operators on Ae of finite rank. Then via the left regular representation, we may regard A algebraically as a subalgebra of the uniform closure of F_0 which contains F_0 (see [4], Ths. 9 and 10).

If $a \in eA$, $u \in Ae$, then $au = eaue = \lambda e = \phi_a(u)e$, and $a \rightarrow \phi_a$ defines an isomorphism and homeomorphism between eA and the bounded linear functionals on Ae [4, Th. 13]. Hence there exists a constant L such that $\|a\| \leqq (L/2)\|\phi_a\|$ for all $a \in eA$.

Let $x \in eA$ and $f = f^2 \in A$. Then $x(1 - f) \in eA$, and e is minimal, so range$(x(1 - f))$ is one-dimensional. Let $M = (x(1 - f))^{-1}(0)$. Then M is a closed subspace of co-dimension one in Ae, so there exists a bounded linear functional β on Ae such that $\|\beta\| = 1$ and $\beta^{-1}(0) = M$. Let $w \in Ae$ such that $\|w\| \leqq 2$ and $\beta(w) = \|\beta\| = 1$. Now $w = (1 - f)w + fw$, and $fw \in (1 - f)^{-1}(0) \subset M$, so $\beta((1 - f)w) = \beta(w) = 1$.

Let $G(u) = \beta(u)(1 - f)w$, $u \in Ae$. Then G is a bounded operator on Ae with one-dimensional range and $G = G^2$, so there exists an idempotent $g \in A$ such that $gu = \beta(u)(1 - f)w$, $u \in Ae$. If $u \in Ae$, then $u - \beta(u)(1 - f)w \in \beta^{-1}(0) = M = (x(1 - f))^{-1}(0)$, so $x(1 - f)u = x(1 - f)\beta(u)(1 - f)w = x\beta(u)(1 - f)w = xgu$. Therefore $x(1 - f) = xg$. Thus $x(1 - f)g = xg^2 = xg = x(1 - f)$. This establishes (1).

To prove (2), we see that $(1 - f)w \in f^{-1}(0)$, so $fg = 0$, and range$(f) = (1 - f)^{-1}(0) \subset M = g^{-1}(0)$, so $gf = 0$.

To establish (3), let $h \in eA$ such that $\phi_h = \beta$. If $u \in Ae$, then

$$(1 - f)whu = (1 - f)w\beta(u)e = \beta(u)(1 - f)we = \beta(u)(1 - f)w = gu.$$

Therefore $(1 - f)wh = g$, so

$$\|g\| \leqq \|h\|(1 + \|f\|)\|w\| \leqq (L/2)(1 + \|f\|)2 \leqq L(1 + \|f\|) .$$

3. The ideals I_L and I_R. In this section we discuss the ideals which enter into the main theorem. Throughout this section, we assume that A is a Banach algebra and that ν is a homomorphism of A into a Banach algebra.

DEFINITION 3.1. Let $I_L = \{x \in A \mid y \to \nu(xy)$ is continuous on $A\}$ and let $I_R = \{x \in A \mid y \to \nu(yx)$ is continuous on $A\}$.

These sets were introduced by Stein, who shows they are two-sided ideals in A [11]. Another useful concept is that of the *separating ideal*, S, which is defined to be the set of $s \in \mathrm{cl}(\nu(A))$ such that $\inf_{x \in A}\{\|x\| + \|s - \nu(x)\|\} = 0$. The separating ideal was introduced in the form above by Yood [13]. It is a closed two-sided ideal in $\mathrm{cl}(\nu(A))$.

In [12], Stein notes that $I_L \subset \{x \in A \mid \nu(x)S = (0)\}$ and similarly for I_R. One actually has equality: For suppose $\nu(x)S = (0)$. If $x_n \to 0$ in A, then by [8, Lemma 2.1], $\nu(x_n) + S \to S$ in $\mathrm{cl}(\nu(A))/S$. Hence there exists $\{s_n\} \subset S$ such that $\nu(x_n) + s_n \to 0$. Thus $\nu(xx_n) = \nu(x)\nu(x_n) = \nu(x)(\nu(x_n) + s_n) \to 0$, so $x \in I_L$.

4. Homomorphisms of annihilator algebras. In this section we establish the main results of this paper. We will make frequent use of the "Main Boundedness Theorem" of Badé and Curtis.

THEOREM 4.1. *Suppose that A is a Banach algebra, and that ν is a homomorphism of A into a Banach algebra. Let $\{x_n\}$ and $\{y_n\}$ be sequences in A such that $x_n y_m = 0$, $n \neq m$. Then*

$$\sup_n \frac{\|\nu(x_n y_n)\|}{\|x_n\|\,\|y_n\|} < \infty .$$

Proof. This is Theorem 3.1 of [5]. The statement there includes the unnecessary hypothesis that $y_n y_m = 0$, $n \neq m$.

Throughout the remainder of this section, A will denote a semi-simple annihilator Banach algebra with socle F, and ν will denote a homomorphism of A into a Banach algebra. We first prove:

LEMMA 4.2. *If A is topologically-simple, and e is a minimal idempotent in A, then $\nu \mid eA$ and $\nu \mid Ae$ are continuous.*

Proof. (For $\nu \mid eA$). Let L be as in Lemma 2.2. Suppose the conclusion fails. Choose $x_1 \in eA$ such that $\|\nu(x_1)\| > L\|x_1\|$. By Lemma 2.2, with $f = 0$, there exists $g_1 = g_1^2 \in A$ such that $\|g_1\| \leqq L$ and $x_1 g_1 = x_1$. Thus $\|\nu(x_1)\| > \|x_1\|\,\|g_1\|$.

Assume that elements $x_i \in eA$, $g_i \in A$ have been chosen such that $x_i g_i = x_i$,

$$g_i g_j = 0\ , \qquad i \neq j\ ,$$

and $\|\nu(x_i)\| > i\|x_i\|\,\|g_i\|$, $1 \leqq i,\ j \leqq n$.

Let $f = g_1 + \cdots + g_n$. Then $f = f^2$, $g_i f = g_i = f g_i$, and $x_i \in eAf$, $1 \leqq i \leqq n$. Since f can be expressed as the sum of minimal idempotents [1, Th. 4.5], eAf is finite-dimensional, so let K be the norm of $\nu \mid eAf$. Now choose $u \in eA$ such that $\|\nu(u)\| > (1+\|f\|)^2 L(n+1)\|u\| + K\|f\|\,\|u\|$. Then

$$\begin{aligned}\|\nu(u)\| &\leqq \|\nu(uf)\| + \|\nu(u(1-f))\| \\ &\leqq K\|u\|\,\|f\| + \|\nu(u(1-f))\|\ ,\end{aligned}$$

so

$$\begin{aligned}\|\nu(u(1-f))\| &> (1 + \|f\|)^2 L(n+1)\|u\| \\ &\geqq (1 + \|f\|)L(n+1)\|u(1-f)\|\ .\end{aligned}$$

Let $x_{n+1} = u(1-f) \in eA$. By Lemma 2.2, there exists $g_{n+1} = g_{n+1}^2 \in A$ such that $x_{n+1} g_{n+1} = x_{n+1}$, $g_{n+1} f = f g_{n+1} = 0$, and $\|g_{n+1}\| \leqq L(1 + \|f\|)$. Thus

$$g_{n+1} g_i = 0 = g_i g_{n+1}\ , \qquad 1 \leqq i \leqq n$$

and

$$\|\nu(x_{n+1})\| > (n+1)\|x_{n+1}\|\,\|g_{n+1}\|\ .$$

Thus by induction there exist sequences $\{x_n\}$, $\{g_n\}$ such that $x_n g_m = x_n g_n g_m = 0$, $n \neq m$ and $\|\nu(x_n g_n)\| > n\|x_n\|\,\|g_n\|$, which contradicts Theorem 4.1.

We now show that I_L and I_R are dense in A:

LEMMA 4.3. $F \subset I_L \cap I_R$.

Proof. If e is a minimal idempotent, then e is contained in a minimal-closed two-sided ideal M, M is a topologically-simple semi-simple annihilator Banach algebra, and $eM = eA$. The preceding lemma gives that $\nu \mid eA$ is continuous. Thus $x \to \nu(ex)$ is continuous on A, so $e \in I_L$. Hence I_L contains all the minimal idempotents of A. Since I_L is an ideal, this implies that $I_L \supset F$. Similarly, $I_R \supset F$.

LEMMA 4.4. *If* $\|\nu(xy)\| > r\|x\|\,\|y\|$, *and if* $x \in I_L$ *or* $y \in I_R$, *then there exist* $x_1,\ y_1 \in F$ *such that* $\|\nu(x_1 y_1)\| > r\|x_1\|\,\|y_1\|$.

Proof. Suppose $x \in I_L$. Since $w \to \nu(xw)$ is continuous on A and F is dense in A, there exists $y_1 \in F$ such that $\|\nu(xy_1)\| > r\|x\|\,\|y_1\|$. Now $y_1 \in I_R$, so there exists $x_1 \in F$ such that $\|\nu(x_1 y_1)\| > r\|x_1\|\,\|y_1\|$.

We can now prove the main theorem:

THEOREM 4.5. *Let A be a semi-simple annihilator Banach algebra, and let ν be a homomorphism of A into a Banach algebra. Then there exists a constant K such that*

$$\| \nu(xy) \| \leqq K \| x \| \| y \|$$

for all x and y in A such that $x \in I_L$ or $y \in I_R$.

Proof. In view of the preceding lemma (or by symmetry considerations), it is enough to show that there exists a K such that $\| \nu(xy) \| \leqq K \| x \| \| y \|$ whenever $x \in I_L$. Suppose this is not the case. By the preceding lemma, there exist x_1 and y_1 in F such that $\| \nu(x_1 y_1) \| > \| x_1 \| \| y_1 \|$.

Assume that elements $x_i, y_i \in F$ have been chosen such that

$$x_i y_j = 0 , \qquad i \neq j$$

and

$$\| \nu(x_i y_i) \| > i \| x_i \| \| y_i \| , \qquad 1 \leqq i, j \leqq n .$$

By Lemma 2.1, there exist idempotents e and f such that $\{x_1, \cdots, x_n, y_1, \cdots, y_n\} \subset eAf$. By [1, Th. 4.5], e and f are in F, and by Lemma 4.2, $F \subset I_L \cap I_R$. Now an idempotent is in I_L (I_R) if and only if the restriction of ν to the right (left) ideal it generates is continuous, so let L be the maximum of the norms of the continuous mappings $\nu \mid Ae$, $\nu \mid eA$, $\nu \mid Af$, $\nu \mid fA$, and let

$$K' = L^2(\| e \|^2 + \| f \|^2 + \| e \| \| f \|) .$$

If $x, y \in A$, then

$$\begin{aligned} \| \nu((x - xe)(y - fy)) \| &= \| \nu(xy) - \nu(xeey) - \nu(xffy) + \nu(xefy) \| \\ &\geqq \| \nu(xy) \| - \| \nu(xe) \| \| \nu(ey) \| \\ &\quad - \| \nu(xf) \| \| \nu(fy) \| - \| \nu(xe) \| \| \nu(fy) \| \\ &\geqq \| \nu(xy) \| - K' \| x \| \| y \| . \end{aligned}$$

By the preceding lemma, there exist $u, v \in F$ such that

$$\| \nu(uv) \| > \{(n + 1)(1 + \| e \|)(1 + \| f \|) + K'\} \| u \| \| v \| .$$

Let $x_{n+1} = u - ue$, $y_{n+1} = v - fv$. By the above, we have that

$$\begin{aligned} \| \nu(x_{n+1} y_{n+1}) \| &> (n + 1)(1 + \| e \|) \| u \| (1 + \| f \|) \| v \| \\ &\geqq (n + 1) \| x_{n+1} \| \| y_{n+1} \| . \end{aligned}$$

Also, $x_i y_{n+1} = x_i f(v - fv) = 0$, $x_{n+1} y_i = (u - ue)ey_i = 0$, $1 \leqq i \leqq n$, and x_{n+1}, $y_{n+1} \in F$.

Thus by induction there exist sequences $\{x_n\}$, $\{y_n\}$ such that $x_n y_m = 0$, $n \neq m$, and $\| \nu(x_n y_n) \| > n \| x_n \| \| y_n \|$, which contradicts Theorem 4.1.

REMARK 4.6. If $x \in I_L$, let $K(x)$ be the norm of the mapping $y \to \nu(xy)$. Then $\| \nu(xy) \| \leqq (K(x)/\| x \|) \| x \| \| y \|$, $y \in A$. The above theorem shows that $\{(K(x)/\| x \|) \mid x \in I_L\}$ is bounded.

The following corollary is an analog for annihilator algebras of a theorem by Badé and Curtis on homomorphisms of commutative, regular semi-simple Banach algebras [2, Th. 3.7]; it gives Theorem 5.1 of [1] as a special case.

COROLLARY 4.7. *Let A be a semi-simple annihilator Banach algebra, and let ν be a homomorphism of A into a Banach algebra. Then there exists a constant K such that.*

$$\| \nu(x) \| \leqq K \| x \| \| y \|$$

for all x and y in A such that $yx = x$ or $xy = x$.

Proof. If $yx = x$ or $xy = x$, then by [1, Corollary 4.12], $x \in F \subset I_L \cap I_R$.

DEFINITION 4.8. A Banach algebra A is said to have a bounded left (right) approximate identity if there exists a norm-bounded net $\{e_\alpha\} \subset A$ such that $e_\alpha x \to x$ $(xe_\alpha \to x)$ for all $x \in A$.

COROLLARY 4.9. *Let A be a semi-simple annihilator Banach algebra with a bounded left or right approximate identity, and let ν be a homomorphism of A into a Banach algebra. Then ν is continuous on the socle of A.*

Proof. Suppose that A has a bounded left approximate identity. Let $z \in F$. By Cohen's factorization theorem [6], there exists a constant L (independent of z) and elements x and y such that $z = xy$, $\| z - y \| \leqq \| z \|$, $\| x \| \leqq L$, and y is in the closed left ideal generated by z. By [1, Corollary 4.9], there exists an idempotent-generated left ideal, J, containing z. Since J is closed, we have $y \in J \subset F \subset I_R$. Thus if K is as in the above theorem, then

$$\| \nu(z) \| = \| \nu(xy) \| \leqq K \| x \| \| y \| \leqq KL(\| z \| + \| z - y \|) \leqq 2KL \| z \| .$$

We conclude this paper with several remarks:

REMARK 4.10. Let G be a compact topological group and let $A = L_p(G)$, $1 \leqq p < \infty$, or $C(G)$, with convolution for multiplication. Then Theorem 4.5 applies to A and the above corollary applies to $L_1(G)$. Here F is the set of trigonometric polynomials, that is, the set of linear combinations of component functions of strongly continuous irreducible unitary representations of G (see [10, p. 330]).

REMARK 4.11. If X is a reflexive Banach space, if F denotes the bounded operators on X of finite rank, and if $A \subset \mathfrak{B}(X)$ is a Banach algebra containing F as a dense subset, then Theorem 4.5 applies to A [10, pp. 102–104]. Here the socle of A is F.

If A is the uniform closure of F in $\mathfrak{B}(X)$, if A has a bounded left or right approximate identity, and if X has a continued bisection, then Johnson has shown that every homomorphism of A into a Banach algebra is actually continuous [8, Th. 3.5]. His theorem is stated for the algebra of compact operators on X (which may indeed always coincide with A), but his method of proof works equally well for A.[1]

REMARK 4.12. Although examples do exist of discontinuous homomorphisms of annihilator algebras (see [2, p. 597, p. 606] [3, p. 853], and [9]), it is still the case for these examples that $I_L = A$. One might conjecture that this is always true. As a small move in this direction, we show below that, in two special cases, I_L properly contains F the socle of A.

(1) Let $\mathfrak{M} = \{M_\lambda \mid \lambda \in \Lambda\}$ denote the minimal-closed two-sided ideals of A and suppose that $\mathfrak{M}$ forms an unconditional decomposition for A. Then $x \in A$ implies $x = \Sigma_\lambda x_\lambda$, where $x_\lambda \in M_\lambda$, and an equivalent Banach algebra norm for A is given by $|x| = \sup\{\|\sum_{\lambda\in\Lambda_1} x_\lambda\| : \Lambda_1$ is a finite subset of $\Lambda\}$. [1, pp. 231–232]. Thus $|x| = \sup_{\Lambda_1\subset\Lambda} |\sum_{\lambda\in\Lambda_1} x_\lambda|$. For $\mathfrak{N} \subset \mathfrak{M}$, let $A(\mathfrak{N})$ denote those x in A whose summands are all in $\mathfrak{N}$. If $\mathfrak{N}_1$ and $\mathfrak{N}_2$ are disjoint subsets of $\mathfrak{M}$, then $A(\mathfrak{N}_1)\cdot A(\mathfrak{N}_2) = (0)$. If $x \in A(\mathfrak{N})$ and $x \notin I_L$, then given K there exists $y \in A$ such that $\|\nu(xy)\| > K|x||y|$. Since removing the summands of y that are not in members of $\mathfrak{N}$ does not increase its norm and does not affect xy, we may assume that $y \in A(\mathfrak{N})$ as well. Thus if $\{\mathfrak{N}_n\}_{n=1}^\infty$ is any sequence of disjoint subsets of $\mathfrak{M}$, then Theorem 4.1 implies that $A(\mathfrak{N}_n) \subset I_L$ for all but finitely many n.

If A is strongly semi-simple, we can say a bit more. In this case, each $M \in \mathfrak{M}$ is finite-dimensional [1, Proposition 4.7]. Let $\mathscr{P}(\mathfrak{M})$ denote the set of subsets of $\mathfrak{M}$, let $\mathscr{F}$ denote the set of finite subsets of $\mathfrak{M}$, and let $[\mathfrak{N}]$ denote an element of the Boolean algebra

[1] See "Added in proof."

$\mathscr{P}(\mathfrak{M})/\mathscr{F}$. If $A(\mathfrak{N}) \subset I_L$, and $\mathfrak{N}_1 \in [\mathfrak{N}]$, then $\mathfrak{N}_1 \cap \mathfrak{N} \in \mathscr{F}$, so $A(\mathfrak{N}_1) \subset I_L$. Thus $\sum_{\mathfrak{N}_1 \in [\mathfrak{N}]} A(\mathfrak{N}_1) \subset I_L$. (Here "$\Sigma$" denotes the algebraic sum. Note that $F = \sum_{\mathfrak{N} \in \mathscr{F}} A(\mathfrak{N})$.) Let $\mathscr{J} = \{[\mathfrak{N}] \in \mathscr{P}(\mathfrak{M})/\mathscr{F} \mid A(\mathfrak{N}) \subset I_L\}$. Then $\mathscr{J}$ is an ideal in $\mathscr{P}(\mathfrak{M})/\mathscr{F}$. If $[\mathfrak{N}] \neq \mathscr{F}$, then there exists $\mathscr{F} \neq [\mathfrak{N}_1] \leqq [\mathfrak{N}]$ such that $[\mathfrak{N}_1] \in \mathscr{J}$: Otherwise, we could find a pairwise disjoint family $\{\mathfrak{N}_n\}$, with $A(\mathfrak{N}_n) \not\subset I_L$ for any n, which would contradict Theorem 4.1. But this says that the annihilator of $\mathscr{J}$ is $\mathscr{F}$, and thus $\mathscr{J}$ corresponds to a dense open set in the dual space of $\mathscr{P}(\mathfrak{M})/\mathscr{F}$, $\beta(\mathfrak{M}) - \mathfrak{M}$, where $\mathfrak{M}$ has the discrete topology[2] (see [7], pp. 76, 84, and 88). Since dividing by $\mathscr{F}$ in effect "mods out the socle", we see in this case that I_L is significantly larger than F.

(2) Suppose that A has proper involution $x \rightarrow x^*$ and that $I \oplus \mathfrak{R}(I)^* = A$ for all closed left ideals I. Let $\{e_\lambda \mid \lambda \in \Lambda\}$ be a maximal family of orthogonal hermitian idempotents. Then $x \in A$ implies $x = \Sigma_\lambda e_\lambda x = \Sigma_\lambda x e_\lambda$, and we may assume $\| x \| = \sup_{\Lambda_1 \subset \Lambda} \| \sum_{\lambda \in \Lambda_1} e_\lambda x \|$. [1, pp. 231-233]. For $\Lambda_1 \subset \Lambda$, let $A(\Lambda_1) = \{x \in A \mid x e_\lambda = 0, \lambda \in \Lambda_1\}$. If $x \in A(\Lambda_1)$ and $\| \nu(xy) \| > K \| x \| \| y \|$, let $y_1 = \sum_{\lambda \in \Lambda_1} e_\lambda y$. Then $xy_1 = xy$, $\| y_1 \| \leqq \| y \|$, and $x' y_1 = 0$ if $x' \in A(\Lambda_2)$ and $\Lambda_1 \cap \Lambda_2 = \varnothing$. Thus, given any sequence $\{\Lambda_n\}$ of disjoint subsets of Λ, Theorem 4.1 implies that $A(\Lambda_n) \subset I_L$ for all but finitely many n. (Of course there may exist $x \in F$ such that $x e_\lambda \neq 0$ for infinitely many λ, but clearly $A(\Lambda_1) \not\subset F$ if Λ_1 is infinite.) Since $\nu \mid A e_\lambda$ is continuous, remarks similar to those in the above paragraph can be made in this situation, with $\mathfrak{M}$ replaced by $\{A e_\lambda \mid \lambda \in \Lambda\}$.

Added in Proof. (continuation of Remark 4.11) If X is a Hilbert space and $A = \mathfrak{F}_1$, the algebra of trace class operators, or $\mathfrak{F}_2$, the algebra of Hilbert-Schmidt operators, then the methods of [8, Th. 3.3] can be adapted to show that $A^2 \subset I_L$. The statement in [8] that these methods imply continuity is in error. The following example (communicated to the author by Professor Johnson) illustrates this: If ν is a discontinuous linear functional on $\mathfrak{F}_2$ which vanishes on $\mathfrak{F}_2^2 (= \mathfrak{F}_1)$, then by defining zero multiplication in the complex numbers, one obtains a discontinuous homomorphism of $\mathfrak{F}_2$.

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