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CONICALLY BOUNDED SETS IN BANACH SPACES

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A condition on subsets of a Banach space E is introduced, intermediate to those of norm and linear boundedness, which depends in an essential way on the topological as well as the linear structure of E. It is shown that this notion, called conical boundedness, is a strictly weaker notion than that of boundedness in some Banach spaces (including infinite dimensional reflexive spaces and infinite dimensional Banach spaces with separable duals) and coincides with that of boundedness in others (including ℓ_1 and all finite dimensional spaces). After a discussion of some of the consequences of the condition of conical boundedness and a result on general structure of convex sets in reflexive spaces in terms of this notion, a construction is given which is valid in any nonreflexive Banach space and which yields two characterizations of reflexive Banach spaces. The first is in terms of (the nonexistence of) certain nonconically bounded convex sets, and the other descibes nonreflexive spaces via the restriction of any nonzero continuous linear functional to the unit balls of equivalent norms.

This latter result was first proved by Klee although his proof differs significantly from ours.

Since the notions of weak and norm boundedness in a Banach space coincide (and coincide with that of weak* boundedness if it is a dual Banach space) these seemingly disparate notions give, in fact, only one handle on the size of a set in such a space. Moreover since the condition of linear boundedness fails to take the topological structure into account, conical boundedness provides a proper *topological* linear space relaxation of boundedness.

I am very much indebted to Professor Z. Zielezny for many stimulating conversations on the subject of this paper. (In particular a specific example of his contained the ideas involved in the proof of Proposition 13.) I also wish to thank Professor V. Drobot who was instrumental in the development of Proposition 7, and the referee for many helpful suggestions and especially for his "cleaning up" the construction.

We begin with a definition of conical boundedness.

1. DEFININION. Let B be a Banach space and $x \in B$. Let $\mathscr{C}(x) = \{C \subset B\colon C$ is norm closed bounded and convex and $x \notin C\}$ and for each C in $\mathscr{C}(x)$ denote by $K_x(C)$ the cone over C with vertex x. (That is, $K_x(C) = \{tc + (1-t)x\colon t \geqq 0$ and $c \in C\}$.) A set $D \subset B$ is said to

be conically bounded at x if $D \cap K_x(C)$ is a bounded set for each $C \in \mathscr{C}(x)$. The set D is conically bounded if there is a point $x \in B$ such that D is conically bounded at x.

When $x = 0$ we will henceforth write $K(C)$ in place of $K_0(C)$.

The first result concerns the role of the vertex x of the cones in Definition 1.

2. PROPOSITION. *A set D is conically bounded at one point if and only if it is conically bounded at any other point. Hence D is conically bounded if and only if it is conically bounded at* 0.

Proof. By symmetry it suffices to show that if D is conically bounded at 0 then it is conically bounded at $x \neq 0$. This will be accomplished by proving that whenever $C \in \mathscr{C}(x)$ there is a set $C_1 \in \mathscr{C}(0)$ for which $K_x(C)\backslash K(C_1)$ is bounded. Let $d_1 = \inf\{||c - x||: c \in C\}$ and $t = 3d_1^{-1}||x||$. Then the set $C' = \{tc + (1 - t)x: c \in C\}$ is in $\mathscr{C}(x)$, $K_x(C') = K_x(C)$, and $\inf\{||c'||: c' \in C'\} \geqq 2||x||$. Let C_1 be the closed convex hull of $C' \cup (C' - x)$. Note first that if $0 \leqq r \leqq 1$ and $c_1, c_2 \in C'$ then

$$\begin{aligned} ||r(c_1 - x) + (1 - r)c_2|| &\geqq ||rc_1 + (1 - r)c_2|| - ||x|| \\ &\geqq 2||x|| - ||x|| = ||x|| > 0 \end{aligned}$$

so that $0 \notin C_1$. Next let $d_2 = \sup\{||c' - x||: c' \in C'\}$. Then if $y \in K_x(C')$ and $||y - x|| > d_2$, there is a number s, $0 < s < 1$, such that $sy + (1 - s)x \in C'$. Since

$$sy = s(sy + (1 - s)x) + (1 - s)(sy + (1 - s)x - x)$$

the point sy is a convex combination of a point of C' and one of $C' - x$ so that $sy \in C_1$ and $y \in K(C_1)$. It follows that $K_x(C)\backslash K(C_1) = K_x(C')\backslash K(C_1) \subseteqq \{y: ||y - x|| \leqq d_2\}$ which completes the proof.

Later results provide large classes of examples of conically bounded sets (and of nonconically bounded linearly bounded convex sets) but the following elementary examples point out the strong dependence of this notion on the topological structure of the underlying Banach space.

3. EXAMPLE. Let B be any of the Banach spaces c_0 or l_p, $1 \leqq p < \infty$. Let $e_n = (0, 0, \cdots, 1, 0, \cdots)$ be the point of B with one in the nth place and 0 elsewhere and let $D = \{ne_n: n = 1, 2, \cdots\}$.

First of all suppose that B is c_0 or l_p, $1 < p < \infty$. If $C \in \mathscr{C}(0)$

assume that $K(C) \cap D = \{ne_n: n \in A\}$ and that A is infinite. For each $n \in A$ choose any positive number d_n for which $d_n e_n \in C$ (so that $\{d_n: n \in A\}$ is bounded). Since $\{e_n\}_{n=1}^{\infty}$ converges in the weak topology on B to 0 so does $\{d_n e_n\}_{n \in A}$ and hence $0 \in C$. From this contradiction we conclude that $K(C) \cap D$ is finite and hence D is an unbounded, conically bounded set in c_0 and l_p, $1 < p < \infty$.

If $B = l_1$ then $D \subset K(C)$ where $C \in \mathscr{C}(0)$ is the set $C = \{(x_i) \in l_1: x_i \geqq 0$ for each i and $\sum_{j=1}^{\infty} x_j = 1\}$. Hence D is not conically bounded as a subset of l_1. (See Proposition 7 and the discussion following it.)

Among the operations which preserve bounded sets, many also preserve conically bounded ones although there are some important differences. (See Example 6). The next Proposition gives several such properties as well as an elementary but useful criterion on sets equivalent to that of conical boundedness. We will first need some notation.

4. NOTATION. Let B be a Banach space, $\varepsilon > 0$ and f any non-zero element of B^*. Let

$$C(f; \varepsilon) = \{x \in B: \|x\| \leqq 1 \quad \text{and} \quad f(x) \geqq \varepsilon\}$$

and

$$K(f; \varepsilon) = K(C(f; \varepsilon)) .$$

Furthermore, if $A \subset B$ let $\operatorname{conv} A$ [respectively, $\overline{\operatorname{conv}}\, A$] denote the convex hull [respectively, closed convex hull] of A, and $\overline{\operatorname{sp}}\, A$ the closed linear span of A.

5. PROPOSITION. *Let B be a Banach space. The following statements are all concerned with subsets of B.*

(a) *Any subset of a conically bounded set is conically bounded.*

(b) *Finite unions of conically bounded sets are conically bounded.*

(c) *D is conically bounded if and only if each of its countably infinite unbounded subsets is conically bounded.*

(d) *If D is conically bounded, $t \in \boldsymbol{R}$, and $x_0 \in B$, then $tD + x_0$ is conically bounded.*

(e) *If D is conically bounded, $x_0 \in B$, and N any real number, then $\{tx + (1 - t)x_0: t$ is between 0 and N and $x \in D\}$ is conically bounded.*

(f) *If D is conically bounded and D_1 is of finite distance from D then D_1 is conically bounded. In particular the closure of a conically bounded set is conically bounded. (D_1 is of finite distance from D if there is a number M for which $\inf\{\|x - y\|: y \in D\} \leqq M$ for each*

x in D_1).

(g) *The closed convex hull of a bounded set and a conically bounded convex set is conically bounded.*

(h) *D is conically bounded if and only if $D \cap K(f; \varepsilon)$ is bounded for each $\varepsilon > 0$ and nonzero f in B^*.*

Proof. Statements (a)—(e) follow directly from Definition 1 and, perhaps, an application of Proposition 2.

(f): For $C \in \mathscr{C}(0)$ define C_1 in $\mathscr{C}(0)$ by $C_1 = \{3Md^{-1}c: c \in C\}$ where M is as in the statement of part (f) and $d = \inf\{||c||: c \in C\} > 0$. Let $C_2 = \{x \in B: \inf\{||x - y||: y \in C_1\} \leqq 2M\}$ so that $C_2 \in \mathscr{C}(0)$. If $x \in D_1 \cap K(C)$ either $||x|| \leqq N_1 = \sup\{||c||: c \in C_1\}$ or, if $||x|| > N_1$ there is a number t, $0 < t < 1$, such that $tx \in C_1$. But by hypothesis there is a point y in D with $||x - y|| < 2M$ so that $||tx - ty|| = t||x - y|| < 2M$. That is, $ty \in C_2$ and hence $y \in K(C_2)$. Since $D \cap K(C_2)$ is bounded (say $\sup\{||z||: z \in D \cap K(C_2)\} = N_2$) we conclude that $||x|| \leqq ||y|| + 2M \leqq N_2 + 2M$. Hence, in either case, if $x \in K(C) \cap D$ then $||x|| \leqq \max(N_1, N_2 + 2M)$ which completes the proof of this part.

(g): Let A be a bounded set and D a conically bounded convex set. For any point x_0 in D let $d = \sup\{||x_0 - y||: y \in A\}$. It is evident that the convex set $D_1 = \{z: \inf\{||z - x||: x \in D\} \leqq d\}$ contains $D \cup A$. Since D_1 is closed and convex it contains $\overline{\text{conv}}(D \cup A)$. Finally, D_1 is conically bounded (by part (f)) and hence so is $\overline{\text{conv}}(D \cup A)$ (by part (a)).

(h): For any set C in $\mathscr{C}(0)$ choose $f \in B^*$ and $\delta > 0$ such that $0 < \delta < \inf\{f(x): x \in C\}$. Let $M = \sup\{||c||: c \in C\}$ and let $\varepsilon = M^{-1}\delta$. If $x \in K(C)$ then $tx \in C$ for some $t > 0$ and hence $||M^{-1}tx|| \leqq 1$. But $f(M^{-1}tx) = M^{-1}f(tx) \geqq \varepsilon$ so $x \in K(f; \varepsilon)$. Thus $K(C) \subseteqq K(f; \varepsilon) = K(C(f; \varepsilon))$ where $C(f; \varepsilon)$ is obviously in $\mathscr{C}(0)$. This proves part (h).

6. EXAMPLE. The hypotheses of Proposition 5, part (g) are not superfluous as the following example (of a conically bounded set whose convex hull is not conically bounded) demonstrates.

Let B be the Banach space c_0 and for $n = 1, 2 \cdots$ let $e_n \in c_0$ denote the n^{th} unit vector (as in the previous Example 3). Then the set $D = \{n(e_1 \pm ne_n); n = 1, 2 \cdots\}$ is conically bounded. In fact if $0 \neq f = (f_i) \in \mathscr{L}_1 = c_0^*$ and $\varepsilon > 0$, clearly $\{n: (1 + n)^{-1}f(e_1 \pm ne_n) \geqq \varepsilon\}$ is finite since $\lim_{i\to\infty} f_i = 0$. From $n(e_1 \pm ne_n)/||n(e_1 \pm ne_n)|| = (1 + n)^{-1}(e_1 \pm ne_n)$ we conclude that there are only finitely many n for which either $n(e_1 + ne_n)$ or $n(e_1 - ne_n)$ belongs to $K(f; \varepsilon)$. It follows from Proposition 5 part (h) that D is conically bounded. On the other hand $1/2[n(e_1 + ne_n)] + 1/2[n(e_1 - ne_n)] = ne_1 \in K(\{e_1\})$ so that conv (D) is not conically bounded.

We come now to a characterization of those Banach spaces in which each conically bounded set is bounded.

7. PROPOSITION. *In a Banach space B the following conditions are equivalent:*

(a) *Each conically bounded subset of B is bounded;*

(b) *Weak and norm convergences of sequence coincide.*

Proof. not (b) $\Rightarrow$ not (a): If there is a sequence $\{x_n\}$ which converges weakly to x and yet for each n we have $\|x_n - x\| > \delta$ for some $\delta > 0$ then taking $y_n = \|x_n - x\|^{-1}(x_n - x)$, the sequence $\{y_n\}$ converges weakly to 0 and each y_n has norm one. For any positive ε and nonzero $f \in B^*$ certainly $ny_n \in K(f; \varepsilon)$ implies that $y_n \in C(f; \varepsilon)$ and this can happen for at most finitely many n by choice of the sequence $\{y_n\}$. Thus $\{ny_n\} \cap K(f; \varepsilon)$ is finite (hence bounded) and $\{ny_n\}$ is thus an unbounded, conically bounded set.

not (a) $\Rightarrow$ not (b): If there is a conically bounded but unbounded set in B, it contains a countable conically bounded subset $\{x_n\}_{n=1}^{\infty}$ with $\|x_n\| \geqq n$ for each n. Thus $\{x_n\} \cap K(f; \varepsilon)$ must be finite for each $\varepsilon > 0$ and $f \in B^*$, $f \neq 0$, so that $f(\|x_n\|^{-1}x_n) < \varepsilon$ for all but finitely many n. That is, for each $f \in B^*$ we have $\limsup f(\|x_n\|^{-1}x_n) \leqq 0$ and hence $\{\|x_n\|^{-1}x_n\}_{n=1}^{\infty}$ converges weakly to 0. Since each point of this sequence has norm one, the proof is complete.

From Proposition 7 it follows that in finite dimensional spaces and in l_1 (see for example [2, Cor. 2, p. 33]) every conically bounded set is bounded, while every Banach space which has an infinite dimensional subspace with separable dual (and hence each infinite dimensional reflexive space) contains an unbounded, conically bounded subset.

The next result (and Corollary) provide a simple but descriptive restriction on the convex subsets of reflexive spaces.

8. PROPOSITION. *Let E be a reflexive Banach space and $W \subset E$ a closed convex set which contains no (infinite) rays. Then W is conically bounded.*

Proof. Since W is conically bounded if and only if $W - y$ is (for any point $y \in W$) we assume without loss of generality that $0 \in W$. Then it suffices to show that $K(f; \varepsilon) \cap W$ is bounded whenever $f \in B^*$, $f \neq 0$ and $\varepsilon > 0$. If $\{x_n: n = 1, 2, \cdots\}$ lies in this intersection and $\|x_n\| \geqq n$ for each n then $\{\|x_n\|^{-1}x_n: n = 1, 2, \cdots\}$ is a sequence in $C(f; \varepsilon)$, a weakly compact set, so there is a point x_0 in $C(f; \varepsilon)$ which is a cluster point of the sequence. (Of course $x_0 \neq 0$ since $f(x_0) \geqq \varepsilon$.) For any integer N the point Nx_0 is a cluster point of the sequence $\{N\|x_n\|^{-1}x_n: n = N, N+1, \cdots\}$ and since W is convex, contains x_n and 0 and $N\|x_n\|^{-1} \leqq 1$ for $n \geqq N$, the sequence $\{N\|x_n\|^{-1}x_n: n = N, N+1, \cdots\}$ lies in W. Since W is (weakly) closed the point Nx_0 must belong to W for each positive integer N. That is, W contains the ray $\{tx_0: t \geqq 0\}$, a contradiction which completes the proof.

9. COROLLARY. *A convex set in a reflexive Banach space is conically bounded if an only if its closure contains no rays.*

Proof. If W is conically bounded then by Proposition 5, part (f) so is its closure, $\mathrm{cl}(W)$. But it is evident that no set containing a ray can be conically bounded, so in fact $\mathrm{cl}(W)$ contains no rays. The converse follows from Proposition 9, and from Proposition 5 part (a).

Proposition 8 shows that, in particular, if W is any linearly bounded convex body which is symmetric about 0 in a reflexive Banach space E then W is conically bounded. (By 'convex body' we mean 'closed convex set with nonempty interior'.) Such sets are the unit balls of continuous norms on E, and the construction presented below shows that no such statement is possible for nonreflexive Banach spaces. In fact, we will prove somewhat more.

10. PROPOSITION. *Let B be a nonreflexive Banach space and $0 \neq f \in B^*$. Then there is a continuous norm for B whose unit ball W satisfies: $W \cap K(f;\varepsilon)$ is unbounded for an appropriate $\varepsilon > 0$. (Hence W is not conically bounded.)*

The proof of this proposition is a corollary of the construction. First, recall that a sequence $(x_i)_{i=1}^{\infty}$ is a *basic sequence* in a Banach space if to each point $x \in \overline{\mathrm{sp}}\,(x_i)_{i=1}^{\infty}$ there corresponds a unique sequence $(f_i(x))_{i=1}^{\infty}$ of real numbers such that $x = \sum f_i(x)x_i$. It may be shown that the functionals f_i so defined (called the *associated biorthogonal functionals*) are continuous linear functionals on $\overline{\mathrm{sp}}\,(x_i)_{i=1}^{\infty}$ and

$$\sup\{\|x_n\|\,\|f_n\|: n = 1, 2, \cdots\} < \infty\ .$$

(See [7, p. 1, p. 17, p. 23, and Th. 3.1, p. 20] for details.

11. **The construction.** Let B be a nonreflexive Banach space and $0 \neq f \in B^*$. Note that $f^{-1}(0)$, being of codimension one in B, must itself be nonreflexive. It follows from a result of Pełczynski [5, Th. 2, p. 374] that there is a basic sequence $(y_i)_{i=1}^{\infty}$ with $0 < \inf\|y_i\| \leqq \sup\|y_i\| \leqq M < \infty$ for $\overline{\mathrm{sp}}\,(y_i)_{i=1}^{\infty}$ and $g_1 \in [\overline{\mathrm{sp}}\,(y_i)]^*$ for which $\limsup|g_1(y_i)| \neq 0$. By passing to a subsequence if necessary and by

possibly replacing g_1 by $-g_1$ one finds an $\varepsilon > 0$ and a subsequence (y_i') of (y_i) for which $g_1(y_i') \geqq \varepsilon$ for $i = 1, 2, \cdots$. Let $x_i = \varepsilon(Mg_1(y_i'))^{-1}y_i'$ and $g = M\varepsilon^{-1}g_1$. Then $(x_i)_{i=1}^{\infty}$ is a basis for $\overline{\mathrm{sp}}\,(x_i)_{i=1}^{\infty}$ with $\|x_i\| \leqq 1$ for each i, $g \in [\mathrm{sp}\,(x_i)_{i=1}^{\infty}]^*$, and $g(x_i) = 1$ for $i = 1, 2, \cdots$.

Choose $x_0 \in B$ such that $f(x_0) > 0$ and note that $(x_i)_{i=0}^{\infty}$ is also a basic sequence. Let $(f_i))_{i=0}^{\infty}$ be the associated sequence of biorthogonal functionals (in $[\overline{\mathrm{sp}}\,(x_i)_{i=0}^{\infty}]^*$). Furthermore, let $\tilde{f}_i \in B^*$ denote any Hahn-Banach extension of f_i (so that $\|f_i\| = \|\tilde{f}_i\|$) and choose an extension $\tilde{g} \in B^*$ of g so that $\tilde{g}(x_0) = -1$. Finally, denote by q the natural quotient map $q: B \to B/\overline{\mathrm{sp}}(x_i)_{i=0}^{\infty}$.

It is evident that

$$p(x) = |\tilde{g}(x)| + \sup_{n=1,2,\cdots} |n^{-1}\tilde{f}_n(x)| + \|q(x)\|$$

defines a seminorm on B. Since $\inf\{\|x_i\|: i = 1, 2, \cdots\} > 0$ (since $\tilde{g}(x_i) = 1$ for each such i) and $\sup\{\|x_i\|\,\|f_i\|: i = 1, 2, \cdots\} < \infty$, we conclude that $\lim_{n\to\infty}(1/n)\|\tilde{f}_n\| = 0$. Consequently p is continuous.

To see that p is in fact a norm, suppose that $x \neq 0$. If $x \notin \overline{\mathrm{sp}}\,(x_i)_{i=0}^{\infty}$ then $q(x) \neq 0$ and thus $p(x) \neq 0$. If $x \in \overline{\mathrm{sp}}\,(x_i)_{i=0}^{\infty}$ then $x = \sum_{i=0}^{\infty} \tilde{f}_i(x)x_i$. If $\tilde{f}_n(x) \neq 0$ for some $n \geqq 1$ then $|n^{-1}\tilde{f}_n(x)| \neq 0$ and hence $p(x) \neq 0$. Otherwise $0 \neq x = \tilde{f}_0(x)x_0$. Consequently $\tilde{g}(x) = -\tilde{f}_0(x) \neq 0$ and again $p(x) \neq 0$. This completes the construction.

Proof of Proposition 10. With the notation developed in the Construction above, let $W = \{x: p(x) \leqq 1\}$. Then the set $S = \{n(x_0 + x_n): n = 1, 2, \cdots\}$ is unbounded and $S \subset W \cap K(f; f(x_0)/(\|x_0\| + 1))$ since

$$f(n(x_0 + x_n)) = nf(x_0) \geqq \frac{f(x_0)}{\|x_0\| + 1}\|n(x_0 + x_n)\|$$

for each $n \geqq 1$, thus completing the proof.

Propositions 8 and 10 combine to yield the following characterization of reflexive Banach spaces.

12. COROLLARY. *A Banach space is reflexive if and only if the unit ball of each continuous norm is conically bounded. That is, B is reflexive if and only if each linearly bounded closed convex body symmetric about* 0 *is conically bounded.*

Turning now to our second characterization of reflexive Banach spaces, recall that Bishop and Phelps [1] proved that for any Banach space B, the collection of elements of B^* which attain their norm is a norm dense subset of B^*, while James [3] proved that if each $f \in B^*$ attains its norm then B is reflexive. A natural question related to these results is: What linear functionals on a Banach space B attain their norms for each equivalent norm on B. If B is reflexive then of course each $f \in B^*$ attains its norm for each equivalent norm. The complete answer for nonreflexive spaces is given in Proposition 13. We wish to thank Professor R. R. Phelps for informing us that Klee [4, Th. 1, p. 16] first proved this result (by different methods).

13. PROPOSITION. *A Banach space B is reflexive if and only if there is a nonzero element of B^* which attains its norm for each equivalent norm on B.*

Proof. Suppose that (B, n) is not reflexive and $0 \neq f \in B^*$. Choose $x_0 \in B$ for which $f(x_0) = 1$ and let

$$||x|| = \max\{n(x - f(x)x_0), |f(x)|\} .$$

It is easy to check that $||\cdot||$ is an equivalent norm on B and that $||x_0|| = ||f|| = f(x_0) = 1$. Define $(x_i)_{i=1}^{\infty}$, $(\tilde{f}_i)_{i=0}^{\infty}$, $\tilde{g}$ and p as in Construction 11, and note that $p(x_0 + x_k) = k^{-1}$ and $||x_0 + x_k|| = 1$ (since $n(x_k) \leqq 1$ for $k \geqq 1$).

Let $|||x||| = p(x) + ||x||$. Then $|||\cdot|||$ is an equivalent norm for B and we now show that f does not attain its $|||\cdot|||$ norm. If $|||x||| \leqq 1$ then $||x|| < 1$ and hence $|f(x)| < 1$. But for $k \geqq 1$ we have $1 = f(x_0 + x_k)$ and

$$|||x_0 + x_k||| = p(x_0 + x_k) + ||x_0 + x_k|| = k^{-1} + 1 .$$

It follows that $|||f||| = 1$, which completes the proof.

14. REMARKS. (1). A notion bearing some resemblance to conical boundedness has been implicit in some work of Phelps [6]. His condition and ours are quite different, though, since he only requires boundedness in the direction of some $K(f; \varepsilon)$ while we require boundedness in the direction of each $K(f; \varepsilon)$.

(2). The definition of conical boundedness evidently carries over without change to general topological vector spaces, and thus a spectrum of problems are immediately raised.

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