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# ATRIODIC HOMOGENEOUS CONTINUA

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*Dedicated to Professor F. Burton Jones*

**In answer to a question of T. Mackowiak and E. D. Tymchatyn [20] we prove that every atriodic homogeneous continuum is 1-dimensional. This is accomplished by showing that every atriodic homogeneous continuum that is not a solenoid and has a decomposable subcontinuum admits a continuous decomposition to a solenoid and that all elements of this decomposition are homeomorphic tree-like hereditarily indecomposable homogeneous continua. It follows from this decomposition theorem that every tree-like atriodic homogeneous continuum is hereditarily indecomposable. This decomposition theorem also provides another proof of the author's theorem [11] that every indecomposable homogeneous plane continuum is hereditarily indecomposable.**

**1. Introduction.** A space is *homogeneous* if for each pair $p$, $q$ of its points there is a homeomorphism of the space onto itself that takes $p$ to $q$. A *continuum* is a compact connected nondegenerate metric space. A continuum $M$ is a *triod* if $M$ has a subcontinuum $H$ such that $M \setminus H$ is the union of three nonempty disjoint opens sets. When a continuum does not contain a triod it is *atriodic*. A continuum $M$ is *tree-like* if for each positive number $\varepsilon$ there is an open covering of $M$ with mesh less than $\varepsilon$ whose nerve is a tree. A continuum is *decomposable* if it is the union of two of its proper subcontinua; otherwise, it is *indecomposable*. When a continuum does not have a decomposable subcontinuum it is *hereditarily indecomposable*. Note that every hereditarily indecomposable continuum is atriodic.

In 1951 R. H. Bing [2] proved that every finite-dimensional hereditarily indecomposable homogeneous continuum is 1-dimensional. Recently J. T. Rogers, Jr. [25] proved that every hereditarily indecomposable homogeneous continuum is tree-like and, therefore, 1-dimensional. Mackowiak and Tymchatyn [20, Theorem 13.4] proved that every finite-dimensional atriodic homogeneous continuum is 1-dimensional. In §13 of [20], Mackowiak and Tymchatyn asked if every atriodic homogeneous continuum is 1-dimensional. Corollary 1 of §4 (below) answers this question in the affirmative.

Our arguments involve a decomposition theory for homogeneous continua that was originated in 1951 by F. B. Jones [15]. Recently Rogers [24] surveyed this area and presented a general decomposition theory for homogeneous spaces. Theorem 2 of §4 (below) solves a problem of Jones' that Rogers [**24**, page 142] called the outstanding problem in decompositions of homogeneous continua.

Mackowiak and Tymchatyn [**20**, Theorem 14.7] proved that every decomposable atriodic homogeneous continuum that is not a simple closed curve has a continuous decomposition to a circle and that the elements of this decomposition are homeomorphic indecomposable homogeneous continua. In §14 of [**20**] they asked if every atriodic homogeneous continuum that is not a solenoid and has a decomposable subcontinuum admits a continuous decomposition to a solenoid such that all elements of the decomposition are homeomorphic tree-like hereditarily indecomposable homogeneous continuum. Theorem 2 of §4 (below) answers this question in the affirmative.

Bing [**4**, Theorem 10] proved that no tree-like atriodic homogeneous continuum contains an arc. Mackowiak and Tymchatyn [**20**, Theorem 14.8] generalized Bing's theorem by proving that no tree-like atriodic homogeneous continuum has a hereditarily decomposable subcontinuum. According to Corollary 2 of §4 (below), no tree-like atriodic homogeneous continuum has a decomposable subcontinuum.

The known examples of atriodic homogeneous continua are the solenoids [**12**], the pseudo-arc [**1**], and the solenoids of pseudo-arcs [**23**] [**13**]. By Rogers' theorem [**25**] and Theorem 2 of §4 (below), if every tree-like homogeneous continuum is a pseudo-arc, then there are no other examples of atriodic homogeneous continua. Unfortunately, it is not known whether the pseudo-arc is the only tree-like continuum that is homogeneous. For additional information and unsolved problems involving 1-dimensional homogeneous continua see C. E. Burgess' expository article [**7**].

**2. More definitions and related results.** A *chain* is a finite collection $\{L_i\colon 1 \leq i \leq n\}$ of open sets such that $L_i \cap L_j \neq \varnothing$ if and only if $|i-j| \leq 1$. If $L_1$ also intersects $L_n$ the collection is called a *circular chain*. Each $L_i$ is called a *link*. A chain (circular chain) is called an *$\varepsilon$-chain* (*$\varepsilon$-circular chain*) if each of its links has diameter less than $\varepsilon$. A continuum is *arc-like* (*circle-like*) if for each positive number $\varepsilon$, it can be covered by an $\varepsilon$-chain ($\varepsilon$-circluar chain). Bing [**1**] [**3**] proved that a continuum is a pseudo-arc if and only if it is homogeneous and arc-like.

A continuum is a *solenoid* if it is homeomorphic to an inverse limit of circles with covering maps as the bonding maps. Note that simple closed curves are solenoids. The author [**12**] proved that a continuum $M$ is a solenoid if and only if $M$ is homogeneous and every proper subcontinuum of $M$ is an arc. Rogers [**24**, Theorem 3] proved that every atriodic homogeneous 1-dimensional continuum that contains an arc is a solenoid. In [**20**, Theorem 14.8], Mackowiak and Tymchatyn generalized these results by showing that every atriodic homogeneous continuum that contains a hereditarily decomposable continuum is a solenoid.

A continuum $M$ is a *solenoid of pseudo-arcs* if $M$ is circle-like and there exists a continuous decomposition $\mathfrak{D}$ of $M$ to a solenoid such that each element of $\mathfrak{D}$ is a pseudo-arc. In 1959 Bing and Jones [**5**] constructed the circle of pseudo-arcs. Rogers [**23**] used this continuum to construct a solenoid of pseudo-arcs for each solenoid. The author and Rogers [**13**] proved that every circle-like homogeneous continuum is either a solenoid, a pseudo-arc, or a solenoid of pseudo-arcs.

Following K. Kuratowski [**18**] we define a continuum $M$ to be of *type* $\lambda$ if $M$ is irreducible and every indecomposable continuum in $M$ is a continuum of condensation. Type $\lambda$ continua are studied by E. S. Thomas in [**26**]. There they are called continua of type $A'$.

If a continuum $M$ is of type $\lambda$, then $M$ admits a unique monotone upper semi-continuous decomposition $\mathfrak{D}$ such that $M/\mathfrak{D}$ is an arc and each element of $\mathfrak{D}$ has a void interior relative to $M$ [**19**, Theorem 3, page 216] [**26**, Theorem 10, page 15]. We shall refer to $\mathfrak{D}$ as simply the decomposition of $M$.

## 3. Preliminaries.

Throughout this section $M$ is an atriodic homogeneous continuum with metric $\rho$.

Let $\varepsilon$ be a positive number. A homeomorphism $h$ of $M$ onto $M$ is called an *$\varepsilon$-homeomorphism* if $\rho(x, h(x)) < \varepsilon$ for each point $x$ of $M$.

NOTATION. Let $x$ be a point of $M$. We denote $\{y \in M$: an $\varepsilon$-homeomorphism of $M$ onto $M$ takes $x$ to $y\}$ by $W(x, \varepsilon)$. Let $X$ be a subset of $M$. We denote $\bigcup\{W(x, \varepsilon): x \in X\}$ by $W(X, \varepsilon)$.

LEMMA 1. *For every positive number $\varepsilon$ and every point $x$ of $M$, the set $W(x, \varepsilon)$ is open in $M$.*

*Proof.* Lemma 1 follows from a short argument [**10**, Lemma 4, proof] involving E. G. Effros' topological transformation group theorem [**8**, Theorem 2.1].

A continuum is *unicoherent* provided that if it is the union of two subcontinuum $H$ and $K$, then $H \cap K$ is connected.

LEMMA 2. *Every proper subcontinuum of $M$ is unicoherent* [**20**, *Theorem* 13.8].

In the remainder of this section we assume there is a continuum $E$ of type $\lambda$ in $M$. Let $k$: $E \to [0, 1]$ be a quotient map associated with the decomposition of $E$. We call $k^{-1}(0)$ and $k^{-1}(1)$ the *end sets* of $E$.

LEMMA 3. *Let $Y$ be an element of the decomposition of $E$ distinct from $k^{-1}(0)$ and $k^{-1}(1)$. Let $F$ be a type $\lambda$ subcontinuum of $M$ with ends $T$ and $V$, and let $U$ be an element of the decomposition of $F$ distinct from $T$ and $V$. Suppose $h$ is a homeomorphism of $M$ onto $M$ such that $U \cap h[Y] \neq \varnothing$ and $U \cap h[k^{-1}(0) \cup k^{-1}(1)] = \varnothing = h[Y] \cap (T \cup V)$. Then $h[Y] = U$.*

*Proof.* Lemma 3 follows from the argument given in paragraphs 9 through 11 in the proof of Theorem 1 of [**10**].

A subcontinuum $F$ of $M$ is called an *extension* of $E$ away from $k^{-1}(0)$ if $F$ is a continuum of type $\lambda$ that contains $E$ and has $k^{-1}(0)$ as an end set.

NOTATION. We denote the set of all extensions of $E$ away from $k^{-1}(0)$ by $\mathfrak{S}(k^{-1}(0), E)$.

LEMMA 4. *The set $\mathfrak{S}(k^{-1}(0), E)$ is linearly ordered by inclusion and does not have a maximal element* [**11**, *Lemma* 4].

LEMMA 5. *The decomposition of each continuum of type $\lambda$ in $M$ is continuous* [**11**, *Lemma* 5].

A continuum $H$ in $E$ is an *essential subcontinuum* of $E$ if $H$ intersects more than one element of the decomposition of $E$.

LEMMA 6. *If $H$ is an essential subcontinuum of $E$, then $H$ is a continuum of type $\lambda$ and every element of the decomposition of $H$ is an element of the decomposition of $E$.*

*Proof.* Lemma 6 follows from Lemma 5 and the irreducibility properties of $E$ [**26**, Theorem 8, page 14].

LEMMA 7. *Suppose $F$ is a continuum of type $\lambda$ in $M$ such that $E \setminus F \neq \varnothing \neq F \setminus E$. Suppose $F \cap k^{-1}(r) \neq \varnothing$ for some number $r$ $(0 \leq r \leq 1)$. Then $k^{-1}(r)$ is an element of the decomposition of $F$.*

*Proof.* By Lemma 4, there is a continuum $H$ of type $\lambda$ in $M$ such that

(1) $E$ is an essential subcontinuum of $H$ that misses both end sets of $H$.

Let $I$ be a continuum of type $\lambda$ in $M$ such that $F$ is an essential subcontinuum of $I$ that misses both end sets of $I$.

Observe that

(2) $k^{-1}(r) \cap (I \setminus F) = \varnothing$.

To see this note that since $F \setminus E \neq \varnothing$, the continuum $F$ is not in $k^{-1}(r)$. Since $M$ is atriodic, it follows from (1) that there is a point $x$ of $F$ in $H \setminus k^{-1}(r)$. Let $J$ be the continuum of type $\lambda$ in $H$ such that $x$ belongs to one end set of $J$ and the other end set of $J$ is $k^{-1}(r)$. By Lemma 2, $F \cap J$ is a subcontinuum of $J$.

Suppose that (2) is false. Then $F \cap J$ does not contain $k^{-1}(r)$. Hence $F \cap J$ is a proper subcontinuum of $J$. Since $x \in F \cap J$ and $F \cap J$ intersects $k^{-1}(r)$, this contradicts the fact that $J$ is irreducible between $x$ and $k^{-1}(r)$. Hence (2) is true.

Since $M$ is atriodic and $F \cap k^{-1}(r) \neq \varnothing$, it follows from (2) that $k^{-1}(r) \subset F$. Therefore, since $M$ is atriodic and $E \setminus F \neq \varnothing$, there is a point $y$ of $E \setminus k^{-1}(r)$ in $I \setminus F$.

Let $K$ be the continuum of type $\lambda$ in $E$ such that $y$ belongs to one end set of $K$ and the other end set of $K$ is $k^{-1}(r)$. Let $L$ be a subcontinuum of $I$ that is irreducible between $y$ and $k^{-1}(r)$. It follows from Lemma 2 and the irreducibility of $K$ and $L$ that $K = K \cap L = L$. Hence $L$ is a continuum of type $\lambda$ and $k^{-1}(r)$ is an element of the decomposition of $L$. Since $k^{-1}(r) \subset F$ and $y \in I \setminus F$, the continuum $L$ is an essential subcontinuum of $I$. By Lemma 6, $k^{-1}(r)$ is an element of the decomposition of $I$. Since $F$ is an essential subcontinuum of $I$ and $k^{-1}(r) \subset F$, it follows from Lemma 6 that $k^{-1}(r)$ is an element of the decomposition of $F$.

LEMMA 8. *Suppose $F$ is a continuum of type $\lambda$ in $M$ that intersects $E$ and misses either $k^{-1}(0)$ or $k^{-1}(1)$. Then $\bigcup(\mathfrak{S}(k^{-1}(0), E) \cup \mathfrak{S}(k^{-1}(1), E))$ contains $F$.*

*Proof*. The conclusion follows immediately if $E$ contains $F$. Therefore we assume that $F \not\subset E$. Assume without loss of generality that $F$ misses $k^{-1}(0)$. By Lemma 7, for each number $r$ ($0 < r \leq 1$) if $k^{-1}(r)$ intersects an element $Y$ of the decomposition of $F$, then $k^{-1}(r) = Y$. Hence the union $\mathfrak{D}$ of the decompositions of $E$ and $F$ is a monotone continuous (Lemma 5) decomposition of the continuum $E \cup F$. Each element of $\mathfrak{D}$ has void interior relative to $E \cup F$. The quotient space $(E \cup F)/\mathfrak{D}$ is the union of two arcs. Since $M$ is atriodic, $(E \cup F)/\mathfrak{D}$ is atriodic. Moreover $(E \cup F)/\mathfrak{D}$ is not a simple closed curve since $F$ misses $k^{-1}(0)$. It follows from Lemma 2 that $(E \cup F)/\mathfrak{D}$ does not contain a simple closed curve. Thus $(E \cup F)/\mathfrak{D}$ is an arc and $E \cup F$ is a continuum of type $\lambda$. Furthermore $k^{-1}(0)$ is an end set of $E \cup F$. Hence $E \cup F$ belongs to $\mathfrak{S}(k^{-1}(0), E)$. This completes the proof of Lemma 8.

LEMMA 9. *Every element of the decomposition of E is homogeneous.*

*Proof.* Lemma 9 follows from paragraphs 5 through 12 in the proof of Theorem 1 of **[10]**.

LEMMA 10. *Suppose N is an indecomposable subcontinuum of M that contains E. Then N contains* $\bigcup \mathfrak{S}(k^{-1}(0), E)$.

*Proof.* Assume $N$ does not contain $\bigcup \mathfrak{S}(k^{-1}(0), E)$. Let $F$ be an element of $\mathfrak{S}(k^{-1}(0), E)$ that intersects $M \setminus N$. Let $A$ be the composant of $N$ that contains $E$. Let $Y$ be the end set of $F$ opposite $k^{-1}(0)$. It follows from Lemma 2 and the irreducibility of $F$ that $N \cap Y = \varnothing$. Let $\varepsilon = \rho(N, Y)$. By Lemma 1, there exist two $\varepsilon$-homomorphisms $f$ and $g$ of $M$ onto $M$ and two composants $B$ and $C$ of $N$ distinct from $A$ such that $f[F] \cap B \neq \varnothing \neq g[F] \cap C$. By Lemma 2, $F, f[F]$, and $g[F]$ are disjoint. Since $f$ and $g$ are $\varepsilon$-homeomorphisms, $M \setminus N$ contains $f[Y] \cup g[Y]$. Hence $N \cup F \cup f[F] \cup g[F]$ is a triod. This contradicts the assumption that $M$ is atriodic. Therefore $N$ contains $\bigcup \mathfrak{S}(k^{-1}(0), E)$.

NOTATION. Let $X$ be a subset of $M$. We denote the boundary of $X$ and the closure of $X$ in $M$ by Bd $X$ and Cl $X$, respectively.

LEMMA 11. *If N is an indecomposable subcontinuum of M that contains E, then* Cl $\bigcup \mathfrak{S}(k^{-1}(0), E)$ *is an indecomposable subcontinuum of N.*

*Proof.* By Lemma 10, Cl $\bigcup \mathfrak{S}(k^{-1}(0), E)$ is a subcontinuum of $N$. The argument given in paragraphs 2 through 11 in the proof of Lemma 6 of **[11]** proves that Cl $\bigcup \mathfrak{S}(k^{-1}(0), E)$ is indecomposable.

Suppose $\mathfrak{L} = \{L_i\colon 1 \leq i \leq 5\}$ is a 5-linked chain in $M$.

NOTATION. We denote the 3-linked subchain $\{L_i\colon 2 \leq i \leq 4\}$ of $\mathfrak{L}$ by $\mathfrak{L}'$.

The chain $\mathfrak{L}$ is *free* if $\mathrm{Bd}(L_1 \cup L_5) \setminus \mathrm{Cl} \bigcup \mathfrak{L}'$ contains Bd $\bigcup \mathfrak{L}$.

The chain $\mathfrak{L}$ is *normal* if Cl $L_i \cap$ Cl $L_j = \varnothing$ whenever $|i - j| > 1$.

The continuum $E$ *runs straight through* $\mathfrak{L}$ provided

(1) $E \subset \bigcup \mathfrak{L}$,

(2) $k^{-1}(0) \subset L_1 \setminus L_2$,

(3) $k^{-1}(1) \subset L_5 \backslash L_4$,

(4) if $0 \leq r \leq 1$ and $k^{-1}(r) \cap \operatorname{Bd} L_i \neq \varnothing$, then $k^{-1}(r) \subset \operatorname{Bd} L_i$, and

(5) if $0 \leq r < t \leq 1$ and $k^{-1}(r) \cup k^{-1}(t) \subset L_i$, then $k^{-1}[[r, t]] \subset L_i$.

The chain $\mathfrak{L}$ is *regular* if for each component $K$ of $\cup \mathfrak{L}'$, the set $\operatorname{Cl} K$ is a continuum of type $\lambda$ that runs straight through $\mathfrak{L}$.

A chain $\{P_i \colon 1 \leq i \leq 5\}$ is an *ordered refinement* of $\mathfrak{L}$ if for each $i$, the link $L_i$ contains $P_i$.

LEMMA 12. *Suppose $E$ runs straight through a normal regular chain $\mathfrak{L} = \{L_i \colon 1 \leq i \leq 5\}$. Suppose $k^{-1}(0) \subset L_1 \backslash \operatorname{Cl} L_2$ and $k^{-1}(1) \subset L_5 \backslash \operatorname{Cl} L_4$. Then $E$ runs straight through a free normal regular ordered refinement of $\mathfrak{L}$.*

*Proof.* Let $A$ and $B$ denote the open sets $L_1 \backslash \operatorname{Cl} L_2$ and $L_5 \backslash \operatorname{Cl} L_4$, respectively. Since $\mathfrak{L}$ is regular, $E \backslash (A \cup B)$ is an essential subcontinuum of $E$. Since $M$ is atriodic, $E \backslash (A \cup B)$ is a component of $M \backslash (A \cup B)$. Since no component of $M \backslash (A \cup B)$ intersects both $E \backslash (A \cup B)$ and $M \backslash \cup \mathfrak{L}$, there exist disjoint open sets $C$ and $D$ in $M$ such that (1) $C$ contains $E \backslash (A \cup B)$, (2) $D$ contains $M \backslash \cup \mathfrak{L}$, and (3) $C \cup D$ contains $M \backslash (A \cup B)$ [**22**, Theorem 44, page 15].

Let $P_1 = L_1$ and $P_5 = L_5$. For $i = 2$, 3, and 4, let $P_i = C \cap L_i$. The chain $\mathfrak{P} = \{P_i \colon 1 \leq i \leq 5\}$ is a free normal regular ordered refinement of $\mathfrak{L}$. Note that $E$ runs straight through $\mathfrak{P}$.

LEMMA 13. *Suppose $\varepsilon$ is a positive number, $A$ is a closed set that misses $E$, and $B$ is an open set that contains $k^{-1}(0) \cup k^{-1}(1)$. Then $E$ runs straight through a free normal regular chain $\mathfrak{P} = \{P_i \colon 1 \leq i \leq 5\}$ in $M \backslash A$ with the property that $B$ contains $\operatorname{Cl}(P_1 \cup P_5)$ and for each component $K$ of $\cup \mathfrak{P}'$ there is an $\varepsilon$-homeomorphism $h$ of $M$ onto $M$ such that $\operatorname{Cl} K$ is an essential subcontinuum of $h[E]$.*

*Proof.* Let $\{x_i \colon 0 \leq i \leq 15\}$ be a set of numbers such that $x_0 = 0$, $x_{15} = 1$, $x_i < x_{i+1}$ for each $i$ $(0 \leq i \leq 14)$, and $k^{-1}[[0, x_3] \cup [x_{12}, 1]] \subset B$. Let $\delta$ be a positive number less than $\varepsilon$, $\rho(A, E)$, $\rho(M \backslash B, k^{-1}[[0, x_3] \cup [x_{12}, 1]])$, and the minimum of $\{\frac{1}{2}\rho(k^{-1}[[0, x_i]], k^{-1}[[x_{i+1}, 1]]) \colon 1 \leq i \leq 13\}$.

For each integer $i$ $(1 \leq i \leq 5)$, let $L_i$ be the open set $W(k^{-1}[[x_{3i-3}, x_{3i}]], \delta)$ (Lemma 1). Let $\mathfrak{L} = \{L_i \colon 1 \leq i \leq 5\}$. Note that $\mathfrak{L}$ is a normal chain in $M \backslash A$.

Next we prove that $\mathfrak{L}$ is regular. To accomplish this let $h$ be a $\delta$-homeomorphism of $M$ onto $M$.

Note that

(1) $h[E] \subset \cup \mathfrak{L}$, and

(2) $h[k^{-1}(0)] \subset L_1 \backslash \mathrm{Cl}\, L_2$ and $h[k^{-1}(1)] \subset L_5 \backslash \mathrm{Cl}\, L_4$.

For each number $r$ $(0 \leq r \leq 1)$ and each integer $i$ $(1 \leq i \leq 5)$

(3) *if* $h[k^{-1}(r)] \cap L_i \neq \varnothing$, *then* $h[k^{-1}(r)] \subset L_i$.

To prove (3) assume for some numbers $r$ and $i$, $h[k^{-1}(r)]$ intersects both $L_i$ and $M \backslash L_i$. It follows from the definition of $\delta$ that $r \in [x_2, x_{13}] \backslash [x_{3i-3}, x_{3i}]$. There exist a number $s$ in $[x_{3i-3}, x_{3i}] \cap [x_2, x_{13}]$ and a $\delta$-homeomorphism $g$ of $M$ onto $M$ such that $g[k^{-1}(s)] \cap h[k^{-1}(r)] \neq \varnothing$. By Lemma 3, $g[k^{-1}(s)] = h[k^{-1}(r)]$. This contradicts the fact that $L_i$ contains $g[k^{-1}(s)]$. Hence (3) holds.

For each number $r$ $(0 \leq r \leq 1)$

(4) if $h[k^{-1}(r)] \cap \mathrm{Bd}\, L_i \neq \varnothing$, then $h[k^{-1}(r)] \subset \mathrm{Bd}\, L_i$.

To see this let $p$ be a point of $h[k^{-1}(r)] \cap \mathrm{Bd}\, L_i$ and assume that $\mathrm{Bd}\, L_i$ misses a point $q$ of $h[k^{-1}(r)]$. By (3), $q \notin L_i$. Let $\mu = \rho(q, L_i)$. By Lemma 1, there exists a $\mu$-homeomorphism $f$ of $M$ onto $M$ such that $fh$ is a $\delta$-homeomorphism and $f(p) \in L_i$. It follows from the argument for (3) that $fh[k^{-1}(r)] \subset L_i$. This contradicts the fact that $f(q) \notin L_i$. Hence (4) holds.

Note that

(5) if $0 \leq r < t \leq 1$ and $k^{-1}(r) \cup k^{-1}(t) \subset L_i$, then $k^{-1}[[r, t]] \subset L_i$.

To see this assume the contrary. By (1), (2), and (3), there exist numbers $r$, $s$, and $t$ $(0 \leq r < s < t \leq 1)$ and an integer $i$ $(1 \leq i \leq 4)$ such that $h[k^{-1}(s)] \subset L_i$ and $h[k^{-1}(r) \cup k^{-1}(t)] \subset L_{i+1} \backslash L_i$. Let $u$ be a number in $[x_{3i-3}, x_{3i}]$ and $g$ be a $\delta$-homeomorphism of $M$ onto $M$ such that $g[k^{-1}(u)] \cap h[k^{-1}(s)] \neq \varnothing$. Since $M$ is atriodic and $g[k^{-1}(0, x_{3i})]]$ misses $L_{i+1} \backslash L_i$, it follows that $g[k^{-1}[0, x_{3i}]] \subset h[k^{-1}[[r, t]]$. This contradicts the definition of $\delta$. Hence (5) holds.

It follows from (1), (2), (4), and (5) that

(6) $h[E]$ runs straight through $\mathfrak{L}$.

Let $K$ be a component of $\cup \mathfrak{L}'$. Let $h$ be a $\delta$-homeomorphism of $M$ onto $M$ such that $K \cap h[E] \neq \varnothing$. Since $M$ is atriodic, it follows from (6) and Lemma 6 that $\mathrm{Cl}\, K$ is an essential subcontinuum of $h[E]$ that runs straight through $\mathfrak{L}$. Hence $\mathfrak{L}$ is regular.

Since $h$ in (2) and (6) can be the identity, $k^{-1}(0) \subset L_1 \backslash \mathrm{Cl}\, L_2$, $k^{-1}(1) \subset L_5 \backslash \mathrm{Cl}\, L_4$, and $E$ runs straight through $\mathfrak{L}$. By Lemma 12, $E$ runs straight through a free normal regular ordered refinement $\mathfrak{P} = \{P_i\colon 1 \leq i \leq 5\}$ of $\mathfrak{L}$.

Since $\cup \mathfrak{L} \subset M \backslash A$ and $\mathrm{Cl}(L_1 \cup L_5) \subset B$, it follows that $\cup \mathfrak{P} \subset M \backslash A$ and $\mathrm{Cl}(P_1 \cup P_5) \subset B$. Let $K$ be a component of $\cup \mathfrak{P}'$. Since $\delta < \varepsilon$,

$M$ is atriodic, and $\mathcal{P}$ is a regular ordered refinement of $\mathcal{L}$, there is an $\varepsilon$-homeomorphism $h$ of $M$ onto $M$ such that Cl $K$ is an essential subcontinuum of $h[E]$. This completes the proof of Lemma 13.

NOTATION. Suppose $\mathcal{P} = \{P_i\colon 1 \le i \le 5\}$ is a regular chain. Let $\Omega(\mathcal{P})$ denote the collection $\{Z\colon Z$ is an element of the decomposition of the closure of a component of $\bigcup \mathcal{P}'\}$. Let $\Delta(\mathcal{P})$ be $\{Z\colon Z \in \Omega(\mathcal{P})$ and $Z \cap \mathrm{Cl}(P_1 \cup P_5) = \varnothing\}$. Note that since $\mathcal{P}$ is regular, $\Delta(\mathcal{P})$ is a decomposition of $(\bigcup \mathcal{P}) \backslash \mathrm{Cl}(P_1 \cup P_5)$.

LEMMA 14. *Suppose* $\mathcal{P} = \{P_i\colon 1 \le i \le 5\}$ *is a free regular chain. Then the decomposition* $\Delta(\mathcal{P})$ *of* $(\bigcup \mathcal{P}) \backslash \mathrm{Cl}(P_1 \cup P_5)$ *is continuous.*

*Proof.* Suppose $\{Z_i\}$ and $\{z_i\}$ are sequences such that (1) $Z_i \in \Delta(\mathcal{P})$ and $z_i \in Z_i$ for each positive integer $i$, and (2) $\{z_i\}$ converges to a point $z$ of $\bigcup \Delta(\mathcal{P})$. Let $Z$ be the element of $\Delta(\mathcal{P})$ that contains $z$. It suffices to show that $\{Z_i\}$ converges to $Z$.

Let $\varepsilon$ be any positive number less than $\frac{1}{2}\rho(\bigcup \Delta(\mathcal{P}), M \backslash \bigcup \mathcal{P}')$. Let $K_0$ be the $z$-component of $\bigcup \mathcal{P}'$. For each positive integer $i$, let $K_i$ be the $z_i$-component of $\bigcup \mathcal{P}'$. Since $\mathcal{P}$ is regular, for each non-negative integer $i$, the end sets of Cl $K_i$ are in $M \backslash \bigcup \mathcal{P}'$. By Lemma 3, if $z_i$ belongs to the open set $W(z, \varepsilon)$ (Lemma 1), then there is an $\varepsilon$-homeomorphism $h$ of $M$ onto $M$ such that $h[Z] = Z_i$. Since $\varepsilon$ is arbitrarily small, $\{Z_i\}$ converges to $Z$. Therefore $\Delta(\mathcal{P})$ is continuous.

LEMMA 15. *Suppose* $\mathcal{P} = \{P_i\colon 1 \le i \le 5\}$ *is a free regular chain. Then there is a positive number* $\varepsilon$ *such that if* $Z \in \Delta(\mathcal{P})$ *and* $h$ *is an* $\varepsilon$*-homeomorphism of* $M$ *onto* $M$, *then* $h[Z] \in \Omega(\mathcal{P})$.

*Proof.* Let $\varepsilon = \frac{1}{2}\rho(\bigcup \Delta(\mathcal{P}), M \backslash \bigcup \mathcal{P}')$. By Lemma 3, if $Z \in \Delta(\mathcal{P})$ and $h$ is an $\varepsilon$-homeomorphism of $M$ onto $M$, then $h[Z] \in \Omega(\mathcal{P})$.

LEMMA 16. *Suppose* $\mathcal{P} = \{P_i\colon 1 \le i \le 5\}$ *is a free regular chain. Suppose* $K$ *is a component of* $\bigcup \mathcal{P}'$ *and* $X$ *is an end set of* Cl $K$. *Suppose* $F$ *is an extension of* Cl $K$ *away from* $X$ *and* $\mathcal{D}$ *is the decomposition of* $F$. *Then no element of* $\mathcal{D}$ *intersects three consecutive links of* $\mathcal{P}$.

*Proof.* Assume the contrary. Let $d\colon F \to [0, 1]$ be a quotient map associated with $\mathcal{D}$ such that $d^{-1}(0) = X$. Let $s$ be the greatest lower bound of $S = \{r \in [0, 1]\colon d^{-1}(r)$ intersects three consecutive links of $\mathcal{P}\}$. Since no element of the decomposition of Cl $K$ intersects three consecutive links of $\mathcal{P}$, it follows that $s > 0$. Since $\mathfrak{D}$ is continuous (Lemma 5), $s \notin S$ and $s \neq 1$.

Since $\mathcal{P}$ is free, there exist a 3-linked subchain $\mathcal{Q}$ of $\mathcal{P}$ and numbers $t$, $w$ greater than $s$ such that

(1) for each number $u$ ($t \leq u \leq w$), $d^{-1}(u)$ intersects each link of $\mathcal{Q}$ and no element of $\mathcal{P} \setminus \mathcal{Q}$ that intersects a link of $\mathcal{Q}$.

Let $v$ be a number between $t$ and $w$.

Since $\mathcal{P}$ is regular, there exists a component $H$ of $\bigcup \mathcal{P}'$ such that $H \cap d^{-1}(v) \neq \varnothing$ and

(2) Cl $H$ runs straight through $\mathcal{P}$.

Since Cl $H$ intersects each link of $\mathcal{P}$, the continuum $d^{-1}[[t, w]]$ does not contain Cl $H$. Therefore, since $M$ is atriodic, Cl $H$ intersects either $d^{-1}(t)$ or $d^{-1}(w)$. Assume without loss of generality that Cl $H \cap d^{-1}(t) \neq \varnothing$. By (1) and (2), there exist distinct elements $Y$ and $Z$ of the decomposition of Cl $H$ such that $Y \cap d^{-1}(t) \neq \varnothing$ and $Z \cap d^{-1}(v) \neq \varnothing$. Let $I$ be the essential subcontinuum of Cl $H$ that is irreducible between $Y$ and $Z$. It follows from Lemma 2 and the irreducibility of $I$ and $d^{-1}[[t, v]]$ that $I = I \cap d^{-1}[[t, v]] = d^{-1}[[t, v]]$. Since no element of the decomposition of $I$ intersects all three links of $\mathcal{Q}$, each element of the decomposition of $I$ is properly contained in an element of the decomposition of $d^{-1}[[t, v]]$. This contradicts the fact that the decomposition of $d^{-1}[[t, v]]$ is unique. Hence Lemma 16 is true.

## 4. Principal results.

THEOREM 1. *Suppose $M$ is an atriodic homogeneous continuum. Suppose $N$ is an indecomposable subcontinuum of $M$ that contains a decomposable continuum. Then $M = N$. Furthermore $M$ admits a continuous decomposition $\mathfrak{D}$ such that $M/\mathfrak{D}$ is a solenoid and the elements of $\mathfrak{D}$ are homeomorphic. Moreover if the elements of $\mathfrak{D}$ are not points, then they are tree-like hereditarily indecomposable homogeneous continua.*

*Proof.* By the argument in paragraphs 1 and 2 in the proof of Theorem 1 of [10], $N$ has a subcontinuum $E$ of type $\lambda$. Let $k: E \to [0, 1]$ be a quotient map associated with the decomposition of $E$.

Let $\varepsilon$ be a positive number.

By Lemma 13, $E$ runs straight through a free normal regular chain $\mathcal{P} = \{P_i: 1 \leq i \leq 5\}$ with the property that

(1) for each component $K$ of $\bigcup \mathcal{P}'$ there is an $\varepsilon$-homeomorphism $h$ of $M$ onto $M$ such that Cl $K$ is an essential subcontinuum of $h[E]$.

For each element $Z$ of $\Omega(\mathcal{P})\backslash\Delta(\mathcal{P})$, let $K(Z)$ be the component of $\bigcup\mathcal{P}'$ that contains $Z$. Let $X(Z)$ be the end set of $\mathrm{Cl}\,K(Z)$ that is separated from $Z$ in $\mathrm{Cl}\,K(Z)$ by $P_3$. By Lemma 11, $\mathrm{Cl}\bigcup\mathfrak{S}(X(Z), \mathrm{Cl}\,K(Z))$ is an indecomposable subcontinuum of $N$. Thus $\bigcup\mathfrak{S}(X(Z), \mathrm{Cl}\,K(Z))$ intersects $P_3\backslash K(Z)$. Hence there is an element $F(Z)$ of $\mathfrak{S}(X(Z), \mathrm{Cl}\,K(Z))$ that intersects $P_3\backslash K(Z)$. By Lemmas 7 and 16, each element of the decomposition of $F(Z)$ that intersects $\bigcup\mathcal{P}'$ is an element of the decomposition of the closure of a component of $\bigcup\mathcal{P}'$. Hence there is an essential subcontinuum $J(Z)$ of $F(Z)$ that contains $Z$, misses $\mathrm{Cl}\,P_3$, and has one end set in $K(Z)\cap(\bigcup\Delta(\mathcal{P}))$ and the other end set in $(\bigcup\Delta(\mathcal{P}))\backslash K(Z)$.

By Lemma 13, $J(Z)$ runs straight through a free normal regular chain $\mathcal{P}(Z) = \{P(Z)_i\colon 1 \le i \le 5\}$ such that

(2) $\bigcup\mathcal{P}(Z)$ misses $\mathrm{Cl}\,P_3$, and

(3) $\bigcup\Delta(\mathcal{P})$ contains $\mathrm{Cl}(P(Z)_1 \cup P(Z)_5)$.

(The sets $A$ and $B$ in Lemma 13 are $\mathrm{Cl}\,P_3$ and $\bigcup\Delta(\mathcal{P})$, respectively.)

Let $\mathfrak{B}$ be the collection of open sets $\{\bigcup\Delta(\mathcal{P})\} \cup \{\bigcup\Delta(\mathcal{P}(Z))\colon Z \in \Omega(\mathcal{P})\backslash\Delta(\mathcal{P})\}$. Since $\mathcal{P}$ is free and since $\mathcal{P}(Z)$ is free and (3) holds for each element $Z$ of $\Omega(\mathcal{P})\backslash\Delta(\mathcal{P})$, it follows that $\bigcup\mathfrak{B}$ is a closed open subset of $M$. Hence $\mathfrak{B}$ covers $M$.

Let $S$ be $\bigcup(\mathfrak{S}(k^{-1}(0), E) \cup \mathfrak{S}(k^{-1}(1), E))$.

Next we prove that

(4) $\rho(p, S) < \varepsilon$ for every point $p$ of $M$.

If $p \in \bigcup\Delta(\mathcal{P})$, then (4) follows from (1). Therefore we assume that $p \notin \bigcup\Delta(\mathcal{P})$. Let $Z$ be an element of $\Omega(\mathcal{P})\backslash\Delta(\mathcal{P})$ such that $p \in \bigcup\Delta(\mathcal{P}(Z))$. Let $H$ be the $p$-component of $\bigcup\Delta(\mathcal{P}(Z))$. Let $X$ be an end set of $\mathrm{Cl}\,H$. Let $K$ be the component of $\bigcup\mathcal{P}'$ that contains $X$. By (1), there is an $\varepsilon$-homeomorphism $h$ of $M$ onto $M$ such that $\mathrm{Cl}\,K$ is an essential subcontinuum of $h[E]$. By (2) and Lemma 2, $\mathrm{Cl}\,H$ does not intersect both $h[k^{-1}(0)]$ and $h[k^{-1}(1)]$. Hence, by Lemma 8, $H$ is a subset of $T = \bigcup(\mathfrak{S}(h[k^{-1}(0)], h[E]) \cup \mathfrak{S}(h[k^{-1}(1)], h[E]))$. Note that $T = h[S]$. Since $p \in H \subset h[S]$ and $h$ is an $\varepsilon$-homeomorphism, (4) holds.

Since $\varepsilon$ may be arbitrarily small, it follows from (4) that $S$ is dense in $M$. Hence, by Lemma 10, $M = N$.

For convenience we define $\mathcal{P}(Z_0)$ to be $\mathcal{P}$.

Since $M$ is compact, there exists a finite subcollection $\mathcal{C} = \{\bigcup\Delta(\mathcal{P}(Z_i))\colon 0 \le i \le n\}$ of $\mathfrak{B}$ that covers $M$.

Let $\mathfrak{D}$ be $\bigcup\{\Delta(\mathcal{P}(Z_i))\colon 0 \le i \le n\}$.

We must show that $\mathfrak{D}$ is a decomposition of $M$. Since $\mathcal{C}$ covers $M$, it suffices to show that the elements of $\mathfrak{D}$ are disjoint. Let $A$ and $B$ be intersecting elements of $\mathcal{D}$. If $A \cup B$ intersects $\bigcup\Delta(\mathcal{P})$, then, by (2), (3), and Lemma 7, $A = B$. Hence we assume that $A \cup B$ misses $\bigcup\Delta(\mathcal{P})$. Let $i$ and $j$ be positive integers such that $A \in \Delta(\mathcal{P}(Z_i))$ and $B \in \Delta(\mathcal{P}(Z_j))$. Let $I$ be the component of $\bigcup\mathcal{P}(Z_i)'$ that contains $A$. Let $J$ be the component of $\bigcup\mathcal{P}(Z_j)'$ that contains $B$. Since $A \cup B$ misses $\bigcup\Delta(\mathcal{P})$, it follows from (3) that $A \cup B$ misses both end sets of $\mathrm{Cl}\, I$ and both end sets of $\mathrm{Cl}\, J$. Thus, by Lemma 3, $A = B$. Hence $\mathcal{D}$ is a decomposition of $M$.

It follows from Lemma 14 that $\mathcal{D}$ is continuous. According to Lemma 9, each element of $\mathcal{D}$ is either a point or a homogeneous continuum. Since $M$ is atriodic, it follows that the quotient space $M/\mathcal{D}$ is an atriodic continuum.

The quotient space $M/\mathcal{D}$ is homogeneous and the elements of $\mathcal{D}$ are homeomorphic. To see this first note that for each integer $i$ $(1 \leq i \leq n)$, since $\mathcal{P}(Z_i)$ is a free regular chain, it follows from (2) and (3) that each component of $\bigcup\Delta\mathcal{P}(Z_i)$ misses $P_3$ and intersects $M \setminus \bigcup \mathcal{P}'$.

Since $\mathcal{P}$ is a free regular chain, it follows from (3) and Lemma 7 that

(5) for each integer $i$ $(0 \leq i \leq n)$, $\Omega(\mathcal{P}(Z_i)) \subset \mathcal{D}$.

By Lemma 15, for each integer $i$ $(0 \leq i \leq n)$ there is a positive number $\varepsilon_i$ such that if $Z \in \Delta(\mathcal{P}(Z_i))$ and $h$ is an $\varepsilon_i$-homeomorphism of $M$ onto $M$, then $h[Z] \in \Omega(\mathcal{P}(Z_i))$. Let $\delta$ be the minimum of $\{\varepsilon_i\colon 0 \leq i \leq n\}$.

It follows from (5) that

(6) if $Z \in \mathcal{D}$ and $h$ is a $\delta$-homeomorphism of $M$ onto $M$, then $h[Z] \in \mathcal{D}$.

Let $X$ and $Y$ be elements of $\mathcal{D}$. Since $M$ is a continuum, there is a finite subset $\{x_i\colon 1 \leq i \leq m\}$ of $M$ such that $Y \cap W(x_1, \delta/2) \neq \varnothing$, $X \cap W(x_m, \delta/2) \neq \varnothing$, and $W(x_i, \delta/2) \cap W(x_{i+1}, \delta/2) \neq \varnothing$ for each integer $i$ $(1 \leq i < m)$.

Define a set $\{v_i\colon 0 \leq i \leq m\}$ such that $v_0 \in Y \cap W(x_1, \delta/2)$, $v_m \in X \cap W(x_m, \delta/2)$, and $v_i \in W(x_i, \delta/2) \cap W(x_{i+1}, \delta/2)$ for each integer $i$ $(1 \leq i < m)$. For each $i$ $(1 \leq i \leq m)$, let $h_i$ be a $\delta$-homeomorphism of $M$ onto $M$ such that $h_i(v_i) = v_{i-1}$. By (6), each $h_i$ maps each element of $\mathcal{D}$ onto an element of $\mathcal{D}$. Therefore $h_1 h_2 \cdots h_m$ induces a homeomorphism of $M/\mathcal{D}$ onto itself that takes $X$ to $Y$. Hence $M/\mathcal{D}$ is homogeneous. Since $h_1 h_2 \cdots h_m[X] = Y$, it follows that the elements of $\mathcal{D}$ are homeomorphic.

Since $M/\mathcal{D}$ is an atriodic homogeneous continuum that contains an arc, $M/\mathcal{D}$ is a solenoid [**20**, Theorem 14.4].

Suppose $\mathcal{D}$ has a nondegenerate element $Z$. By Lemma 2, $Z$ is hereditarily unicoherent. Since $Z$ is homogeneous, it follows that $Z$ is indecomposable [**15**] [**9**]. In fact, $Z$ is hereditarily indecomposable; for if $Z$

has a decomposable subcontinuum, then, by the above argument, $M = Z$ and this is impossible. Hence $Z$ is tree-like [**25**]. Therefore, if the elements of $\mathcal{D}$ are not points, they are homeomorphic tree-like hereditarily indecomposable homogeneous continua.

THEOREM 2. *Suppose $M$ is an atriodic homogeneous continuum that is not a solenoid and has a decomposable subcontinuum. Then $M$ admits a continuous decomposition $\mathcal{D}$ such that $M/\mathcal{D}$ is a solenoid and the elements of $\mathcal{D}$ are homeomorphic tree-like hereditarily indecomposable homogeneous continua.*

*Proof.* If $M$ is indecomposable, the conclusion follows immediately from Theorem 1. Therefore we assume that $M$ is decomposable. According to Theorem 14.7 of [**20**], $M$ admits a continuous decomposition $\mathcal{D}$ such that $M/\mathcal{D}$ is a simple closed curve and the elements of $\mathcal{D}$ are homeomorphic indecomposable homogeneous continua. Let $Z$ be an element of $\mathcal{D}$. The indecomposable continuum $Z$ is hereditarily indecomposable; for if $Z$ has a decomposable subcontinuum, it follows from Theorem 1 that $Z = M$ and this is impossible. Hence $Z$ is tree-like [**25**] and the proof is complete.

The following corollary to Theorem 2 answers in the affirmative Mackowiak and Tymchatyn's question [**20**, § 13].

COROLLARY 1. *If $M$ is an atriodic homogeneous continuum, then $M$ is 1-dimensional.*

*Proof.* If $M$ is hereditarily indecomposable, then $M$ is tree-like [**25**], and, therefore, 1-dimensional. Furthermore, if $M$ is a solenoid, then $M$ is 1-dimensional. Hence we assume that $M$ is not a solenoid and has a decomposable subcontinuum. By Theorem 2, $M$ admits a continuous decomposition $\mathcal{D}$ such that $M/\mathcal{D}$ is 1-dimensional and the elements of $\mathcal{D}$ are 1-dimensional continua. According to the second inequality in the proof of Theorem VI 7 on page 92 of [**14**], the dimension of $M$ is either 1 or 2. Hence, by Theorem 13.4 of [**20**], $M$ is 1-dimensional.

COROLLARY 2. *If $M$ is a tree-like atriodic homogeneous continuum, then $M$ is hereditarily indecomposable.*

*Proof.* Suppose $M$ has a decomposable subcontinuum. By Theorem 2, $M$ admits a monotone continuous decomposition $\mathcal{D}$ such that $M/\mathcal{D}$ is a solenoid. Since $M$ is indecomposable [**15**] [**9**], $M/\mathcal{D}$ is not a simple closed curve. This contradicts the fact that no tree-like continuum can be mapped onto a solenoid that is not a simple closed curve **[6]** **[17]**. Therefore $M$ is hereditarily indecomposable.

In 1968 F. B. Jones **[16]** suggested the following method for proving that every indecomposable homogeneous plane continuum $M$ is hereditarily indecomposable. Assume that $M$ has a decomposable subcontinuum. Find a monotone decomposition $\mathfrak{D}$ of $M$ such that $M/\mathfrak{D}$ is a homogeneous plane continuum that contains an arc. It follows from a theorem of Bing **[4]** that $M/\mathfrak{D}$ is a simple closed curve and this contradicts the fact that $M$ is indecomposable.

Every indecomposable homogeneous plane continuum is atriodic [**10**, Lemma 1]. In Theorem 2 (above), since the elements of $\mathfrak{D}$ are tree-like, if $M$ is planar, then $M/\mathfrak{D}$ is planar **[21]**. Hence Theorem 2 provides the decomposition that Jones requested and we obtain the following:

COROLLARY 3. *Every indecomposable homogeneous plane continuum is hereditarily indecomposable* **[11]**.

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