PACIFIC JOURNAL OF MATHEMATICS
Vol. 145, No. 2, 1990

CONSTANT MEAN CURVATURE SURFACES ON A STRIP

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For bounded planar domains there is a notion of extremal domain for which the constant mean curvature equation $\operatorname{div}(Du/\sqrt{1+|Du|^2}) = 2$ has a unique solution under no boundary conditions whatsoever. Extending to unbounded domains, it was expected that an infinite strip of unit width has the analogous property of admitting only cylinders as solutions. We show here that in fact other (distinct) solutions can appear.

The idea is to construct a super-solution $\operatorname{div}(Dv/\sqrt{1+|Dv|^2}) < 2$ and a subsolution $w \leq v$ with the same boundary values on the strip $\Omega = \{(x, y)| -\frac{1}{2} \leq y \leq \frac{1}{2}\}$. Implicitly we describe the function $w = w(x, y)$ by a parametric surface

$$\begin{aligned} x &= \xi - \frac{1}{2}(1-\cos\theta)\frac{\xi}{\sqrt{1+\xi^2}}, \\ y &= \frac{1}{2}\sin\theta, \\ z &= \frac{1}{2}\xi^2 + \frac{1}{2}(1-\cos\theta)\frac{1}{\sqrt{1+\xi^2}}, \end{aligned}$$

which is a parabolic trough with semi-circular cross section. Clearly its mean curvature > 2 and a moment reflection shows that it is a graph. Also the two boundary curves are congruent and have curvature ≤ 1. Therefore the parabolic cylinder containing these two curves is the required super-solution v.

The author would like to express his gratitude to Mr. Jenn-Fang Hwang for bringing this problem to his attention.

REFERENCES

For an affirmative result under extra boundary condition, the reader is referred to:

Luen-Fai Tam, *On the Uniqueness of Capillary Surfaces*, in Variational Methods for Free Surface Interfaces (P. Concus & R. Finn, eds.), Springer-Verlag, 1987.

For the existence of a constant mean curvature surface in between a sub-solution and a super-solution, the reader is referred to:

Enrico Giusti, *Minimal Surfaces and Functions of Bounded Variation*, Birkhäuser, 1984.

Received September 26, 1988. Partially supported by Grant NSC 79-0208-M002-19.

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