PACIFIC JOURNAL OF MATHEMATICS
Vol. 153, No. 2, 1992

REDUCTION OF TOPOLOGICAL STABLE RANK IN INDUCTIVE LIMITS OF C^*-ALGEBRAS

MARIUS DĂDĂRLAT, GABRIEL NAGY,
ANDRÁS NÉMETHI AND CORNEL PASNICU

We consider inductive limits A of sequences $A_1 \to A_2 \to \cdots$ of finite direct sums of C^*-algebras of continuous functions from compact Hausdorff spaces into full matrix algebras. We prove that A has topological stable rank (tsr) one provided that A is simple and the sequence of the dimensions of the spectra of A_i is bounded. For unital A, $\operatorname{tsr}(A) = 1$ means that the set of invertible elements is dense in A. If A is infinite dimensional, then the simplicity of A implies that the sizes of the involved matrices tend to infinity, so by general arguments one gets $\operatorname{tsr}(A_i) \leq 2$ for large enough i whence $\operatorname{tsr}(A) \leq 2$. The reduction of tsr from two to one requires arguments which are strongly related to this special class of C^*-algebras.

The problem of reduction of real rank (see [**6**]) for these algebras was recently studied in [**2**] in connection with some interesting features revealed in several papers ([**3**], [**1**], [**15**], [**5**], [**12**], [**11**]). The reduction of tsr and real rank for other classes of C^*-algebras was studied in [**22**], [**21**], [**8**], [**24**], [**17**], [**25**].

The paper consists of three sections:

1. Preliminaries and Notation
2. Local aspects of the connecting homomorphisms
3. The Main Result.

1.

1.1. For a unital C^*-algebra A and a finitely generated projective A-module E, we denote by $\operatorname{End}_A(E)$ the algebra of A-linear endomorphisms of E and by $\operatorname{GL}_A(E)$ the group of units of $\operatorname{End}_A(E)$. For $E = A^n$ we shall write $\operatorname{GL}(n, A)$ for $\operatorname{GL}_A(A^n)$ and $\operatorname{GL}^0(n, A)$ for the connected component of 1. Let $\operatorname{U}(A)$ denote the unitary group of A and $\operatorname{U}(n) := \operatorname{U}(\mathbf{C}^n)$. A selfadjoint idempotent element of a C^*-algebra will be simply called projection.

Recall some definitions from [**23**]. For a unital C^*-algebra A and a natural number n let $Lg_n(A)$ denote the set of n-tuples of elements of A which generate A as a left ideal. The topological stable rank of A is the least n (if it does not exist it will be taken by definition to be ∞) such that $Lg_n(A)$ is dense in A^n. One denotes by $\operatorname{csr}(A)$ the least integer n such that $\mathrm{GL}^0(m, A)$ acts transitively by right multiplication on $Lg_m(A)$ for any $m \geq n$. (If no such integer exists one takes $\operatorname{csr}(A) = \infty$.) For nonunital A one takes $\operatorname{tsr}(A) := \operatorname{tsr}(\widetilde{A})$ and $\operatorname{csr}(A) := \operatorname{csr}(\widetilde{A})$ where $\widetilde{A}$ is the algebra obtained from A by adjoining a unit.

For a compact Hausdorff space X of finite covering dimension one has:

$$\operatorname{tsr}(C(X)) = \left\lceil \frac{\dim X}{2} \right\rceil + 1,$$

$$\operatorname{csr}(C(X)) \leq \left\lceil \frac{\dim X + 1}{2} \right\rceil + 1$$

(see **[23]** and **[18]**).

1.2. We consider C^*-inductive limits

$$A = \varinjlim (A_i, \Phi_{ij}).$$

The A_i's are C^*-algebras of the form

$$A_i = \bigoplus_{t=1}^{s(i)} C(X_{it}) \otimes M_{n(i,t)}$$

where X_{it} is a Hausdorff compact space, $s(i)$, $n(i, t)$ are positive integers and $M_{n(i,t)}$ is the C^*-algebra of complex $n(i, t) \times n(i, t)$ matrices. The $*$-homomorphisms $\Phi_{ij}\colon A_i \longrightarrow A_j$ are not assumed to be unital or injective. We denote by Φ_i the natural map $A_i \longrightarrow A$ and by $X_i = \bigsqcup_{t=1}^{s(i)} X_{it}$ the spectrum of A_i.

We begin with a brief discussion on the $*$-homomorphisms between certain homogeneous C^*-algebras.

1.3. For given C^*-algebras C, D we denote by $\operatorname{Hom}(C, D)$ the space of all $*$-homomorphisms from C to D with the point-norm topology. $\operatorname{Hom}^1(C, D)$ stands for the subspace of unital $*$-homomorphisms. We shall identify

$$\operatorname{Hom}(C(X), C(Y) \otimes M_n) \quad \text{with } \operatorname{Map}(Y, \operatorname{Hom}(C(X), M_n))$$

where for topological spaces Y, Z, $\operatorname{Map}(Y, Z)$ denotes the space of continuous functions from Y to Z endowed with the compact-open topology.

Each $\psi \in \operatorname{Hom}(C(X), M_n)$ has the form

$$\psi(f) = \sum f(x_r) p_r, \qquad f \in C(X),$$

for suitable points $x_r \in X$ and mutually orthogonal projections p_r in M_n. Let L_ψ be the set of all x_r's that appear in the above formula. More generally, each $\Phi \in \operatorname{Hom}(C(X), C(Y) \otimes M_n)$ is identified with a map $\Phi: Y \to \operatorname{Hom}(C(X), M_m)$ and we define for each $y \in Y$, $L_\Phi(y) := L_{\Phi(y)}$. In the same way for given

$$\Phi \in \operatorname{Hom}\left(\bigoplus C(X_\alpha) \otimes M_{n(\alpha)}, \bigoplus C(Y_\beta) \otimes M_{m(\beta)}\right)$$

and $y \in Y$ we define

$$L_\Phi(y) = \bigsqcup_\alpha L_{\Phi_{\alpha,\beta}}(y)$$

where $\Phi_{\alpha,\beta}$ denotes the component of Φ acting from $C(X_\alpha) \subset C(X_\alpha) \otimes M_{n(\alpha)}$ to $C(Y_\beta) \otimes M_{m(\beta)}$.

Note that $\Phi(f)(y) = \Phi(g)(y)$ whenever $f = g$ on $L_\Phi(y)$.

The map $y \mapsto L_\Phi(y)$ has useful semicontinuity properties:

(a) if $L_\Phi(y)$ is contained in some open set U then $L_\Phi(z) \subset U$ for any z in some neighborhood of y,

(b) the set $\{y: L_\Phi(y) \cap U \neq \varnothing\}$ is open for each open set U (see **[9]** and **[19]**).

2. We begin by giving two criteria of simplicity for C^*-algebras A as above, which extend the corresponding results for AF-algebras **[4]** and Bunce-Deddens algebras **[7]**.

2.1. PROPOSITION. *Let* $A = \varinjlim (A_i, \Phi_{ij})$ *be as in* 1.1 *and assume that the connecting homomorphisms* Φ_{ij} *are injective. Then the following conditions are equivalent*:

(i) A *is simple.*

(ii) *For any positive integer* i *and any open nonempty subset* U *of* X_i *there is a* j_0 *such that* $L_{\Phi_{ij}}(x) \cap U \neq \varnothing$ *for any* $j \geq j_0$ *and* $x \in X_j$.

(iii) *For any nonzero* $a \in A_i$ *there is a* j_0 *such that*

$$\Phi_{ij}(a)(x) \neq 0 \quad \textit{for each } j \geq j_0 \textit{ and } x \in X_j.$$

Proof. (ii) $\Leftrightarrow$ (iii). This is clear since for given $a \in A_i$ one has

$$\Phi_{ij}(a)(x) = 0 \quad \text{if and only if} \quad a = 0 \text{ on } L_{\Phi_{ij}}(x).$$

(i) $\Rightarrow$ (ii). Assume that (ii) does not hold for some i and some open nonempty $U \subsetneqq X_i$. Passing to a subsequence, if necessary, we may assume that for any $j \geq i$ the set $F_j = \{x \in X_j; L_{\Phi_{ij}}(x) \cap U = \varnothing\}$

is nonempty and $F_j \neq X_j$. By the last part of 1.3 F_j is closed. Therefore the family $(J_j)_{j \geq i}$ where

$$J_j = \{a \in A_j \colon a = 0 \text{ on } F_j\}$$

defines a closed two sided ideal J in A. (Note that $\Phi_{jk}(J_j) \subset J_k$ since $L_{\Phi_{ij}}(y) \subset L_{\Phi_{ik}}(x)$ for any $y \in L_{\Phi_{jk}}(x)$.) Also $J \neq A$ since if e_i is the unit of A_i then $\operatorname{dist}(\Phi_{ij}(e_i), J_j) = 1$ for any $j \geq i$ and so $e_i \notin J$. The existence of J contradicts (i).

(iii) $\Rightarrow$ (i). Let J be a two-sided closed nonzero ideal of A. One has $J = \overline{\bigcup (J \cap A_i)}$ (see **[4]**). We shall prove that $J \cap A_j = A_j$ for large enough j. Take $a \in J \cap A_i$, $a \neq 0$. By (iii) there is a j_0 such that $\Phi_{ij}(a)(x) \neq 0$ for all $j \geq j_0$ and $x \in X_j$. Since $\Phi_{ij}(J \cap A_i) \subset J \cap A_j$ we find that $\Phi_{ij}(a) \in J \cap A_j$ for $j \geq j_0$. Since $\Phi_{ij}(a)$ does not vanish at any point of X_j this forces $J \cap A_j = A_j$. □

Let $A = \varinjlim (A_i, \Phi_{ij})$ be as above. For a noninvertible element $a \in A_i$ there are $x_0 \in X_i$, $u \in U(A_i)$ and a projection $p \in A_i$ (both u and p "scalars") such that $ua(x_0)p = pua(x_0) = 0$.

For simple A the following two lemmas enable us to obtain something similar for $\Phi_{ij}(a)$ (for some $j \geq i$) locally around any point of X_j, after a small perturbation of a.

2.2. LEMMA. *Let $\Phi \in \operatorname{Hom}\left(\bigoplus_{i=1}^{s} C(X_i) \otimes M_{n(i)}, C(Y) \otimes M_m\right)$, let $k \geq 1$, let U be an open subset of X_1 and let $y \in Y$ such that $L_\Phi(y) \cap U$ has at least k points. Then there is $p_W \in C(Y) \otimes M_m$ such that $p_W(z)$ is a projection of rank greater than or equal to k for all z in some neighborhood W of y and*

$$\Phi(a)p_W = p_W \Phi(a)$$

for any $a \in \bigoplus_{i=1}^{s} C(X_i) \oplus M_{n(i)}$ satisfying

$$a(x)e_{11} = e_{11}a(x) = 0$$

for all $x \in U$. (Here (e_{ij}) stands for a system of matrix units of $M_{n(1)}$.)

Proof. Take U_1, U_2 open subsets of $X = \bigcup_{i=1}^{s} X_i$ having disjoint closures such that

$$L_\Phi(y) \cap U \subset U_1 \subset U, \qquad L_\Phi(y) \cap (X_1 - U) \subset U_2.$$

Using the continuity of L_Φ (see 1.3) we find a neighborhood W of y such that $L_\Phi(z) \subset U_1 \cup U_2$ for all $z \in W$. Take a continuous map $g: X_1 \to [0, 1]$ such that $g = 1$ on U_1 and $g = 0$ on U_2 and define $p_W = \Phi(g \otimes E_{11})$. If $z \in W$ then $p_W(z) = p_W^2(z) = p_W^*(z)$ since $g = g^2 = g^*$ on $L_\Phi(W)$. One has $\operatorname{rank} p_W(z) \geq k$ since $L_\Phi(y) \cap U_1$ has at least k elements and $g = 1$ on U_1. Finally if $a(x)e_{11} = e_{11}a(x) = 0$ for all $x \in U$ then $(g \otimes e_{11})a = a(g \otimes e_{11}) = 0$. This implies $p_W \Phi(a) = \Phi(a) p_W = 0$. □

2.3. LEMMA. *Let $C = C(X) \otimes M_n$ and let $a \in C$ such that $\det a(x) = 0$ for some $x \in X$. Then for any $\varepsilon > 0$ there exist u, $v \in \mathrm{GL}(C)$ and $b \in C$ such that*

$$\|uav - b\| < \varepsilon \quad \text{and} \quad be_{11} = e_{11}b = 0 \text{ on a neighbourhood of } x .$$

Proof. Take u, $v \in \mathrm{Gl}(n, \mathbf{C})$ such that the matrix $ua(x)v$ has only zero entries on the first row and on the first column. Now b is easily found since continuous functions vanishing at x can be uniformly approximated by continuous functions vanishing on a neighbourhood of x. □

3. The next step toward the main result is based on the following theorem which follows from Michael's paper **[16]**.

3.1. THEOREM. *Let X be a Hausdorff compact space of dimension d, let T be a complete metric space and let Y be a map from X to the family of the nonempty closed subsets of T.*

Suppose that

(a) *Y is lower semicontinuous, i.e. for each open subset U of T the set $\{x \in X : Y(x) \cap U \neq \varnothing\}$ is open;*

(b) *Each $Y(x)$ is $(d+1)$-connected;*

(c) *There is an $\varepsilon > 0$ such that for any $0 < r < \varepsilon$ and $x \in X$ the intersection of $Y(x)$ with any closed ball of radius r in T is a contractible space.*

Then there is a continuous map $\sigma: X \to T$ such that $\sigma(x) \in Y(x)$ for all $x \in X$.

Proof. The theorem follows from Theorem 1.2 in **[16]** using the comments from the second part of the same paper.

3.2. PROPOSITION. *Let X be a Hausdorff compact space, let $k' \geq k \geq 1$ integers, let $\mathscr{W}$ be an open cover of X and assume that for each $W \in \mathscr{W}$ there is given a continuous projection valued map $p_W: W \to M_n$ such that* $\operatorname{rank} p_W(x) \geq k'$ *for $x \in W$. If* $\dim(X) \leq 2(k' - k) - 1$

then there is a continuous projection valued map $p\colon X \longrightarrow M_n$ *such that for* $x \in X$:

$$\operatorname{rank} p(x) \geq k,$$
$$p(x) \leq \bigvee\{p_W(x)\colon W \in \mathscr{W},\ x \in W\}.$$

Proof. For $x \in X$ define $\mathscr{W}(x) = \{W \in \mathscr{W}\colon x \in W\}$ and $H(x) = \operatorname{span}\{p_W(x)\mathbf{C}^n\colon W \in \mathscr{W}(x)\}$.

For any linear subspace H of $\mathbf{C}^n$ let $V(H, k)$, $k \leq \dim(H)$, denote the Stiefel manifold of k-orthogonal frames in H (see **[14]**). For any $x \in X$ define $Y(x) = V(H(x), k) \subset V(\mathbf{C}^n; k)$. We check that Y satisfies the conditions of Theorem 3.1.

(a) The lower semicontinuity of Y follows from the lower semicontinuity of the map $x \mapsto H(x) \subset \mathbf{C}^n$ which is almost obvious having in mind the definition of $H(x)$.

(b) $V(H, k)$ is $2(\dim(H) - k)$-connected (see **[14]**). Therefore $V(H(x), k)$ is $2(k' - k)$-connected since $\dim H(x) \geq k'$.

(c) For any m, $n \geq m \geq k$, there is $\varepsilon_m > 0$ such that any closed ball of radius at most ε_m in $V(\mathbf{C}^m, k)$ is contractible. (We consider $V(\mathbf{C}^m, k)$ with the metric induced by the restriction of a $\mathrm{U}(n)$-invariant Riemann structure on $V(\mathbf{C}^n, k)$.) In this situation $V(\mathbf{C}^m, k)$ is a totally geodesic submanifold of $V(\mathbf{C}^n, k)$ and the same is true for any $V(H, k)$ with $H \subset \mathbf{C}^n$. Therefore the induced metric form from $V(\mathbf{C}^n, k)$ coincides with the metric given by the induced Riemann structure of $V(H, k)$ (see **[13]**). Having also the $\mathrm{U}(n)$-invariance of this metric one can take

$$\varepsilon = \min\{\varepsilon_m\colon k \leq m \leq n\}. \qquad \square$$

We also need the following approximation results:

3.3. Lemma. *Let* B *be a unital* C^**-algebra and let*

$$k \geq \max(\operatorname{tsr}(B), \operatorname{csr}(B)).$$

Then for any positive integer m *and any* $a \in M_m(B)$, *the matrix* $\left(\begin{smallmatrix} a & 0 \\ 0 & 0_k \end{smallmatrix}\right)$ *belongs to the closure of* $\mathrm{GL}(m+k, B)$.

Proof. If $m \leq k$ one can take

$$b_\varepsilon = \begin{pmatrix} a & \varepsilon 1_m & 0 \\ \varepsilon 1_m & 0_m & 0 \\ 0 & 0 & \varepsilon 1_{k-m} \end{pmatrix} \in \mathrm{GL}(m+k, B)$$

and $b_\varepsilon \to a$ as $\varepsilon \to 0$.

For $m \geq k$ we proceed by induction. Assume the statement holds for a fixed $m \geq k$ and let a $a \in M_{m+1}(B)$. Since

$$m \geq \max(\mathrm{tsr}(B), \mathrm{csr}(B))$$

it follows from **[23]** that for each $\varepsilon > 0$ there are $t \in \mathrm{GL}(m+1, B)$, $a_1 \in M_m(B)$ and $b \in B^m$ such that

$$\left\| a - \begin{pmatrix} 1 & 0 \\ b & a_1 \end{pmatrix} \cdot t \right\| < \varepsilon .$$

By the induction hypothesis one can approximate

$$\begin{pmatrix} 1 & 0 & 0 \\ b & a_1 & 0 \\ 0 & 0 & 0_k \end{pmatrix}$$

with an invertible matrix of the form

$$\begin{pmatrix} 1 & 0 & 0 \\ b & \multicolumn{2}{c}{\multirow{2}{*}{c}} \\ 0 & & \end{pmatrix}$$

Hence $\begin{pmatrix} a & 0 \\ 0 & 0_k \end{pmatrix}$ will be approximated by

$$\begin{pmatrix} 1 & 0 & 0 \\ b & \multicolumn{2}{c}{\multirow{2}{*}{c}} \\ 0 & & \end{pmatrix} \cdot \begin{pmatrix} t & 0 \\ 0 & 1_k \end{pmatrix} . \qquad \square$$

3.4. REMARK. Suppose B, k are as above. Let F, G, H be finitely generated projective B-modules and put $E = F \oplus G \oplus H$. If F, G are free and $G \simeq B^k$, then a slight modification of the above arguments shows that $\mathrm{End}_B(F) \subset \overline{\mathrm{GL}_B(E)}$.

In the proof of the main result we shall invoke the following straightforward approximation device:

3.5. LEMMA. *Let $B = \overline{\bigcup B_i}$ where the B_i's form an increasing sequence of unital C^*-algebras. Let e_i be the unit of B_i. If for any $a \in B_i$ and $\varepsilon > 0$ there is $j \geq i$ and $b \in \mathrm{GL}(e_i B_j e_i)$ such that $\|a - b\| < \varepsilon$ then* $\mathrm{tsr}(B) = 1$.

Proof. Let $\widetilde{B} = B + \mathbf{C} \cdot 1$ be the algebra obtained by adjoining a unit to B. Let $x + \lambda 1 \in \widetilde{B}$ with $x \in B_i$. By hypothesis there is $j \geq i$ and $y \in \mathrm{GL}(e_i B_j e_i) \subset \mathrm{GL}(e_i B e_i)$ such that $\|x + \lambda e_i - y\|$ is small. Choosing a non zero scalar λ' close to λ, the element $y + \lambda'(1 - e_i)$ is invertible and approximates $x + \lambda \cdot 1$. Therefore $\mathrm{GL}(\widetilde{B})$ is dense in $\widetilde{B}$ which means $\mathrm{tsr}(B) = 1$. $\square$

3.6. THEOREM. *Let* $A = \varinjlim (A_i, \Phi_{ij})$ *where* $A_i = \bigoplus_{t=1}^{s(i)} C(X_{it}) \otimes M_{n(i,t)}$, *each* X_{it} *being a Hausdorff compact space such that* $d = \sup \dim(X_{it}) < \infty$.

If A *is simple then* $\operatorname{tsr}(A) = 1$.

Proof. Replacing each A_i by its image in A one may suppose that all the Φ_{ij}'s are injective. We shall verify the conditions from Lemma 3.5. Let $a \in A_i$ be a noninvertible element and put $Z = \{x \in X_i : \det a(x) = 0\}$. If Z consists only of isolated points of X_i then it is obvious that $a \in \overline{\mathrm{GL}(A_i)}$. Thus we may assume that there is $x \in Z$ such that each neighbourhood of x is an infinite set.

Moreover by Lemma 2.3 we may suppose that $ae_{11}^t = e_{11}^t a = 0$ on some neighbourhood U of x for some t. Fix integers k', k such that

$$k \geq 2d + 4, \qquad 2(k' - k) + 1 \geq d.$$

Since U is an infinite open set and the C^*-algebra A is simple it follows by Proposition 2.1 that there is $j \geq i$ such that $L_{\Phi_{ij}}(y) \cap U$ has at least k' elements for any $y \in X_j$. This enables us by using Lemma 2.2 to find an open covering $\mathscr{W}$ of X_j such that for each $W \in \mathscr{W}$ there is $p_W \in A_j$ satisfying

(1) p_W is projection valued on W,
(2) $\operatorname{rank} p_W(y) \geq k'$ for any $y \in W$,
(3) $p_W \Phi_{ij}(a) = \Phi_{ij}(a) p_W = 0$ on W,
(4) $p_W \leq \Phi_{ij}(e_i)$ where e_i is the unit of A_i.

Proposition 3.2 provides us a projection $p \in A_j$ such that
(a) $p(x) \leq \bigvee \{p_W(x) : W \in \mathscr{W}, x \in W\}$ for all $x \in X_j$.
(b) $\operatorname{rank} p(x) \geq k$ for all $x \in X_j$.

Of course (4) and (a) imply that $p \leq \Phi_{ij}(e_i)$.

We have also
(c) $\Phi_{ij}(a) p = p \Phi_{ij}(a) = 0$

as a consequence of (3) and (a).

Let $b := \Phi_{ij}(a)$ have the components (b_t) with $b_t \in C(X_{jt}) \otimes M_{n(j,t)}$. We shall use Remark 3.4 in order to approximate each b_t by invertible elements in $\operatorname{End}_{C(X_{jt})}(E_t)$ where $E_t := \Phi_{ij}(e_i) C(X_{jt})^{n(j,t)}$. Consider also the finitely generated projective $C(X_{jt})$-modules

$$P_t = pC(X_{jt})^{n(j,t)}, \qquad Q_t = (\Phi_{ij}(e_i) - p) C(X_{jt})^{n(j,t)}.$$

It is clear that $E_t \simeq P_t \oplus Q_t$.

Since $\operatorname{rank} P_t \geq k \geq 2d + 4$, by using the stability properties of vector bundles (see **[14]**), one can split P_t as a direct sum of finitely generated projective $C(X_{jt})$-modules $P_t = R_t \oplus G_t \oplus H_t$ such that $Q_t \oplus R_t$ and G_t are free and

$$\operatorname{rank} G_t \geq [(d+1)/2] + 1 \geq \max\{\operatorname{tsr} C(X_{jt}), \operatorname{csr} C(X_{jt})\}.$$

Let $F_t = Q_t \oplus R_t \oplus G_t$. By equation (c) above one can regard b_t as an element of $\operatorname{End}_{C(X_{jt})}(F_t)$ that vanishes on G_t. Since both F_t and G_t are free it follows from Lemma 3.3 that b_t belongs to the closure of $\mathrm{GL}(F_t)$. As F_t is a direct summand in E_t, this implies that b_t belongs to the closure $\mathrm{GL}(E_t)$. It follows that $\Phi_{ij}(a)$ belongs to the closure of $\mathrm{GL}(\bigoplus_t E_t) = \mathrm{GL}(\Phi_{ij}(e_i)A_j\Phi_{ij}(e_i))$. The proof is complete by virtue of Lemma 3.5. □

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Received October 5, 1990 and in revised form June 7, 1991.

UNIVERSITY OF CALIFORNIA
LOS ANGELES, CA 90024-1555

UNIVERSITY OF CALIFORNIA
BERKELEY, CA 94720

OHIO STATE UNIVERSITY
COLUMBUS, OH 43210

AND

INSTITUTE OF MATHEMATICS
BD. PACII 220
79622 BUCHAREST, ROMANIA