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SPHERES NOT ADMITTING SMOOTH ODD-FIXED-POINT ACTIONS OF S_5 AND $SL(2,5)$

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Abstract

Let G be a finite group and Σ a homology sphere with smooth G-action. If the G-fixed-point set of Σ consists of odd-number points then the dimension of Σ could be restrictive. In this article we confirm the claim in the cases where $G = S_5$ or $SL(2,5)$.

1. Introduction

Let G be a finite group. For a smooth manifold M, we refer to a smooth G-action on M simply as a G-action on M. A G-action on M is called an *odd-fixed-point action*, or *odd-f.p. action* for short, (resp. *one-fixed-point action*, or *one-f.p. action* for short) if the G-fixed-point set of M consists of odd-number points (resp. exactly one point). It is easy to see that for a G-action on the sphere of dimension 0, 1, or 2, if the G-fixed-point set contains at least one isolated point, there are exactly two G-fixed points. Contrary to an expectation by D. Montgomery–H. Samelson [14, Remarks 7], E. Stein [26] found one-f.p. actions on S^7 of $SL(2,5)$, and next T. Petrie [22] constructed one-f.p. actions on spheres of odd-order abelian groups having at least three non-cyclic Sylow subgroups.

Let A_5 and S_5 denote the alternating group and the symmetric group on five letters, respectively. The fact that the n-dimensional sphere S^n, $n \geq 6$, has one-f.p. actions of A_5 was shown by the first author in [15, 17] for $n = 6$; in [17, 18] for $n \geq 9$; and by A. Bak and the first author in [1, 2] for $n = 7, 8$. On the other hand, M. Furuta [10] proved the non-existence of one-f.p. actions on S^4 for any finite group, cf. [16]. M. Furuta, S. Demichelis [8] and N.P. Buchdahl–S. Kwasik–R. Schultz [6] showed the non-existence of one-f.p. actions on S^n, where $n = 3, 4, 5$, for any finite group.

A closed smooth manifold is called a *homotopy sphere* if it is homotopy equivalent to the sphere of same dimension. E. Laitinen–P. Traczyk [13] showed that for $n = 5$ or ≥ 7, if $G \not\cong A_5$ then there never exist one-f.p. actions of G on n-dimensional homotopy spheres Σ such that $\dim \Sigma^H \leq 2$ for all subgroups $H \neq \{e\}$ of G. For a commutative ring R, a closed smooth manifold M is called an *R-homology sphere* if the homology groups $H_i(M;R)$ are isomorphic to the homology groups $H_i(S^n;R)$ of the sphere S^n of same dimension, for all $i \geq 0$. A $\mathbb{Z}$-homology sphere is usually called a *homology sphere*. A. Borowiecka [3, Theorem 1.1] showed that any $\mathbb{Z}$-homology sphere of dimension 8 does not admit one-f.p. actions of $SL(2,5)$.

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In this paper we will prove the following theorem.

Theorem 1.1. *Let G be the group S_5 (resp. $SL(2,5)$) and Σ a $\mathbb{Z}$-homology sphere. If* $\dim\Sigma \in \{0,1,2,3,4,5,7,8,9,13\}$ *(resp.* $\{0,1,2,3,4,5,6,8,9\}$*) then Σ does not admit effective odd-f.p. actions of G.*

Recall that E. Stein [26] proved the existence of an effective one-f.p. $SL(2,5)$-action on S^7. On the other hand, it has not yet proved in a literature that there is a one-f.p. S_5-action on S^6.

Theorem 1.1 follows from Theorems 4.2 and 5.2 in the present article.

Remark. After the submission of the first draft of this paper, we were informed that a recent work [4] of A. Borowiecka–P. Mizerka shows that for some finite Oliver groups G of order up to 216, and for $G = A_5 \times C_k$ with $k = 3$, 5, or 7, there is no one-f.p. G-action on a sphere S^n, where n is chosen within a subset of $\{6,7,8,9,10\}$, and the choice depends on G.

2. Necessary conditions for odd-f.p. actions on spheres

Let G be a finite group. The following three well-known facts are useful for our study.

Proposition 2.1 (cf. [5, Chapter III, Theorem 4.3])**.** *Let p be a prime, G a finite group of p-power order, and X a finite G-CW complex. Then the Euler characteristic $\chi(X^G)$ is congruent modulo p to the Euler characteristic $\chi(X)$.*

Lemma 2.2 (Smith's Theorem [25], cf. [5, Chapter III, Theorem 5.1])**.** *Let p be a prime, G a finite group of p-power order, and X a $\mathbb{Z}_p$-homology sphere with G-action. Then X^G is a $\mathbb{Z}_p$-homology sphere.*

Lemma 2.3 (Lefschetz' Formula, cf. [9, p.225, Exercises (6.17), 3])**.** *Let X be a finite G-CW complex and $g \in G$. Then the Lefschetz number $L(g : X \to X)$ coincides with the Euler characteristic $\chi(X^g)$. Therefore if X is $\mathbb{Q}$-acyclic then $\chi(X^g) = 1$.*

Let $\mathcal{S}(G)$ denote the set of all subgroups of G. For a prime p, let $\mathcal{P}_p(G)$ denote the family of all subgroups of G with p-power order and set $\mathcal{P}(G) = \bigcup_p \mathcal{P}_p(G)$ where p ranges over all primes. After [20], for p and q each of which is a prime or 1, we denote by $\mathcal{G}_p^q$ the family of finite groups G having a sequence $P \trianglelefteq H \trianglelefteq G$ such that $|P|$ is a power of p, H/P is cyclic, and $|G/H|$ is a power of q. Set $\mathcal{G}_p = \bigcup_q \mathcal{G}_p^q$, $\mathcal{G}^q = \bigcup_p \mathcal{G}_p^q$, $\mathcal{G} = \bigcup_{p,q} \mathcal{G}_p^q$, $\mathcal{G}_p^q(G) = \mathcal{G}_p^q \cap \mathcal{S}(G)$, $\mathcal{G}_p(G) = \mathcal{G}_p \cap \mathcal{S}(G)$, $\mathcal{G}^q(G) = \mathcal{G}^q \cap \mathcal{S}(G)$, and $\mathcal{G}(G) = \mathcal{G} \cap \mathcal{S}(G)$.

Let Σ be a $\mathbb{Z}$-homology sphere with G-action. For $x \in \Sigma^G$, let $T_x(\Sigma)$ denote the tangent space at x in Σ with induced linear G-action.

Proposition 2.4. *Let Σ be a $\mathbb{Z}$-homology sphere with G-action and $x_0 \in \Sigma^G$. Then the following properties hold.*

(1) *For every $H \in \mathcal{G}^1(G)$, $\chi(\Sigma^H)$ is equal to $1 + (-1)^k$, where $k = \dim T_{x_0}(\Sigma)^H$.*
(2) *For every $K \in \mathcal{G}^q(G)$, $\chi(\Sigma^K)$ is congruent modulo q to $1 + (-1)^k$, where $k = \dim T_{x_0}(\Sigma)^K$.*

Proof. Let U be a closed disk G-neighborhood of x_0 in Σ and take the pinching map

$$f : \Sigma \to \Sigma/(\Sigma \smallsetminus \mathrm{int}(U)) \cong S(\mathbb{R} \oplus T_{x_0}(\Sigma)).$$

Clearly, f is a $\mathbb{Z}$-homology equivalence. Let C_f denote the mapping cone of f. It follows that C_f is $\mathbb{Z}$-acyclic. Thus by Lemmas 2.2 and 2.3, we obtain the property (1). Applying Proposition 2.1 to K/H-spaces Σ^H and $S(\mathbb{R} \oplus T_{x_0}(\Sigma))^H$ for $H \trianglelefteq K$ such that $|K/H|$ is a power of q and $H \in \mathcal{G}^1$, we obtain the property (2) from the property (1). □

Corollary 2.5 (cf. [20, 11]). *Let Σ be a $\mathbb{Z}$-homology sphere with G-action. If Σ^G consists of exactly one point then G does not belong to $\mathcal{G}$.*

Lemma 2.6 (P. Conner–E. Floyd [7, Theorem 25.1]). *Let C be a group of order 2 and M a connected closed smooth manifold of positive dimension with C-action. If M^C is a non-empty finite set then $|M^C|$ is an even integer.*

Proposition 2.7. *Let Σ be a $\mathbb{Z}$-homology sphere with G-action and L a subgroup of G with $|G : L| = 2$. If $|\Sigma^G| \equiv 1 \mod 2$ then there exists a G-fixed point x_0 such that $\dim T_{x_0}(\Sigma)^L = 0$, hence the point x_0 is an isolated L-fixed point of Σ.*

Proof. There is a connected component M of Σ^L such that $|M \cap \Sigma^G| \equiv 1 \mod 2$. Note that G/L acts on M and $M^{G/L} = M \cap \Sigma^G$ $(= M^G)$. By Lemma 2.6, M does not have positive dimension. Thus $\dim M = 0$ and M consists of one point, say x_0. Then x_0 is a G-fixed point and $T_{x_0}(\Sigma)$ satisfies the desired property. □

Corollary 2.8 (cf. [19, Lemma 2.1]). *Let Σ be a $\mathbb{Z}$-homology sphere with G-action and K the intersection of all subgroups H of G with $|G : H| \le 2$. If $\Sigma^G = \{x_0\}$ then $\dim T_{x_0}(\Sigma)^K = 0$, hence the point x_0 is an isolated K-fixed point of Σ.*

Proof. Without any loss of generality, we may assume $G \neq K$. If $\dim T_{x_0}(\Sigma)^K > 0$ then there exists a subgroup L of G with $|G : L| = 2$ such that $\dim T_{x_0}(\Sigma)^L > 0$. But this contradicts Proposition 2.7. □

Proposition 2.9. *Let Σ be a $\mathbb{Z}$-homology sphere with G-action. Suppose G satisfies the following conditions.*

(1) *Each element of G is of prime-power order.*
(2) *For every element $g \in G$ of 2-power order, the order of g is less than or equal to 4, cf. the 8-condition in* [21].

Then the tangential G-representations at G-fixed points of Σ are mutually isomorphic as real G-modules.

Proof. If $\Sigma^G = \{x, y\}$ then by a slight generalization of Sanchez' theorem on the Smith equivalence, see [24, Corollary 1.11], [23, p.194, Theorem 0.4], [12], [21, 8-condition Lemma], $T_x(\Sigma)$ is isomorphic to $T_y(\Sigma)$. Now we suppose $\Sigma^G \supsetneq \{x, y\}$ $(x \neq y)$. For an arbitrary element $g \in G$ of prime-power order, say $\mathrm{ord}(g) = p^k$ (p a prime), the g-fixed-point set Σ^g is a $\mathbb{Z}_p$-homology sphere. Since $|\Sigma^g| \ge 3$, we see that Σ^g is connected and that $\mathrm{res}^G_{\langle g\rangle} T_x(\Sigma)$ is isomorphic to $\mathrm{res}^G_{\langle g\rangle} T_y(\Sigma)$. Therefore $T_x(\Sigma)$ is isomorphic to $T_y(\Sigma)$ as real G-modules. □

Note that the conditions (1) and (2) in Proposition 2.9 are fulfilled for $G = A_5$. Also remark that $|S_5/A_5| = 2$.

Proposition 2.10. *Let G and L be the groups S_5 and A_5, respectively, and Σ a $\mathbb{Z}$-homology sphere with G-action. If $|\Sigma^G| \equiv 1 \mod 2$ then the following holds.*

(1) $T_x(\Sigma) \cong T_y(\Sigma)$ *as real L-modules for all $x, y \in \Sigma^L$.*
(2) $|\Sigma^L| \equiv 1 \mod 2$.

Proof. The property (1) follows from Proposition 2.9. By Proposition 2.7, there is a G-fixed point x_0 of Σ being isolated in Σ^L. By the property (1), all points in Σ^L are isolated, and hence $|\Sigma^L| < \infty$. By Proposition 2.1, we obtain $|\Sigma^L| \equiv 1 \mod 2$. □

3. A_5-actions on homology spheres

In this section we let $L = A_5$. There are precisely five irreducible real L-modules (up to isomorphisms), say $\mathbb{R}$, $U_{3,1}$, $U_{3,2}$, U_4 and U_5, which are mutually non-isomorphic and $\dim \mathbb{R} = 1$, $\dim U_{3,1} = \dim U_{3,2} = 3$, $\dim U_4 = 4$, and $\dim U_5 = 5$.

Theorem 3.1. *Let Σ be a $\mathbb{Z}$-homology sphere with L-action. If $|\Sigma^L| \equiv 1 \mod 2$ then for any $x \in \Sigma^L$, $T_x(\Sigma)$ contains an irreducible real L-submodule of dimension* 3.

Proof. The data of $\dim U^H$ for irreducible real L-modules U and subgroups H of L are as in the next table. In the table A_4 stands for the alternating group on four letters.

Table 3.1.

	E	C_2	C_3	C_5	D_4	D_6	D_{10}	A_4	A_5
$U_{3,i}$	3	1	1	1	0	0	0	0	0
U_4	4	2	2	0	1	1	0	1	0
U_5	5	3	1	1	2	1	1	0	0

Assume that $T_x(\Sigma)$ is isomorphic to $U_4^{\oplus a} \oplus U_5^{\oplus b}$ for non-negative integers a and b. Then Σ^{C_2} is a $\mathbb{Z}_2$-homology sphere, and the equalities $\dim \Sigma^{C_2} = 2a + 3b$, $\dim \Sigma^{D_4} = a + 2b$, $\dim \Sigma_0^{D_6} = a+b$ hold, where $\Sigma_0^{D_6}$ is the union of all connected components of Σ^{D_6} intersecting with Σ^{D_4}. The intersection form

$$H_{a+2b}(\Sigma^{C_2}; \mathbb{Z}_2) \times H_{a+b}(\Sigma^{C_2}; \mathbb{Z}_2) \longrightarrow \mathbb{Z}_2$$

is trivial. On the other hand, $|\Sigma^{D_4} \cap \Sigma^{D_6}| \equiv 1 \mod 2$, which shows a contradiction. Thus $T_x(\Sigma)$ contains an L-submodule isomorphic to $U_{3,1}$ or $U_{3,2}$. □

According to [2, 16, 17], the spheres S^n, $n \geq 6$, have one-f.p. actions of L.

Proposition 3.2. *Let Σ be an n-dimensional homology sphere with L-action such that $|\Sigma^L| < \infty$. Then the following holds.*

(1) *If* $n = 3$ *or* 6 *then* $|\Sigma^L| \leq 2$.
(2) *If* $n = 9$ *and* $|\Sigma^L| \equiv 1 \mod 2$ *then* $|\Sigma^L| = 1$.

Proof. Let Σ be a $\mathbb{Z}$-homology sphere with an L-action of dimension 3, 6 or 9 such that $0 < |\Sigma^L| < \infty$ and $x_0 \in \Sigma^L$. Then $T_{x_0}(\Sigma)$ is isomorphic to a direct sum of

(Case 1) 3-dimensional irreducible real L-modules, or
(Case 2) 4- and 5-dimensional irreducible real L-modules.

In Case 1, we have $\dim T_{x_0}(\Sigma)^{D_4} = 0$ and hence, by Smith's theorem, $|\Sigma^{D_4}| = 2$, which shows $|\Sigma^L| \leq 2$. For Σ satisfying $\dim \Sigma = 9$ and $|\Sigma^L| \equiv 1 \mod 2$, Theorem 3.1 implies that Case 2 does not happen. Therefore we conclude $|\Sigma^L| = 1$. □

4. S_5-actions on homology spheres

In this section we let $G = S_5$ and $L = A_5$. There are precisely seven irreducible real G-modules (up to isomorphisms), say $\mathbb{R}$, $\mathbb{R}_\pm$, $V_{4,1}$, $V_{4,2}$, $V_{5,1}$, $V_{5,2}$, V_6, where $\dim \mathbb{R} = \dim \mathbb{R}_\pm = 1$, $\dim V_{k,i} = k$, and $\dim V_6 = 6$.

Theorem 4.1. *Let Σ be a $\mathbb{Z}$-homology sphere with G-action. If $|\Sigma^G| \equiv 1 \mod 2$ then for any $x \in \Sigma^G$, $T_x(\Sigma)$ contains an irreducible real G-submodule of dimension* 6*.*

Proof. We remark $\mathrm{res}^G_L V_{4,i} \cong U_4$, $\mathrm{res}^G_L V_{5,i} \cong U_5$, and $\mathrm{res}^G_L V_6 \cong U_{3,1} \oplus U_{3,2}$, where U_4, U_5, $U_{3,1}$, and $U_{3,2}$ are irreducible real L-modules in the previous section. Since $|\Sigma^G| \equiv 1 \mod 2$, we obtain $|\Sigma^L| \equiv 1 \mod 2$ by Proposition 2.10. Theorem 3.1 implies that $\mathrm{res}^G_L T_x(\Sigma)$ contains an L-submodule isomorphic to $U_{3,1}$ or $U_{3,2}$. Therefore $T_x(\Sigma)$ contains a G-submodule isomorphic to V_6. □

Theorem 4.2. *Let Σ be a $\mathbb{Z}$-homology sphere with G-action. If $|\Sigma^G| \equiv 1 \mod 2$ then $\dim \Sigma$ does not lie in $T = \{0, 1, 2, 3, 4, 5, 7, 8, 9, 13\}$.*

Proof. Set $n = \dim \Sigma$ and let a, b, c be non-negative integers such that $n = 4a + 5b + 6c$. If $n \in T$ then we get $c = 0$. Thus Theorem 4.2 follows from Theorem 4.1. □

5. $SL(2, 5)$-actions on homology spheres

Let G be the group $SL(2, 5)$ and Z the center of G. There are precisely nine irreducible real G-modules (up to isomorphisms), say $\mathbb{R}$, $U_{3,1}$, $U_{3,2}$, U_4, U_5, $W_{4,1}$, $W_{4,2}$, W_8, and W_{12}, where $\dim \mathbb{R} = 1$,

$$\dim U_{3,i} = 3,\ U^Z_{3,i} = U_{3,i},\ \dim W_{4,i} = 4 \text{ and } W^Z_{4,i} = 0 \text{ for } i = 1, 2,$$
$$\dim U_k = k \text{ and } U^Z_k = U_k \text{ for } k = 4, 5,$$
$$\dim W_k = k \text{ and } W^Z_k = 0 \text{ for } k = 8, 12.$$

For an integer $m \geq 2$, let C_m denote the cyclic group of order m and Q_m the quaternion group or generalized quaternion group of order m. We remark that for a real G-module W with $W^Z = 0$, the equality $W^H = 0$ holds for all subgroups H of G isomorphic to C_4, Q_8, or Q_{12}.

The next result is a generalization of [3, Theorem 1.2 (ii)].

Theorem 5.1. *Let Σ be a $\mathbb{Z}_2$-homology sphere with G-action. If $|\Sigma^G| \equiv 1 \mod 2$ then the set of all points $x \in \Sigma^G$ such that $T_x(\Sigma)$ contains an irreducible real G-submodule of dimension* 3 *consists of odd-number points.*

Proof. By the hypothesis $|\Sigma^G| \equiv 1 \mod 2$, it suffices to consider the case $\dim \Sigma > 0$. Let F denote the set consisting of points $x \in \Sigma^G$ such that each tangential representation $T_x(\Sigma)$ contains an irreducible real G-submodule of dimension 3, and set $F' = \Sigma^G \setminus F$. It suffices to prove $|F'| \equiv 0 \mod 2$.

If F' is empty then it is obvious that $|F| = |\Sigma^G| \equiv 1 \mod 2$. Therefore we may assume that F' is not empty. Let x and y be points in F and F', respectively. Then we have

$$T_x(\Sigma^Z) \cong U \oplus U_4^{\oplus a} \oplus U_5^{\oplus b}$$

for a direct sum $U \neq 0$ of 3-dimensional irreducible real G-modules, say $\dim U = 3k > 0$, and non-negative integers a and b, and

$$T_y(\Sigma^Z) \cong U_4^{\oplus s} \oplus U_5^{\oplus t}$$

for some non-negative integers s and t with $4s + 5t > 0$. Note that for $H \cong Z, C_4, Q_8$, the H-fixed point sets Σ^H are $\mathbb{Z}_2$-homology spheres and $G/Z \cong A_5$ acts on Σ^Z. By Table 3.1 we get the equalities

$$\begin{aligned} &\dim \Sigma = 3k + 4a + 5b,\ \dim \Sigma = 4s + 5t, \\ &\dim \Sigma^{C_4} = k + 2a + 3b,\ \dim \Sigma^{C_4} = 2s + 3t, \\ &\dim \Sigma^{Q_8} = a + 2b,\ \dim \Sigma^{Q_8} = s + 2t, \end{aligned}$$

as well as

$$(5.1)\qquad \begin{aligned} &\dim T_x(\Sigma^{Q_8}) + \dim T_x(\Sigma^{Q_{12}}) = \dim \Sigma^{C_4} - k, \\ &\dim T_y(\Sigma^{Q_8}) + \dim T_y(\Sigma^{Q_{12}}) = \dim \Sigma^{C_4}. \end{aligned}$$

It is remarkable that the integers s and t are uniquely determined by $\dim \Sigma^{C_4}$ and $\dim \Sigma^{Q_8}$. In addition, we note

$$\Sigma^{Q_8} \cap \Sigma^{Q_{12}} = \Sigma^G.$$

Let $\Sigma_0^{Q_{12}}$ denote the union of all connected components C of $\Sigma^{Q_{12}}$ such that $C \cap \Sigma^G \neq \emptyset$ and $\dim \Sigma^{Q_8} + \dim C = \dim \Sigma^{C_4}$. The equality (5.1) implies that $F \cap \Sigma_0^{Q_{12}} = \emptyset$ and $F' = \Sigma_0^{Q_{12}} \cap \Sigma^G$, and furthermore that the mod-2 intersection number of Σ^{Q_8} and $\Sigma_0^{Q_{12}}$ in Σ^{C_4} is equal to $|F'|$ in $\mathbb{Z}_2$. Let

$$\varphi : H_{s+2t}(\Sigma^{C_4}; \mathbb{Z}_2) \times H_{s+t}(\Sigma^{C_4}; \mathbb{Z}_2) \longrightarrow \mathbb{Z}_2$$

be the canonical mod-2 intersection form on Σ^{C_4}. Since Σ^{C_4} is a $\mathbb{Z}_2$-homology sphere, the intersection form φ is trivial, hence the mod-2 intersection number of Σ^{Q_8} and $\Sigma_0^{Q_{12}}$ in Σ^{C_4} is zero. Therefore we have proved $|F'| \equiv 0 \mod 2$. □

Theorem 5.2. *Let G be the group $SL(2,5)$ and Σ a $\mathbb{Z}_2$-homology sphere with effective G-action. If* $|\Sigma^G| \equiv 1 \mod 2$ *then* $\dim \Sigma$ *does not lie in* $\{0, 1, 2, 3, 4, 5, 6, 8, 9\}$.

Proof. By Theorem 5.1 there exists a G-fixed point x of Σ such that $T_x(\Sigma)$ contains a 3-dimensional irreducible real G-submodule. Since the action is effective, $T_x(\Sigma)$ contains a G-submodule isomorphic to $W_{4,1}$, $W_{4,2}$, W_8, or W_{12}. Now we can readily see $\dim \Sigma \geq 7$ and $\dim \Sigma \neq 8, 9$. □

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References

[1] A. Bak and M. Morimoto: *Equivariant surgery and applications*, in: Proc. of Conf. on Topology in Hawaii 1990, K.H. Dovermann (ed.), World Scientific Publ., Singapore, 13–25, 1992.

[2] A. Bak and M. Morimoto: *The dimension of spheres with smooth one fixed point actions*, Forum Math. **17** (2005), 199–216.

[3] A. Borowiecka: *$SL(2,5)$ has no smooth effective one-fixed-point action on S^8*, Bull. Pol. Acad. Sci. Math. **64** (2016), 85–94.

[4] A. Borowiecka and P. Mizerka: *Excluding smooth effective one-fixed point actions of finite Oliver groups on low-dimensional spheres*, arXiv:1805.00447v2.

[5] G.E. Bredon: Introduction to Compact Transformation Groups, Academic Press, New York, 1972.

[6] N.P. Buchdahl, S. Kwasik and R. Schultz: *One fixed point actions on low-dimensional spheres*, Invent. math. **102** (1990), 633–662.

[7] P.E. Conner and E.E. Floyd:Differentiable Periodic Maps, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1964.

[8] S. Demichelis: *The fixed point set of a finite group action on a homology four sphere*, L'Enseign. Math. **35** (1989), 107–116.

[9] T. tom Dieck: Transformation Groups, Walter de Gruyter, Berlin-New York, 1987.

[10] M. Furuta: *A remark on a fixed point of finite group action on S^4*, Topology **28** (1989), 35–38.

[11] E. Laitinen and M. Morimoto: *Finite groups with smooth one fixed point actions on spheres*, Forum Math. **10** (1998), 479–520.

[12] E. Laitinen and K. Pawałowski: *Smith equivalence of representations for finite perfect groups*, Proc. Amer. Math. Soc. **127** (1999), 297–307.

[13] E. Laitinen and P. Traczyk: *Pseudofree representations and* 2*-pseudofree actions on spheres*, Proc. Amer. Math. Soc. **97** (1986), 151–157.

[14] D. Montgomery and H. Samelson: *Fiberings with singularities*, Duke Math. J. **13** (1946), 51–56.

[15] M. Morimoto: *On one fixed point actions on spheres*, Proc. Japan Academy, Ser. A Math Sci. **63** (1987), 95–97.

[16] M. Morimoto: *S^4 does not have one fixed point actions*, Osaka J. Math. **25** (1988), 575–580.

[17] M. Morimoto: *Most of the standard spheres have one fixed point actions of A_5*, in: Transformation Groups, K. Kawakubo (ed.), Lecture Notes in Mathematics **1375**, Springer-Verlag, Berlin-Heidelberg, 240–259, 1989.

[18] M. Morimoto: *Most standard spheres have one-fixed-point actions of A_5. II*, K-Theory **4** (1991), 289–302.

[19] M. Morimoto: *Smith equivalent $Aut(A_6)$-representations are isomorphic*, Proc. Amer. Math. Soc. **136** (2008), 3683–3688.

[20] R. Oliver: *Fixed point sets of group actions on finite acyclic complexes*, Comment. Math. Helv. **50** (1975), 155–177.

[21] K. Pawałowski and R. Solomon: *Smith equivalence and finite Oliver groups with Laitinen number* 0 *or* 1, Algebr. Geom. Topol. **2** (2002), 843–895.

[22] T. Petrie: *One fixed point actions on spheres I*, Adv. Math. **46** (1982), 3–14.

[23] T. Petrie and J. Randall: Transformation Groups on Manifolds, Marcel Dekker, Inc., New York-Basel, 1984.

[24] C.U. Sanchez: *Actions of groups of odd order on compact, orientable manifolds*, Proc. Amer. Math. Soc. **54** (1976), 445–448.

[25] P.A. Smith: *Transformations of finite period*, Ann. of Math. **39** (1938), 127–164.

[26] E. Stein: *Surgery on products with finite fundamental group*, Topology **16** (1977), 473–493.

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