NOTICE

The paper was withdrawn by the author after its first appearance online, upon learning that the same result as the main theorem of the paper had already been proved by Murai in *Characterizations of p-nilpotent groups* which appeared in Osaka J. Math. 31 (1994), 1-8. The paper has not been included in the printed version of this volume.

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ON A THEOREM OF MURAI

Dedicated to M. Murai

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Abstract

If B is a p-block of a finite group G, then the intersection of the kernels of the height zero characters in B has a normal p-complement.

If B is a p-block of a finite group G, it was already known to Richard Brauer himself that the group

$$\bigcap_{\chi \in \mathrm{Irr}(B)} \ker(\chi)$$

has order not divisible by p. (See for instance Theorem 6.10 of [3].) If $\mathrm{Irr}_0(B)$ is the subset of height zero characters in $\mathrm{Irr}(B)$, we show the following.

Theorem A. *If B is a Brauer p-block of a finite group G, then*

$$\bigcap_{\chi \in \mathrm{Irr}_0(B)} \ker(\chi)$$

has a normal p-complement.

In fact, Theorem A is an easy consequence of a nice theorem of M. Murai (Theorem 4.4 of [2]) to whom this note is dedicated. The contributions of Murai to block theory are not easily overstated.

Proof of Theorem A. Let K be the intersection of the kernels of the height zero characters in B. Let b be a block of K covered by B. Let D be a common defect group of B and of the Fong-Reynolds correspondent of B over b (Theorem 9.14 of [3]). By Lemma 2.2 of [2], we have that the unique block $\hat{b}$ of KD that covers b has defect group D. Let $\tau \in \mathrm{Irr}_0(\hat{b})$. By Corollary 9.18 of [3], we have that $\tau_K = \theta \in \mathrm{Irr}(K)$, and θ has height zero in b. Now, by Theorem 4.4 of [2], we have that θ lies below some $\chi \in \mathrm{Irr}_0(B)$. By the definition of K, we have that $\theta = 1_K$, and therefore τ is linear. Fix some (linear) $\tau \in \mathrm{Irr}_0(\hat{b})$, and consider

$$\tilde{b} = \{\bar{\tau}\gamma \,|\, \gamma \in \mathrm{Irr}(\hat{b})\} \subseteq \mathrm{Irr}(KD)\,,$$

where $\bar{\tau}$ is the complex conjugate of τ. By using Theorem 3.19 of [3], for instance, we check that $\tilde{b}$ is a block of KD. Furthermore, every height zero character in $\tilde{b}$ is linear. Since 1_{KD} is in $\tilde{b}$, we have that $\tilde{b}$ is the principal block of KD. Now we use a result of Isaacs-Smith,

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Corollary 3 of [1], to conclude that KD has a normal p-complement. Hence, K has a normal p-complement, as desired. □

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References

[1] I.M. Isaacs and S.D. Smith: *A note on groups of p-length one*, J. Algebra **38** (1976), 531–535.

[2] M. Murai: *Block induction, normal subgroups and characters of height zero*, Osaka J. Math. **31** (1994), 9–25.

[3] G. Navarro: Characters and Blocks of Finite Groups, London Mathematical Society Lecture Note Series, **250**, Cambridge University Press, Cambridge, 1998.

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