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THE DUAL THURSTON NORM AND THE GEOMETRY OF CLOSED 3-MANIFOLDS

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Abstract

We investigate closed Riemannian 3-manifolds which satisfy an extremal condition. Using monopole equations and considering the action of the covering transformations, we decide the geometric structure of such 3-manifolds. As a result, we characterize the geometry of 3-manifolds with monopole classes whose dual Thurston norm is equal to one.

1. Introduction

In low dimensional topology, there are a large number of works to find sharp estimates for the genus of embedded surfaces. The most famous one is the proof of the Thom conjecture given by Kronheimer and Mrowka ([9]). Auckly applied their method to the 3-dimensional case ([1]). On the other hand, Kronheimer and Mrowka refined Auckly's result and described the relationship between the dual Thurston norm and scalar curvature ([8], [10]). Moreover, McMullen ([13]), Ozsváth and Szabó ([14]) and Vidussi ([17]) related the Thurston norm and Alexander polynomial, Heegaard Floer homology. However, the study of geometric aspects of the dual Thurston norm still remains open.

In this article, we investigate the Riemannian metrics which satisfy some equality for the L^2-norms of the scalar curvature and a monopole class, and determine the geometric structure of such closed Riemannian 3-manifolds. Geometric characterization of 3-manifolds has recently drawn a great attention. Our study deals with the geometric structure of 3-manifolds by applying the 3-dimensional Seiberg-Witten theory to the dual Thurston norm.

Let M be a closed, oriented 3-manifold with $b_1(M) > 0$, and suppose that M contains neither non-separating 2-spheres nor tori. These assumptions make one to be easy to deal with the 3-dimensional Seiberg-Witten theory. Let $\alpha \in H^2(M;\mathbf{R})$. The dual Thurston norm of α is defined by

$$|\alpha|_* := \sup_{\Sigma} \frac{\langle \alpha, [\Sigma]\rangle}{2g(\Sigma)-2},$$

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the supremum being taken over all connected, oriented surfaces Σ embedded in M whose genus $g(\Sigma) \geq 2$.

We call α a monopole class, when α is the first Chern class c_1 of the complex line bundle L associated with a principal $Spin(3)^c$ bundle P over a closed, oriented 3-manifold M, such that the corresponding monopole equations have a solution for all Riemannian metrics h on M. Kronheimer and Mrowka obtained the following theorems with respect to the dual Thurston norm of a monopole class.

Theorem 1.1 ([10]). *If M is a closed, oriented, irreducible 3-manifold, then the convex hull of the monopole classes is precisely the unit ball for the dual Thurston norm on $H^2(M;\mathbf{R})$.*

Theorem 1.2 ([10]). *Let M be a closed, oriented, irreducible 3-manifold. Then the dual Thurston norm of $\alpha \in H^2(M;\mathbf{R})$ is given by*

$$|\alpha|_* = 4\pi \sup_h \frac{\|\alpha\|_h}{\|s_h\|_h},$$

the supremum being taken over all Riemannian metrics on M.

In Theorem 1.2, $\|\alpha\|_h$ is the L^2-norm of the harmonic representative of α, and $\|s_h\|_h$ is the L^2-norm of the scalar curvature s_h for the given metric h. These theorems have been proved by using the following key lemma:

Lemma 1.3 ([10]). *If $\alpha \in H^2(M;\mathbf{R})$ is a monopole class, then*

$$\|\alpha\|_h \leq \frac{1}{4\pi} \|s_h\|_h$$

for all metrics h.

We consider the metrics h which are extremal, namely, these for which

$$\|\alpha\|_h = \frac{1}{4\pi} \|s_h\|_h$$

holds for a monopole class α. In this case, from Theorem 1.2 and Lemma 1.3, we have then $|\alpha|_* = 1$. Our aim of this article is to investigate this case and to determine completely the geometric structure of M as follows:

Main Theorem. *Let M be a connected, closed, oriented 3-manifold with $b_1(M) > 0$ and $\alpha \in H^2(M;\mathbf{R})$ be a monopole class of M. If there exists a Riemannian metric h on M with $\|\alpha\|_h = \|s_h\|_h/4\pi$, then (1) the universal covering space of (M, h) is isometric to the Riemannian product $(\mathbf{R}, dt^2) \times (H^2, g_H)$ (g_H is a hyperbolic metric)* *so that* $M = (\mathbf{R} \times H^2)/\Gamma$, *where* Γ *is a group of orientation preserving isometries of* $\mathbf{R} \times H^2$, *and* (2) *if, in addition, the image of the projection* $\phi\colon \Gamma \to \mathrm{Isom}(\mathbf{R})$ *is discrete in* $\mathrm{Isom}(\mathbf{R})$, *then* (M, h) *is either a fiber bundle over* S^1 *with closed Riemann surfaces as fibers or is a* $\mathbf{Z}_2$*-quotient of this fiber bundle so that* (M, h) *is a fiber space over* $I = [0, 1]$ *with singular fibers at the end points.*

Main Theorem shows that the universal covering space of (M, h) is $\mathbf{E}^1 \times H^2$. This is one of "the eight model geometries" introduced by Thurston ([16]). From the result of Scott ([15]), M admits a Seifert manifold structure.

We have $b_1(M) = 2g(\Sigma) + 1$ from the Leray-Hirsch theorem ([2]) for Riemann surface Σ appearing in the fibers of M. Therefore we obtain $b_1(M) \geq 5$, because we assume $g(\Sigma) \geq 2$ to define the dual Thurston norm.

The monopole class α can be described as

$$\alpha = \frac{i}{2\pi}\left[C_1 \frac{dx \wedge dy}{y^2}\right] = \left[C_2\, dv_{g_\Sigma}\right],$$

where C_1, C_2 are some constants and dv_{g_Σ} is the area form of Σ.

One of significant invariants is the Yamabe invariant $Y(M)$. For the closed 3-manifold M carrying the geometric structure in Main Theorem, we obtain the following corollary:

Corollary. *Let* M *be a connected, closed, oriented* 3*-manifold with* $b_1(M) > 0$ *and* $\alpha \in H^2(M; \mathbf{R})$ *be a monopole class of* M. *If there exists a Riemannian metric* h *on* M *with* $\|\alpha\|_h = \|s_h\|_h/4\pi$, *then* $Y(M) = 0$.

Next, in Section 2, we review the 3-dimensional Seiberg-Witten theory and finally, in Section 3, we give the proof of Main Theorem and Corollary.

2. The 3-dimensional Seiberg-Witten theory

Let M be a closed, oriented 3-manifold. Then there exists a $Spin(3)^c$ structure which defines the principal $Spin(3)^c$ bundle P associated to the tangent bundle TM of M. Let L be the complex line bundle and W be the spinor bundle associated with P. We consider the monopole equations, namely, equations for a unitary connection A on L and a section Φ of W.

$$\begin{cases} c(*F_A) = \Phi \otimes \Phi^* - \dfrac{1}{2}|\Phi|^2 \mathrm{Id}_W \\ D_A \Phi = 0 \end{cases}$$

In the first equation, c is the Clifford multiplication $T^*M \to \mathrm{End}(W)$, $*$ is the Hodge star operator and F_A is the curvature form of A. In the second, D_A is the Dirac operator

$$\Gamma(W) \xrightarrow{\nabla_A} \Gamma(T^*M \otimes W) \xrightarrow{c} \Gamma(W).$$

Suppose that there are no reducible solutions, and for every metric h, the equations have an irreducible solution, that is, a solution with $\Phi \neq 0$. (In Main Theorem, $b_1(M) > 0$ assures the irreducibility of solutions. See [3], for example.)

If there exists a Riemannian metric h on M with $\|\alpha\|_h = \|s_h\|_h/4\pi$ for a monopole class $\alpha \in H^2(M;\mathbf{R})$, then we have $\nabla_A\Phi = 0$.

To see this, we review the proof of Lemma 1.3 (or for a similar argument, refer to [5]). Let (A, Φ) be an irreducible solution to the monopole equations. Then by using the Bochner-Weitzenböck formula, one obtains

$$0 = D_A^* D_A \Phi = \nabla_A^* \nabla_A \Phi + \frac{s_h}{4}\Phi + \frac{1}{2}c(*F_A)\Phi.$$

Therefore one gets

$$\left\langle \nabla_A^* \nabla_A \Phi, \Phi \right\rangle + \frac{s_h}{4}\langle \Phi, \Phi\rangle + \frac{1}{4}\left\langle |\Phi|^2\Phi, \Phi \right\rangle = 0$$

and integrating

$$\int_M |\nabla_A \Phi|^2\, dv_h + \int_M \frac{s_h}{4}|\Phi|^2\, dv_h + \frac{1}{4}\int_M |\Phi|^4\, dv_h = 0$$

or

$$(\star) \qquad \int_M \frac{s_h}{4}|\Phi|^2\, dv_h + \frac{1}{4}\int_M |\Phi|^4\, dv_h = -\int_M |\nabla_A \Phi|^2\, dv_h \leq 0.$$

Hence by using the Cauchy-Schwarz inequality, one sees

$$\int_M |\Phi|^4\, dv_h \leq \int_M (-s_h)|\Phi|^2\, dv_h \leq \sqrt{\int_M s_h^2\, dv_h}\sqrt{\int_M |\Phi|^4\, dv_h}.$$

Since the solution is irreducible, one obtains

$$\int_M |\Phi|^4\, dv_h \leq \int_M s_h^2\, dv_h.$$

Again by using the first monopole equation, one has $|\Phi|^2 = 2|F_A|$. Therefore one gets

$$\int_M 4|F_A|^2\, dv_h \leq \int_M s_h^2\, dv_h$$

and hence

$$\|F_A\|_h^2 \leq \left\| \frac{s_h}{2} \right\|_h^2.$$

To see $\|\alpha\|_h \leq \|s_h\|_h/4\pi$ for all Riemannian metrics h on M, recall that α is a monopole class and hence $\alpha = c_1(L) = i[F_A]/2\pi$. Considering the L^2-norm of the harmonic representative of α, we get desired result.

By following the argument above, we can easily observe that if there exists a Riemannian metric h on M which satisfy $\|\alpha\|_h = \|s_h\|_h/4\pi$, then the equality holds on the inequality ($\star$). Hence we obtain $\nabla_A \Phi = 0$, from which we get detailed information about M.

3. The geometry of closed 3-manifolds with the extremal metrics

We investigate in this section the structure of M with $\nabla_A \Phi = 0$. First, it is clear that $|\Phi|$ is constant. Second, from the first monopole equation, F_A is parallel, i.e. $\nabla_X F_A = 0$, where ∇ is the Levi-Civita connection of (M, h). Moreover, by using the Bochner-Weitzenböck formula, we obtain $s_h = -|\Phi|^2$, i.e. the scalar curvature of (M, h) is negative constant. Especially from $\nabla_X F_A = 0$, we can prove the following proposition:

Proposition 3.1 ([6]). *Let $\pi : L \to M$ be a $U(1)$-principal bundle over an oriented Riemannian n-manifold (M, h) and F_A is the curvature form of a unitary connection A on L. Let $\mathcal{D}$ be the null distribution defined by*

$$\mathcal{D} := \{D_x\}_{x \in M}, \quad D_x := \{X \in T_x M \mid i_X F_A = 0\},$$

and $\mathcal{D}^\perp$ be the orthogonal complement of $\mathcal{D}$ defined by $\mathcal{D}^\perp := \{D_x^\perp\}_{x \in M}$, where $D_x^\perp$ is the orthogonal complement of D_x. If $\nabla_X F_A = 0$ for the Levi-Civita connection ∇ of (M, h), then $\mathcal{D}$ and $\mathcal{D}^\perp$ are integrable and invariant under the parallel translation.

Proof. To begin with, we will show that $\mathcal{D}$ and $\mathcal{D}^\perp$ are invariant under the parallel translation. Let $c : [0, 1] \to M$ be a piecewise smooth curve with $c(0) = x, c(1) = y$. Let $X \in D_x$ and $Y \in T_x M$ and take parallel vector fields $X(t), Y(t)$ along c such that $X(0) = X$, $Y(0) = Y$. Since F_A is parallel, we have

$$\frac{d}{dt} F_A(X(t), Y(t)) = F_A \left(\frac{\nabla X}{dt}(t), Y(t) \right) + F_A \left(X(t), \frac{\nabla Y}{dt}(t) \right) = 0.$$

Hence we obtain

$$F_A(X(t), Y(t)) = F_A(X(0), Y(0)) = 0$$

because of $X(0) \in D_{c(0)}$. Therefore we obtain $X(t) \in D_{c(t)}$. Similarly, if we have $X(0) \in D_{c(0)}^\perp$, then we can easily check $X(t) \in D_{c(t)}^\perp$, because ∇ is compatible with

h. Therefore for the parallel translation $\tau_c : T_x M \to T_y M$ along c, we obtain $\tau_c(D_x) = D_y$, $\tau_c(D_x^\perp) = D_y^\perp$.

Now we will show that $\mathcal{D}$ and $\mathcal{D}^\perp$ are integrable. For $X, Y \in \Gamma(U; \mathcal{D})$ where U is an open subset of M we have $\nabla_Y X \in \Gamma(U; \mathcal{D})$, because for any smooth vector field Z we have

$$0 = Y(F_A(X, Z)) = F_A(\nabla_Y X, Z) + F_A(X, \nabla_Y Z) = F_A(\nabla_Y X, Z).$$

Hence if we have $X, Y \in \Gamma(U; \mathcal{D})$, then $[X, Y] = \nabla_X Y - \nabla_Y X \in \Gamma(U; \mathcal{D})$. This means that $\mathcal{D}$ is integrable. Similarly, we can easily check $[X, Y] = \nabla_X Y - \nabla_Y X \in \Gamma(U; \mathcal{D}^\perp)$ for $X, Y \in \Gamma(U; \mathcal{D}^\perp)$ so that $\mathcal{D}^\perp$ is integrable. $\square$

Proof of Main Theorem. We first derive the conclusion (1). Since we consider irreducible solutions to the monopole equations, F_A is not identically zero and we have $\dim \mathcal{D}^\perp \neq 0$. Moreover, F_A is a parallel 2-form on $\mathcal{D}^\perp$, therefore we have $\dim \mathcal{D}^\perp = 2$ because of $\dim M = 3$. Since the scalar curvature of (M, h) is negative constant, the integral manifold of $\mathcal{D}^\perp$ is a Riemann surface (H^2, g_H) with negative curvature. On the other hand, the integral manifold of $\mathcal{D}$ is the 1-dimensional Euclidean space $(\mathbf{R}, dt^2)$ because of $\dim \mathcal{D} = 1$. Hence by the de Rham decomposition theorem of Riemannian manifold, the universal covering space of (M, h) is the Riemannian product $(\mathbf{R}, dt^2) \times (H^2, g_H)$.

To obtain the conclusion (2), let $\mathrm{Isom}^+(\mathbf{R} \times H^2)$ be the group of orientation preserving isometries of $\mathbf{R} \times H^2$. Γ is the subgroup of $\mathrm{Isom}^+(\mathbf{R} \times H^2)$. The connected component $\mathrm{Isom}^o(\mathbf{R} \times H^2)$ which includes the identity of $\mathrm{Isom}^+(\mathbf{R} \times H^2)$ has the following decomposition by the de Rham decomposition theorem ([7]):

$$\mathrm{Isom}^o(\mathbf{R} \times H^2) = \mathrm{Isom}^o(\mathbf{R}) \times \mathrm{Isom}^o(H^2).$$

It is clear that

$$\mathrm{Isom}^o(\mathbf{R}) = \mathrm{Isom}^+(\mathbf{R}), \quad \mathrm{Isom}^o(H^2) = \mathrm{Isom}^+(H^2),$$
$$\mathrm{Isom}^+(\mathbf{R}) = \{\alpha \mid \alpha : x \to x + t_\alpha\}, \quad \mathrm{Isom}^-(\mathbf{R}) = \theta_1 \circ \mathrm{Isom}^+(\mathbf{R}), \quad \theta_1(t) := -t.$$

On the other hand, one can easily check the following ([12]):

$$\mathrm{Isom}^+(H^2) = \left\{ f \;\middle|\; f(z) = \frac{az + b}{cz + d}, \;\; a, b, c, d \in \mathbf{R}, \;\; ad - bc = 1 \right\},$$
$$\mathrm{Isom}^-(H^2) = \theta_2 \circ \mathrm{Isom}^+(H^2), \quad \theta_2(z) := -\overline{z}.$$

Hence $\mathrm{Isom}^+(\mathbf{R} \times H^2)$ has the following decomposition:

$$\mathrm{Isom}^+(\mathbf{R} \times H^2) = (\mathrm{Isom}^+(\mathbf{R}) \times \mathrm{Isom}^+(H^2)) \sqcup (\mathrm{Isom}^-(\mathbf{R}) \times \mathrm{Isom}^-(H^2)).$$

Similarly, Γ has the decomposition $\Gamma = \Gamma_0 \sqcup \Gamma_1$ such that

$$\Gamma_0 := \Gamma \cap (\mathrm{Isom}^+(\mathbf{R}) \times \mathrm{Isom}^+(H^2)), \quad \Gamma_1 := \Gamma \cap (\mathrm{Isom}^-(\mathbf{R}) \times \mathrm{Isom}^-(H^2)).$$

REMARK. (i) Γ is fixed point free, i.e. if there exists a point $(x, p) \in \mathbf{R} \times H^2$ with $\gamma(x, p) = (x, p)$ for $\gamma \in \Gamma$, then we have $\gamma = (\mathrm{Id}_{\mathbf{R}}, \mathrm{Id}_{H^2})$.
(ii) Γ is properly discontinuous, i.e. for any compact sets K_1, K_2 on $\mathbf{R} \times H^2$, there are only a finite number of elements $\gamma \in \Gamma$ such that $\gamma(K_1) \cap K_2 \neq \emptyset$.

We consider two cases. One is (a) $\Gamma_1 = \emptyset$, and the other is (b) $\Gamma_1 \neq \emptyset$.

CASE (a). In this case, we have $\Gamma = \Gamma_0 \subset \mathrm{Isom}^+(\mathbf{R}) \times \mathrm{Isom}^+(H^2)$ and we can define the action of $\gamma \in \Gamma$ on $\mathbf{R} \times H^2$ by $\gamma(x, p) := (x + t_\gamma, \varphi_\gamma(p))$, where the map $t : \Gamma \to \mathbf{R}$, $\gamma \mapsto t_\gamma$ is a homomorphism. We consider the normal subgroup $\Gamma' := \{\gamma \in \Gamma \mid t_\gamma = 0\}$ and the exact sequence

$$\{1\} \to \Gamma' \to \Gamma \to \phi(\Gamma) \to \{1\}.$$

Hence $\Gamma/\Gamma' \cong \phi(\Gamma)$ and from the discreteness of $\phi(\Gamma) \subset \mathrm{Isom}^+(\mathbf{R})$, we obtain $\Gamma/\Gamma' \cong \mathbf{Z}$. Therefore we get the following diagram:

$$\begin{tikzcd}
\mathbf{R} \times H^2 \arrow[r, "\Gamma'"] \arrow[d, "\Gamma"] & \mathbf{R} \times (H^2/\Gamma') \arrow[d, "\Gamma/\Gamma' \cong \mathbf{Z}"] \\
(\mathbf{R} \times H^2)/\Gamma \arrow[r, equal] & (\mathbf{R} \times (H^2/\Gamma'))/\mathbf{Z} = \mathbf{R} \times_{(\mathbf{Z},\rho)} (H^2/\Gamma').
\end{tikzcd}$$

Thus we can determine the structure of M as

$$M = (\mathbf{R} \times (H^2/\Gamma'))/\mathbf{Z} = \mathbf{R} \times_{(\mathbf{Z},\rho)} (H^2/\Gamma').$$

From this diagram, M is regarded as a fiber bundle over $S^1 = \mathbf{R}/\mathbf{Z}$ with Riemann surfaces as fibers. Since each fiber is an inverse image of a point in S^1, it is compact.

CASE (b). Denote $(\mathbf{R} \times H^2)/\Gamma_0$ by M_0. By the same argument as Case (a), M_0 can be written as $M_0 = (\mathbf{R} \times (H^2/\Gamma'))/\mathbf{Z}$. Recall that we have $\Gamma = \Gamma_0 \sqcup \Gamma_1$, and by the definition of Γ_0 and Γ_1, we see $\Gamma/\Gamma_0 \cong \mathbf{Z}_2$ so that there exists $\gamma \in \Gamma_1$ such that $[\gamma] \in \Gamma/\Gamma_0$ is a generator of Γ/Γ_0. Therefore we obtain that $M = (\mathbf{R} \times H^2)/\Gamma$ is written as $M = M_0/(\Gamma/\Gamma_0) = M_0/\sim$, where $[x, p] \sim [y, p_1]$ if and only if $(y, p_1) = \gamma'\gamma(x, p)$ for some $\gamma' \in \Gamma_0$. By the result of Case (a), M_0 is a fiber bundle over S^1 with closed Riemann surfaces as fibers. Since

$$\gamma = (\theta_1, \theta_2) \circ \gamma_0, \quad \gamma_0 \in \mathrm{Isom}^+(\mathbf{R}) \times \mathrm{Isom}^+(H^2),$$

we have $(\theta_1 \circ \phi(\gamma_0))^2 = \mathrm{Id}_{\mathbf{R}}$ so that the action of $\mathbf{Z}_2$ descends to an action on the base space S^1 which has two fixed points. In fact, $\theta_1 \circ \phi(\gamma_0) : \mathbf{R} \to \mathbf{R}$ maps x to $-x - t_{\gamma_0}$

so that the points $[x_1], [x_2]$ in $S^1 = \mathbf{R}/\mathbf{Z}$ are exactly fixed points of the $\mathbf{Z}_2$-action, $x_1 = -t_{\gamma_0}/2$, $x_2 = -(t_{\gamma_0}+1)/2$. Consequently, the fibers are singular over these two fixed points and regular over the other points. □

Proof of Corollary. We first take Γ-invariant metrics $\tilde{g}_\delta := \delta dt^2 \oplus g_H$ on $\widetilde{M}$ with parameter $\delta > 0$, and define metrics g_δ on $M = \widetilde{M}/\Gamma$ such that $\tilde{g}_\delta = \pi^* g_\delta$ where $\pi : \widetilde{M} \to M$. For g_δ, we can easily check

$$\inf_{\delta>0} \int_M \left|s_{g_\delta}\right|^{3/2} dv_{g_\delta} = 0.$$

Therefore $Y(M) \geq 0$ by Proposition 5 in [11].

Suppose $Y(M) > 0$. Then there exists a metric h such that the Yamabe constant $Y_{[h]}(M) > 0$, because by definition

$$Y(M) = \sup_{[g]} Y_{[g]}(M) = \sup_{[g]} \inf_{\tilde{g}\in[g]} \frac{\int_M s_{\tilde{g}}\, dv_{\tilde{g}}}{\left(\int_M dv_{\tilde{g}}\right)^{1/3}},$$

the supremum being taken over all conformal classes on M. Hence from the Yamabe problem, we have a metric of positive scalar curvature. However, from the strongly maximum principle $|\Phi|^2 \leq \max\{0, -s_h\}$, Φ vanishes. This implies that the solution of monopole equations is reducible and contradicts the irreducibility of solutions. Hence we obtain $Y(M) = 0$. □

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References

[1] D. Auckly: *The Thurston norm and three-dimensional Seiberg-Witten theory*, Osaka J. Math. **33** (1996), 737–750.

[2] R. Bott and L.W. Tu: Differential Forms in Algebraic Topology, Springer-Verlag, New York-Berlin, 1982.

[3] A.L. Carey and B.L. Wang: *Notes on Seiberg-Witten-Floer Theory*, Contemp. Math. **258** (2000), 71–85.

[4] R. Friedman and J.W. Morgan: Smooth Four-Manifolds and Complex Surfaces, Springer-Verlag, Berlin, 1994.

[5] M. Itoh: *Almost Kähler 4-manifold, L^2-scalar curvature functional and Seiberg-Witten equations*, Internat. J. Math. **15** (2004), 573–580.

[6] M. Itoh and T. Satou: *Circle bundle metrics and the conformal flatness*, preprint (1999).

[7] S. Kobayashi and K. Nomizu: Foundations of Differential Geometry, I, John Wiley & Sons, Inc., New York, 1963.
[8] P.B. Kronheimer: *Embedded surfaces and gauge theory in three and four dimensions*; in Surveys in Differential Geometry, III, Int. Press, Boston, MA, 1998, 243–298.
[9] P.B. Kronheimer and T.S. Mrowka: *The genus of embedded surfaces in the projective plane*, Math. Res. Lett. **1** (1994), 797–808.
[10] P.B. Kronheimer and T.S. Mrowka: *Scalar curvature and the Thurston norm*, Math. Res. Lett. **4** (1997), 931–937.
[11] C. LeBrun: *Kodaira Dimension and the Yamabe Problem*, Comm. in Anal. Geom. **7** (1999), 133–156.
[12] Y. Matsushima: Differentiable Manifolds, Marcel Dekker, Inc., New York, 1972.
[13] C.T. McMullen: *The Alexander polynomial of a* 3*-manifold and the Thurston norm on cohomology*, Ann. Sci. École Norm. Sup. (4) **35** (2002), 153–171.
[14] P. Ozsváth and Z. Szabó: *Holomorphic disks and genus bounds*, Geom. Topol. **8** (2004), 311–334.
[15] P. Scott: *The geometries of* 3*-manifolds*, Bull. London Math. Soc. **15** (1983), 401–487.
[16] W.P. Thurston: Three-Dimensional Geometry and Topology, I, Princeton University Press, Princeton, NJ, 1997.
[17] S. Vidussi: *Norms on the cohomology of a* 3*-manifold and SW theory*, Pacific J. Math. **208** (2003), 169–186.

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