CORRECTION TO MY PAPER "ON THE EXISTENCE OF UNRAMIFIED SEPARABLE INFINITE SOLVABLE EXTENSIONS OF FUNCTION FIELDS OVER FINITE FIELDS" IN NAGOYA MATHEMATICAL JOURNAL VOL. 13 (1958)

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1.1. In the above referred paper we have said that, for the proof of the theorem, it is sufficient to prove lemmas 1 and 2. But it is not correct. A correct proof is given in the followings.

We assume that

1° $q \geqq 11$,

2° $g_K > 1$,

3° L/K is an unramified separable normal extension which is regular over k,

4° $\mathfrak{G}$ is a subgroup of $J_L(\quad, k)$ such that $L(\mathfrak{G})/K$ is normal and $J_L(\quad, k)/\mathfrak{G}$ is of type $(\overbrace{l, \ldots, l}^{t})$, where l is a prime number,

5° $[L(\mathfrak{G}): L] = l^s m$, where $(l, m) = 1$.

Instead of lemma 2, we must prove the following lemmas:

LEMMA 3. *If $G(L(\mathfrak{G})/L)$ is contained in the center of $G(L(\mathfrak{G})/K)$, there exists a subgroup $\mathfrak{G}'$ in $J_L(\quad, k)$ such that* i) *$L(\mathfrak{G}')/K$ is normal and* ii) $[L(\mathfrak{G}): L(\mathfrak{G}')] = l$.

LEMMA 4. *If there exists b in $J_{L(\mathfrak{G})}(\quad, k)$ such that $a(\varepsilon_\nu) + (\delta_{J_{L(\mathfrak{G})}} - \eta(\varepsilon_\nu))b \in A_{L(\mathfrak{G})/L}(\quad, k)$ for every $\varepsilon_\nu \in G(L(\mathfrak{G})/L)$, then there exists* $\mathfrak{G}_1$ in $J_{L(\mathfrak{G})}(\quad, k)$ *such that* i) *$L(\mathfrak{G})(\mathfrak{G}_1)/K$ is normal and* ii) $L(\mathfrak{G})(\mathfrak{G}_1) \supsetneqq L(\mathfrak{G})$.

LEMMA 5. *If $[L(\mathfrak{G}): L] = l$, there exists b in $J_{L(\mathfrak{G})}(\quad, k)$ such that $a(\varepsilon) + (\delta_{J_{L(\mathfrak{G})}} - \eta(\varepsilon))b \in A_{L(\mathfrak{G})/L}(\quad, k)$, where ε is a generator of $G(L(\mathfrak{G})/L)$.*

LEMMA 6. *If $[B_{L(\mathfrak{G})/L}(\quad, k): \{0\}]$ is not coprime to m, then there exists $\mathfrak{G}_1$ in $J_{L(\mathfrak{G})}(\quad, k)$ such that* i) *$L(\mathfrak{G})(\mathfrak{G}_1)/K$ is normal and* ii) $L(\mathfrak{G})(\mathfrak{G}_1) \supsetneqq L(\mathfrak{G})$.

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LEMMA 7. *If* $[B_{L(\mathfrak{G})/L}(\ ,k):\{0\}]$ *is coprime to* m *and there exists no* b *in* $J_{L(\mathfrak{G})}(\ ,k)$ *such that* $a(\varepsilon_\nu)+(\delta_{J_{L(\mathfrak{G})}}-\eta(\varepsilon_\nu))b\in A_{L(\mathfrak{G})/L}(\ ,k)$ *for every* $\varepsilon_\nu\in G(L(\mathfrak{G})/L)$, *then there exist subgroups* $\mathfrak{G}'$ *and* $\mathfrak{G}''$ *of* $J_L(\ ,k)$ *such that* i) $L(\mathfrak{G}')/K$ *and* $L(\mathfrak{G}'')/K$ *are normal,* ii) $\mathfrak{G}'\supsetneqq\mathfrak{G}''$ *and* iii) $G(L(\mathfrak{G}')/L(\mathfrak{G}''))$ *is contained in the center of* $G(L(\mathfrak{G}')/K)$.

2.1. Lemma 3 is clear.

Next we observe a property of $\{a(\sigma)\}$.

LEMMA 8. $a(\sigma\tau\sigma^{-1})-\eta(\sigma)a(\tau)=a(\sigma)-\eta(\sigma\tau\sigma^{-1})a(\sigma)$.

Proof. Since $a(\sigma\tau)=\eta(\sigma)a(\tau)+a(\sigma)$, we have

$$\begin{aligned}a(\sigma\tau\sigma^{-1})-\eta(\sigma)a(\tau)&=a(\sigma)+\eta(\sigma\tau)a(\sigma^{-1})\\&=a(\sigma)+\eta(\sigma\tau)\ (a(e)-\eta(\sigma^{-1})a(\sigma))\\&=a(\sigma)-\eta(\sigma\tau\sigma^{-1})a(\sigma).\end{aligned}$$

2.2. Proof of lemma 4.

By the assumption in the lemma we may assume, after a suitable translation of the origin, that $a(\varepsilon_\nu)\in A_{L(\mathfrak{G})/L}(\ ,k)$ for every $\varepsilon_\nu\in G(L(\mathfrak{G})/L)$. Then, by virtue of lemma 8, we observe that

$$a(\sigma)\in\bigcap_{\varepsilon_\nu\in G(L(\mathfrak{G})/L)}(\delta_{J_{L(\mathfrak{G})}}-\eta(\varepsilon_\nu))^{-1}(A_{L(\mathfrak{G})/L}(\ ,k)).$$

We put $\mathfrak{G}_1=(\delta_{J_{L(\mathfrak{G})}}-\eta(\varepsilon_\nu))^{-1}(A_{L(\mathfrak{G})/L}(\ ,k))\cap J_{L(\mathfrak{G})}(\ ,k)$. Then $\mathfrak{G}_1=\eta(\sigma)\mathfrak{G}_1$ and $a(\sigma)\in\mathfrak{G}_1$ for every σ. Therefore, by virtue of lemma 1, it is sufficient to prove $\mathfrak{G}_1\neq J_{L(\mathfrak{G})}(\ ,k)$.

The order $[(\delta_{J_{L(\mathfrak{G})}}-\eta(\varepsilon_\nu))^{-1}(A_{L(\mathfrak{G})/L}(\ ,k)):\{0\}]$ is not greater than

$$l^{2(g_{L(\mathfrak{G})}-g_L)/l-1}[J_L(\ ,k):0].$$

On the other hand $[J_{L(\mathfrak{G})}(\ ,k):\{0\}]=[B_{L(\mathfrak{G})/L}(\ ,k):\{0\}][J_L(\ ,k):\{0\}]$ and $[B_{L(\mathfrak{G})/L}(\ ,k):\{0\}]\geqq(q-2\sqrt{q}+1)^{\mathfrak{G}_{L(\mathfrak{G})}-\mathfrak{G}_L}$. By the reason stated in the proof of lemma 2, $(q-2\sqrt{q}+1)^{l-1}>l^2$. Hence $[(\delta_{J_{L(\mathfrak{G})}}-\eta(\varepsilon_\nu)^{-1}(A_{L(\mathfrak{G})/L}(\ ,k):\{0\}]\lneqq[J_{L(\mathfrak{G})}(\ ,k):\{0\}]$. This shows that $\mathfrak{G}_1\neq J_{L(\mathfrak{G})}(\ ,k)$.

2.3. In order to prove lemma 5, we prove the following lemma:

LEMMA 9. *If* $L(\mathfrak{G})/L$ *is cyclic, then*

$$(\delta_{J_{L(\mathfrak{G})}}-\eta(\varepsilon))J_{L(\mathfrak{G})}(\ ,k)=B_{L(\mathfrak{G})/L}(\ ,k).$$

Proof. Let b be a point in $(\delta_{J_{L(\mathfrak{S})}} - \eta(\varepsilon))^{-1}(0) \cap J_{L(\mathfrak{S})}(\ \ , k)$ and $\mathfrak{B}$ be a divisor of degree zero of $L(\mathfrak{S})$. Then $\varphi(\mathfrak{B}^{\varepsilon^{-\nu}} - \mathfrak{B}) = \eta(\varepsilon^{\nu})\varphi(\mathfrak{B}) - \varphi(\mathfrak{B}) = 0$. Therefore there exists a system of elements $\{f_{\varepsilon^{\nu}}\}$ in $L(\mathfrak{S})$ such that $(f_{\varepsilon^{\nu}}) = \mathfrak{B}^{\varepsilon^{\nu}} - \mathfrak{B}$. Put $\eta_{\varepsilon^{\nu},\varepsilon^{\mu}} = f_{\varepsilon^{\nu+\mu}}(f_{\varepsilon^{\mu}}^{\varepsilon^{\nu}} f_{\varepsilon^{\nu}})^{-1}$. Then $\{\eta_{\varepsilon^{\nu},\varepsilon^{\mu}}\}$ is a k-valued cocyle. Since k-valued cohomology groups vanish, we may assume that $\{f_{\varepsilon^{\nu}}\}$ is a $L(\mathfrak{S})$-valued 1-cocycle. Since $L(\mathfrak{S})$-valued cohomology groups also vanish, we have an element g in $L(\mathfrak{S})$ such that $f_{\varepsilon} = g^{\varepsilon-1}$. Hence $(\mathfrak{B}^{\varepsilon^{-1}} - \mathfrak{B}) = (g^{\varepsilon^{-1}})^{-1} - (g)$. This shows that $\mathfrak{B} - (g)$ is a divisor of degree zero of L. Hence $b = \varphi(\mathfrak{B}) = \varphi(\mathfrak{B} - (g))$ belongs to $A_{L(\mathfrak{S})/L}(\ \ , k)$. Namely $(\eta(\varepsilon) - \delta_{J_{L(\mathfrak{S})}})^{-1}(0) = A_{L(\mathfrak{S})/L}(\ \ , k)$.

On the other hand $J_{L(\mathfrak{S})}(\ \ , k)/A_{L(\mathfrak{S})/L}(\ \ , k) \cong B_{L(\mathfrak{S})/L}(\ \ , k)$, hence $(\eta(\varepsilon) - \delta_{J_{L(\mathfrak{S})}})J_{L(\mathfrak{S})}(\ \ , k) = B_{L(\mathfrak{S})/L}(\ \ , k)$.

Proof of lemma 5.

We denote by $\rho_{L(\mathfrak{S})/L}$ the cotrace mapping of J_L into $J_{L(\mathfrak{S})}$. Since $\bar{A}_{L(\mathfrak{S})/L}(\ \ , k) \cong J_L(\ \ , k)$, $\bar{\pi}_{L(\mathfrak{S})/L}(J_L(\ \ , k))/A_{L(\mathfrak{S})/L}(\ \ , k) \cong G(L(\mathfrak{S})/L)$. Hence there exists a point $\bar{a}$ in $\bar{A}_{L(\mathfrak{S})/L}$ such that i) $l\bar{a} = \alpha_{L(\mathfrak{S})/L}a(\varepsilon)$ and ii) $\bar{\pi}_{L(\mathfrak{S})/L}\bar{a} \in J_L(\ \ , k)$. Put $a = \rho_{L(\mathfrak{S})/L}\bar{\pi}_{L(\mathfrak{S})/L}\bar{a}$. Then $\alpha_{L(\mathfrak{S})/L}a = l\bar{a} = \alpha_{L(\mathfrak{S})/L}a(\varepsilon)$. This shows that $a(\varepsilon) - a$ belongs to $B_{L(\mathfrak{S})/L}(\ \ , k)$. By virtue of lemma 9, there is a point c in $J_{L(\mathfrak{S})}(\ \ , k)$ such that $a(\varepsilon) - a = (\eta(\varepsilon) - \delta_{J_{L(\mathfrak{S})}})c$. Hence $a(\varepsilon) + (\delta_{J_{L(\mathfrak{S})}} - \eta(\varepsilon)) = a \in A_{L(\mathfrak{S})/L}(\ \ , k)$.

2.4. Proof of lemma 6.

Since $[G(L(\mathfrak{S})/L):\ \{e\}] = l^t$, there exist c_1 and c_2 in $J_{L(\mathfrak{S})}(\ \ , k)$ such that i) $l^{\lambda}c_1 = 0$ with a λ, ii) the order of c_2 is coprime to l and iii) $l^t a(\varepsilon_\nu) = (\delta_{J_{L(\mathfrak{S})}} - \eta(\varepsilon_\nu))\ (l^t c_2 + c_1)$ for $\varepsilon_\nu \in G(L(\mathfrak{S})/L)$. This shows that, after a suitable translation of the origin, we may assume that $l^{t+\lambda}\ a(\varepsilon_\nu) = 0$ for every $\varepsilon_\nu \in G(L(\mathfrak{S})/L)$.

Put $\mathfrak{S}_1 = \{a \mid a \in J_{L(\mathfrak{S})}(\ \ , k),\ l^u a \in A_{L(\mathfrak{S})/L}(\ \ , k)$ with a $u\}$. Then $a(\varepsilon_\nu) \in \mathfrak{S}_1$ for $\varepsilon_\nu \in G(L(\mathfrak{S})/L)$. On the other hand $G(L(\mathfrak{S})/L)$ is normal in $G(L(\mathfrak{S})/K)$, hence by virtue of lemma 8, we have

$$a(\sigma) \in \bigcap_{\varepsilon_\nu \in G(L(\mathfrak{S})/L)} (\eta(\varepsilon_\nu) - \delta_{J_{L(\mathfrak{S})}})^{-1}(A_{L(\mathfrak{S})/L}(\ \ , k)).$$

On the other hand there exists u such that

$$(l^u \delta_{J_{L(\mathfrak{S})}})^{-1}(A_{L(\mathfrak{S})/L}(\ \ , k) \supset \bigcap_{\varepsilon_\nu \in G(L(\mathfrak{S})/L)} (\eta(\varepsilon_\nu) - \delta_{J_{L(\mathfrak{S})}})^{-1}(A_{L(\mathfrak{S})/L}(\ \ , k)),$$

This shows that $\mathfrak{G}_1 \in a(\sigma)$. By virtue of the definition of $\mathfrak{G}_1$ and the assumption in the lemma, we have $\mathfrak{G}_1 = \eta(\sigma)\mathfrak{G}_1$ and $\mathfrak{G}_1 \neq J_{L(\mathfrak{G})}(\ \ ,k)$. Hence by virtue of lemma 1, $L(\mathfrak{G})\ (\mathfrak{G}_1)/K$ is normal and $L(\mathfrak{G})\ (\mathfrak{G}_1) \neq L(\mathfrak{G})$.

2.5. Proof of lemma 7.

Let P be the subset of $G(L(\mathfrak{G})/K)$ consisting of all its elements whose order is coprime to l. Then, by the same reason as in the proof of lemma 6, after a suitable translation of the origin, we may assume that $m^{\lambda}a(\sigma)=0$ with a λ for $\sigma \in P$. By virtue of the assumption in the lemma, we have $a(\sigma) \in A_{L(\mathfrak{G})/L}(\ \ ,k)$ for $\sigma \in P$.

Let P^* be the subgroup generated by P. Then P^* is a normal subgroup of $G(L(\mathfrak{G})/K)$. Since $a(\sigma\tau)=\eta(\sigma)a(\tau)+a(\sigma)$, we observe that $a(\sigma^*) \in A_{L(\mathfrak{G})/L}(\ \ ,k)$ for $\sigma^* \in P^*$. Since $G(L(\mathfrak{G})/L)$ is normal in $G(L(\mathfrak{G})/K)$, $G(L(\mathfrak{G})/L) \cap P^*$ is normal in $G(L(\mathfrak{G})/K)$. From the assumption in the lemma $G(L(\mathfrak{G})/L) \supsetneqq G(L(\mathfrak{G})/L) \cap P^*$. Let $L(\mathfrak{G}')$ be the subfield corresponding to $P^* \cap G(L(\mathfrak{G})/L)$. Put $P^{**} = P^*/G(L(\mathfrak{G})/L) \cap P^*$. Then, since $P^{**} \cap G(L(\mathfrak{G})/L) = \{e\}$, $P^{**}G(L(\mathfrak{G})/L)$ is a direct product $P^{**} \times G(L(\mathfrak{G})/L)$.

On the other hand, we have by virtue of lemma 8, $\alpha_{L(\mathfrak{G}')/L}a(\sigma\varepsilon_\nu\sigma^{-1}) = \eta(\sigma)\alpha_{L(\mathfrak{G}')/L}a(\varepsilon_\nu)$ for $\varepsilon_\nu \in G(L(\mathfrak{G}')/L)$. Since $G(L(\mathfrak{G}')/L)$ is of type $(l,\ldots,l)$, if we take a base $\{\varepsilon_1,\ldots,\varepsilon_s\}$ of $G(L(\mathfrak{G})/L)$ we get a representation $\{N(\bar{\sigma})\}$ of $G(L(\mathfrak{G}')/K)/P^{**}$ in the field with l-elements such that $(\alpha_{L(\mathfrak{G}')/L}a(\varepsilon_1),\ldots,\alpha_{L(\mathfrak{G}')/L}a(\varepsilon_s))N(\bar{\sigma}) = (\overline{\eta(\sigma)}\,\alpha_{L(\mathfrak{G}')/L}a(\varepsilon_1),\ldots,\overline{\eta(\sigma)}\,\alpha_{L(\mathfrak{G}')/L}a(\varepsilon_s))$, where $\bar{\sigma}$ is the class of σ in $G(L(\mathfrak{G})/K)/P^{**}$.

Since $G(L(\mathfrak{G}')/K)/P^{**}$ is an l-group, $\{N(\bar{\sigma})\}$ is equivalent to the following representation:

$$\left\{\begin{pmatrix} 1 & & & & A\sigma \\ & 1 & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & \cdot \\ & & & & 1 \end{pmatrix}\right\}$$

This shows that there exists a non-trivial subgroup $\overline{H}$ in $\{\alpha_{L(\mathfrak{G})/L}a(\varepsilon_\nu)\}$ which is elementwise fixed by $\eta(\sigma)$. Since $\alpha_{L(\mathfrak{G}')/L}$ is an onto isomorphism, we have a nontrivial subgroup H which is contained in the center of $G(L(\mathfrak{G}')/K)$.

Then, if we denote by $\mathfrak{G}''$ the subgroup of $J_L(\quad, k)$ such that $L(\mathfrak{G}'')$ corresponds to H, these $\mathfrak{G}'$ and $\mathfrak{G}''$ satisfy the conditions in the lemma.

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