

ADDENDUM: Π -PRINCIPAL HEREDITARY ORDERS

(Nagoya Math. J. 32 (1968), 41-65)

SUSAN WILLIAMSON

Let R denote a complete discrete rank one valuation ring of unequal characteristic, and let p denote the characteristic of the residue class field \bar{R} of R . Consider the integral closure S of R in a finite Galois extension K of the quotient field k of R . Recall (see Prop. 1.1 of [3]) that the inertia group G_0 of K over k is a semi-direct product $G_0 = J \times G_p$, where J is a cyclic group of order relatively prime to p and G_p is a normal p -subgroup of G .

The author has proved in [3] that if $\Delta(f, S, G)$ is a Π -principal hereditary order, then G_p is Abelian; the purpose of the present note is to extend this result by showing that the inertia group G_0 must also be Abelian. The reader should refer to [3] for definitions and notation.

PROPOSITION. *If the crossed product $\Delta(f, S, G)$ is Π -principal, then*

- i) *the inertia group G_0 is Abelian,*
- ii) *J is a normal subgroup of G .*

Proof. To prove that G_0 is Abelian, it suffices to show that $\Delta(\bar{f}, \bar{S}, G_0)$ is a commutative ring. According to Prop. 1.6 of [3] we may consider a splitting field L of $\Delta(\bar{f}, \bar{S}, G_0)$, so that $\Delta(\bar{f}, L, G_0)$ is isomorphic to the trivial crossed product $\Delta(1, L, G_0)$. Now $\text{rad } \Delta(1, L, G_0)$ is generated by $\text{rad } \Delta(1, L, G_p)$, (see the exercise on p. 435 of [1]), from which it follows that $\Delta(1, L, G_0)/\text{rad } \Delta(1, L, G_0) = \Delta(1, L, J)$. Therefore the factor ring $\Delta(\bar{f}, L, G_0)/\text{rad } \Delta(\bar{f}, L, G_0)$ is isomorphic to $\Delta(1, L, J)$.

We proceed to show that $\Delta(\bar{f}, \bar{S}, G_0)$ is isomorphic to a subring of the commutative ring $\Delta(1, L, J)$. The inclusion $(\text{rad } \Delta(\bar{f}, L, G_0)) \cap \Delta(\bar{f}, \bar{S}, G_0) \subset \text{rad } \Delta(\bar{f}, \bar{S}, G_0)$ follows from Lem. 2.4 of [2], and $\text{rad } \Delta(\bar{f}, \bar{S}, G_0) = (0)$ because $\Delta(f, S, G)$ is Π -principal; these facts combine to show that the natural map $\Delta(\bar{f}, \bar{S}, G_0) \rightarrow \Delta(\bar{f}, L, G_0)/\text{rad } \Delta(\bar{f}, L, G_0)$ is a monomorphism,

and this together with the observations in the preceding paragraph completes the proof of part i).

Assertion ii) follows from i) and Prop. 3.2 of [3].

REFERENCES

- [1] C. Curtis and I. Reiner, Representation Theory of Finite Groups and Associative Algebras, Wiley (1962).
- [2] S. Williamson, Crossed products and ramification, Nagoya Math. J. Vol. 28 (1966), pp. 85-111.
- [3] S. Williamson, H -principal hereditary orders, Nagoya Math. J. Vol. 32 (1968), pp. 41-65.

Regis College