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ERRATUM TO MY PAPER: ON THE INVARIANT DIFFERENTIAL METRICS NEAR PSEUDOCONVEX BOUNDARY POINTS WHERE THE LEVI FORM HAS CORANK ONE

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While the author's article [He] was printed, it turned out, that, unfortunately the function $\mathscr{C}_{2k}$ occuring in the statement of Theorem 1 in [He] was not correctly defined. In particular, the first part of section 5 in [He] must be changed, since part c) of Lemma 3.2 is not correct. In this short note we describe which alterations need to be made in order to get a satisfactory definition of $\mathscr{C}_{2k}$ and proof of Theorem 1.

a) First of all, in the definition of the functions A_l in formula (1.5) of [He] the holomorphic tangential field L_n has to be replaced by a holomorphic tangential field L_* without zeroes on B, with the property $\partial r([L_a, \bar{L}_*]) = 0$ for $2 \leq a \leq n-1$. If we assume that the submatrix $(\mathscr{L}_{a\bar{b}})_{a,b=2}^{n-1}$ is invertible throughout B, then such a holomorphic tangent field always exists. Furthermore, although L_* is determined only up to a multiplicative smooth factor, the estimates (1.7) and (1.9) from [He] hold independently of the choice of L_*.

b) The normalization of the g_a-functions occuring in formula (2.4) of [He] cannot be done exactly as claimed on [He], p. 30, but we can, step by step, eliminate the antiholomorphic terms from the g_a by a series of transformations of the form

$$\begin{aligned} &w_1' \to w_1', \\ &w_a' \to w_a' + \gamma_a w_n'^{m_a}, \quad 2 \leq a \leq n-1 \\ &w_n' \to w_n'. \end{aligned}$$

Then the statement of Theorem 3 remains correct. Furthermore, part c) of Lemma 3.2, together with its proof, should be ignored.

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c) Since the function $\mathscr{C}_{2k}$ has been changed, other computations for the transformation from the normalized coordinates to the original ones are necessary. We now sketch them (The notations are as in [He]): We have to show

$$\sum_{l=2}^{2k}\left(\frac{\| P_l(\cdot\,;q)\|}{t}\right)^{\frac{2}{l}} \approx \mathscr{C}_{2k}(z)^2. \tag{1}$$

(Here we write $f \approx g$ for two functions f, g, to indicate that there is a uniform constant $c > 0$, satisfying $\frac{1}{c}f \leq g \leq cf$). Let C denote the matrix $(\mathscr{L}_{a,\bar{b}})_{a,b=2}^{n-1}$, which is supposed to be invertible on B. Also we write $F = F(\cdot, q)$ for the transformation of [He], Theorem 3 and put $\hat{r} = \hat{r}_q = r \circ F^{-1}$.

We choose L_* as follows:

$$L_* = \sum_{i=2}^{n-1} s_i L_i + L_n,$$

where the functions $s_n, \ldots, s_{n-1}$ are smooth on B and defined by

$$(s_2, \ldots, s_{n-1}) = -\,(\mathscr{L}_{n\bar{2}}, \ldots, \mathscr{L}_{n\overline{n-1}})C^{-1},$$

We use the notations $L' = F_* L_*$, $\hat{\mathscr{L}}_{i\bar{j}} = \partial\hat{r}([\hat{L}_i, \overline{\hat{L}_j}])$, and $\hat{C} = (\hat{\mathscr{L}}_{i\bar{j}})_{i,j=2}^{n-1}$, where

$$\hat{L}_i = \frac{\partial}{\partial w_i} - \frac{\partial\hat{r}/\partial w_i}{\partial\hat{r}/\partial w_1}\frac{\partial}{\partial w_1}, \quad 2 \leq i \leq n.$$

Then

$$\hat{L}_{i\bar{j}} = \frac{\partial^2\hat{r}}{\partial w_i \partial\bar{w}_j} - \frac{\partial^2\hat{r}}{\partial w_i\partial\bar{w}_1}\frac{\partial\hat{r}}{\partial\bar{w}_j} - \frac{\partial^2\hat{r}}{\partial w_1\partial\bar{w}_j}\frac{\partial\hat{r}}{\partial w_i}\Big/\frac{\partial\hat{r}}{\partial w_1} + \frac{\partial^2\hat{r}/\partial w_1\partial\bar{w}_1}{|\,\partial\hat{r}/\partial w_1|^2}\frac{\partial\hat{r}}{\partial w_i}\frac{\partial\hat{r}}{\partial\bar{w}_j} \tag{2}$$

The field L_* transforms under F as follows:

$$L' = F_* L_* = -\sum_{i,j=2}^{n-1} \hat{\mathscr{L}}_{n\bar{j}}\hat{C}^{ji}\,\hat{L}_i + \hat{L}_n.$$

where $\hat{C}^{ij}$ denotes the entries of $\hat{C}^{-1}$. For $l \geq 2$ we introduce the functions

$$A'_l(w) = \max\{|\, L'^{a-1}\overline{L'}^{b-1}\hat{\lambda}(w)\,\|\, a, b \geq 1, a + b = l\},$$

where $\hat{\lambda} = \det(\hat{\mathscr{L}}_{i\bar{j}})_{i,j=2}^n$. From the fact that

$$\left|\det\left(\frac{\partial F_i}{\partial z_a}\right)_{i,a=2}^n\right|^2 \equiv 4\lambda'(q)\left|\frac{\partial r(q)}{\partial z_1}\right|^2$$

and

$$\lambda_{\partial\Omega} = \left| \det \left(\frac{\partial F_i}{\partial z_a}\right)^n_{i,a=2} \right|^2 \hat{\lambda}\circ F$$

we easily see by computation

$$A'_l(F(z)) = \frac{1}{4\lambda'(q)\left|\dfrac{\partial r}{\partial z_1}\right|^2} A_l(z).$$

Let us put

$$\mathscr{C}'_{2k}(w) = \sum_{l=2}^{2k} \left(\frac{A'_l(w)}{|\hat{r}(w)|}\right)^{\frac{1}{l}}.$$

Then it is obvious that the proof of (1) will be complete once we have shown

$$\mathscr{C}'_{2k}(-t, 0') \simeq \frac{1}{R_n(t)} \tag{3}$$

By the mean value theorem together with inf $A_{2k} > 0$, we see that in (3) we may replace $(-t, 0')$ by 0. Now we only need to take into account that

$$\frac{1}{R_n(t)} \simeq \max_{2\le l\le 2k} \max_{a,b\ge 1, a+b=l} \left(\frac{\left|\dfrac{\partial^{a+b}\hat{r}(0)}{\partial w_n^a \partial \bar{w}_n^b}\right|}{t}\right)^{1/l}$$

$$\mathscr{C}'_{2k}(0) \simeq \max_{2\le l\le 2k} \max_{a,b\ge 1, a+b=l} \left(\frac{|L'^{a+b}\bar{L}'^{b-1}\hat{\lambda}(0)|}{t}\right)^{1/l}$$

in order to see that (3) will follow from

LEMMA 5.1. *For any integers $a, b \ge 1$ there exists a constant $C_{ab} > 0$, independent of q, such that for all sufficiently small t one has the estimate*

$$\left| L'^{a-1}\bar{L}'^{b-1}\hat{\lambda}(0) - \frac{\hat{\lambda}'(0)}{|\partial\hat{r}(0)/\partial w_1|^2} \frac{\partial^{a+b}\hat{r}}{\partial w_n^a \partial \bar{w}_n^b} \right| \le C_{ab} \frac{t}{R_n(t)^{a+b-1}}. \tag{4}$$

For the proof of this we need to compare the iterates of L' and its conjugate with the mixed partial derivatives with respect to w_n. In order to state the relevant formulas we introduce the following sets:

For a positive integer p we put

$$M'_p = \left\{\frac{\partial^{\nu+\mu}\hat{r}}{\partial w_n^\nu \partial \bar{w}_n^\mu} \,\middle|\, 1 \le \nu+\mu \le p\right\}$$

and

$$M''_p = \left\{ \frac{\partial^{\nu'+\mu'+1}\hat{r}}{\partial w_j^\alpha \partial \bar{w}_j^\beta \partial w_n^{\nu'} \partial \bar{w}_n^{\mu'}} \frac{\partial^{\nu''+\mu''+1}\hat{r}}{\partial w_s^\gamma \partial \bar{w}_s^\delta \partial w_n^{\nu''} \partial \bar{w}_n^{\mu''}} \,\middle|\, \alpha, \ldots, \delta, \nu', \ldots, \mu'' \geq 0, \right.$$
$$\left. 2 \leq j,\ s \leq n-1,\ \alpha+\beta=1,\ \gamma+\delta=1,\ \nu'+\cdots+\mu'' \leq p \right\}.$$

Let us denote $M_p = M'_p \cup M''_{p+1}$, and call S_p the set of all functions which are smooth on B and which are rational functions in the derivatives of $\hat{r}$ of order $\leq p$. For two sets T_1, T_2 of smooth functions on B we denote by T_1T_2 the set of sums of products of a function from T_1 with a function from T_2.

LEMMA 5.2. *For any positive integers a, b we have*

$$L'^{a-1}\,\bar{L}'^{b-1}\hat{\lambda} - \frac{\lambda'}{\left|\frac{\partial \hat{r}}{\partial w_n}\right|^2} \frac{\partial^{a+b}\hat{r}}{\partial w_n^a \partial \bar{w}_n^b} \in S_{a+b}M_{a+b-1}. \tag{5}$$

Proof. The case $a = b = 1$ follows from (2) and the Leibniz rule for the determinant $\hat{\lambda}$. We observe that for any positive integer p the set M_p satisfies $L'(M_p) \subset M_{p+1}$ and $\bar{L}'(M_p) \subset M_{p+1}$. The proof of the lemma now follows by induction on a. The details will be omitted, since they are based on elementary calculus.

Proof of Lemma 5.1. If we choose in (3.10) of [He] $w_n = R_n(t)$, we obtain for any function $f \in M_p$:

$$|f(0)| \lesssim \frac{t}{R_n(t)^p}$$

Applying this to $p = a + b - 1$ we obtain (4).

d) If we in the definition of the functions $s_a(X)$, $2 \leq a \leq n$ replace the vector field L_n by L_*, also Theorem 2 becomes correct. The computations for converting the formula of Theorem 6 into the term $M_\Omega(z, X)$ are similar to those in c).

REFERENCES

[He] G. Herbort, On the invariant metrics near pseudoconvex boundary points where the Levi form has corank one, Nagoya Math. J., **130** (1993), 25–54.

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