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Mathematical Modeling Analyses for Obtaining an Optimal Railway Track Maintenance Schedule

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Railway track irregularities need to be kept at a satisfactory level by taking appropriate maintenance activities. This paper aims at obtaining an optimal maintenance schedule for improving the railway track irregularities using all-integer linear programming (AILP) optimization model analyses.

Firstly, we try to predict a change of surface irregularities by investigating the transition process through degradation and restoration model analyses. Then we develop an AILP model for obtaining an optimal schedule of multiple tie tamper (MTT) operation. The model takes both maintenance costs and the level of surface irregularities that reflects riding quality and safety into account, then finally gives an optimal tamping schedule of MTT for the whole year. Then we apply the results of this model to solve the optimal MTT's maintenance scheduling problem for the actual railway network system and show that it is effective and useful enough by comparing our model results with actual existing data.

Key words: railway track maintenance schedule, all-integer linear programming model, track irregularities, degradation model, restoration model

1. Introduction

How to deal with deteriorating phenomenon of the ballasted track is one of the most important problems and necessary to be solved urgently in the area of railway engineering as it brings serious consequences on the safety of train operation when it is worsened. The more frequently trains pass on the specific track, the worse track irregularities are getting according as an accumulation of plastic deformation due to the repetitive loading by the train. Thus ballasted tracks accounting for 80% of the present track in Japan require maintenance operations represented by "periodical tamping" in order to maintain track irregularity at the satisfactory level from the viewpoints of riding quality and safety. However, current situation in Japanese railway company shows that necessary annual maintenance costs and work forces are becoming higher and higher every year. Therefore, it is imperative that we need to keep the track irregularities at a satisfactory level through appropriate maintenance activities.

In this paper, we try to obtain an optimal railway track maintenance strategy by building discrete optimization mathematical programming models. In the

1990's several decision supporting systems (DSSs) have been developed and are still being developed and modified [1, 2] in order to help us decide desirable railway track maintenance strategies. Our modeling approach proposed in this paper is an extension with the similar direction by incorporating an additional optimality criterion under certain necessary conditions. This paper consists of mainly three parts such as measuring the transition of track irregularity, predicting maintenance operation effects, and planning an optimal maintenance schedule. Computational procedures of this research are illustrated in Fig. 1.

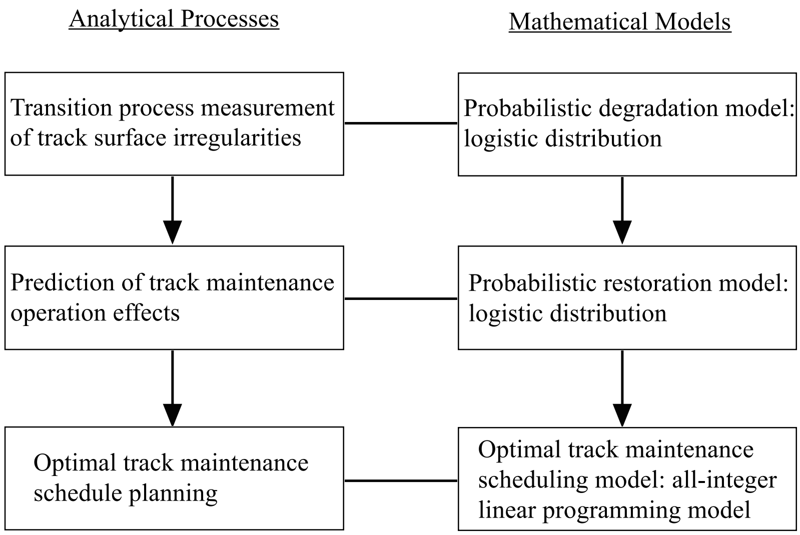

Fig. 1. Flow diagram of the computational procedures.

Firstly, we develop a transition process model for measuring the transition of track surface irregularity and predicting its maintenance effects in the future. This transition process model is composed of two models such as degradation model and restoration model. The former model tries to clarify the transition of degradation process of the track surface irregularity for each track unit "lot" (100m in length) based on historical data by applying the exponential smoothing method. The latter model forecasts maintenance effects obtained from tamping operation by MTT.

Then, we build an all-integer linear programming (AILP) model in order to determine an optimal maintenance schedule for the MTT tamping operation. This model enables us to decide which lot and when we should add maintenance operation of tamping with MTT taking various necessary conditions into consideration.

Finally, we give numerical results obtained from applying the AILP scheduling model to the actual railway network in Japan. From the viewpoint of solving the discrete optimization model, the size of the mathematical programming model, especially represented by the number of integer variables and the number of constraints, is very important as it effects the computation time to obtain an exact optimal solution of the model. We show numerical results obtained from computational procedure in order to solve the model by finding a nearly optimal solution within a reasonable amount of time even when the size of the model is very large. We confirm that the procedure provides us with an approximately optimal annual tamping schedule for MTT in each term for each year. We compare our AILP model solution with the actual data with respect to the standard deviation of surface irregularities, then find out that our schedule brings more advantageous results as its standard deviation of surface irregularities is smaller than those in the past. Thus, we can conclude that our AILP model can be practical and useful enough to be applied to the actual railway track data in order to obtain an optimal tamping operation of MTT for maintaining a maximum improvement of the total track irregularity.

2. Transition Process Model Analyses

In this section, we describe transition process models for explaining degradation and restoration processes of the track surface irregularities. Results obtained from these models are used as input data for the optimal track maintenance scheduling model described in the following section.

2.1. Modeling the condition of surface irregularities

Conditions of track surface irregularities are expressed by measuring the geometry of tracks by 10m-chord versine method, which is illustrated in Fig. 2. In the figure vertical coordinate data y corresponds to the surface irregularity of the tracks. Distribution data of the actual surface irregularities, which consists of 80,706 sampling data obtained from measuring at 25cm interval, is illustrated in Fig. 3. Trying to apply probability distribution model to explain these actual surface irregularity data, we find that the logistic distribution most suitably fits the actual surface irregularity data. Statistical details regarding the comparison of the goodness of fit for several probability distributions are shown in Table 1, where Log, HG, Pois,

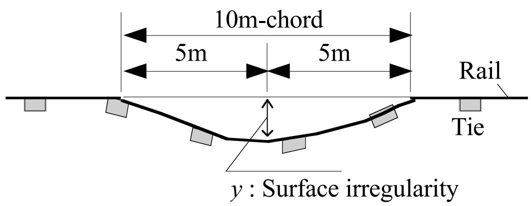

Fig. 2. Surface irregularity.

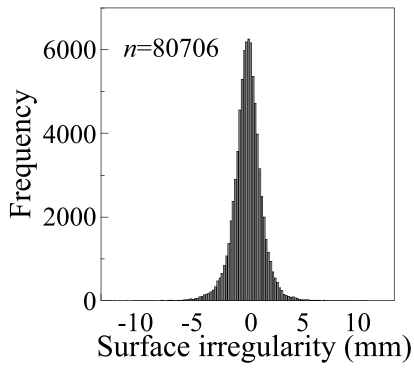

Fig. 3. Distribution of surface irregularities.

Table 1. Results of test of goodness of fit.

Rank	1	2	3	4
Distribution	Log	HG	Pois	Chisq
χ^2 value	0.79	4.28	6.61	11.55

and Chisq stand for logistic, hyper geometric, Poisson and chi-square distributions, respectively. Track surface irregularity data are characterized by the conditions of the tracks such as structure, train operation, maintenance method, and so on. We can conclude that the logistic probability distribution most fits the actual surface irregularities under any track conditions. The probability density function $f(x)$ of the logistic distribution is expressed by the following formula $f(x)$ containing two parameters α and β.

$$f(x) = \frac{\exp\{(x-\alpha)/\beta\}}{\beta[1+\exp\{(x-\alpha)/\beta\}]^2} \tag{2.1}$$

In the above expression of the probability density function of the logistic distribution, parameter α indicates the mean value, and its standard deviation is given by $\pi\beta/3^{0.5}$. As for the surface irregularities, the estimate of the parameter α is nearly equal to zero as we use the normalized data set. In the following analyses we focus on the parameter β which corresponds to the standard deviation of the logistic distribution and also can express most typical characteristics of the track surface irregularities.

2.2. Modeling the transition process of surface irregularities

Transition process of surface irregularities is composed of two processes of degradation and restoration, schematic illustration of which is shown in Fig. 4. We develop a mathematical model for each process of degradation and restoration. In this section we briefly describe the structure and the main framework of these degradation and restoration models, respectively. Details of these models are described in [3], [4], [5], [6], and [7]. Numerical results obtained from these models are used in an optimal track maintenance scheduling model described in the next section.

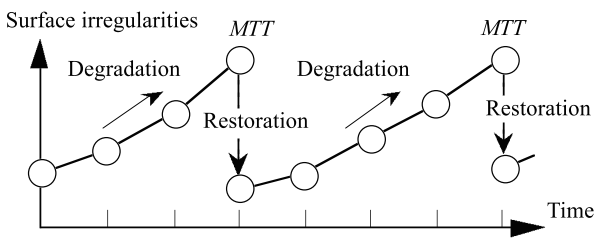

Fig. 4. Schematic illustration of transition process.

(1) Degradation model

Degradation model aims at estimating the future value of parameter β for each lot (100m in length) in the unit of the tracks. We apply the exponential smoothing method (see [8]) described below to each lot with 100m long each in order to predict the increasing trend of parameter β given in (2.1) for the corresponding logistic probability distribution model. Calculating the standard deviation of surface irregularities data at arbitrary time t for each lot, we obtain the estimate value $\hat{\beta}(t)$ of parameter β as follows. Firstly, the initial estimate for parameter β at time t denoted by $\bar{\beta}(t)$ is given by using observed value $\beta(t)$ and estimate value $\bar{\beta}(t-1)$ at time $t-1$ as follows.

$$\bar{\beta}(t) = s\beta(t) + (1-s)\bar{\beta}(t-1) \tag{2.2}$$

where s is a smoothing coefficient appropriately determined in the range [0.3, 0.6]. Then the second estimate variable $\bar{\bar{\beta}}(t)$ is defined using initial estimate $\bar{\beta}(t)$ and the second estimate $\bar{\bar{\beta}}(t-1)$ at time $t-1$ as follows.

$$\bar{\bar{\beta}}(t) = s\bar{\beta}(t) + (1-s)\bar{\bar{\beta}}(t-1) \tag{2.3}$$

Defining the intermediate coefficients at time t as $\hat{a}(t)$ and $\hat{b}(t)$ as follows.

$$\begin{aligned} \hat{a}(t) &= 2\bar{\beta}(t) - \bar{\bar{\beta}}(t-1) \\ \hat{b}(t) &= \frac{1-s}{s}\left\{\bar{\beta}(t) - \bar{\bar{\beta}}(t)\right\} \end{aligned} \tag{2.4}$$

We obtain the following extrapolation formula in order to predict the value for parameter β at L periods after t, which is denoted by $\hat{\beta}(t+L)$ with $L \geq 0$ as follows.

$$\hat{\beta}(t+L) = \bar{\beta}(t) + LT(t) \tag{2.5}$$

Fig. 5 shows the relation between actually observed data obtained from a maintenance division of the railway company and predicted parameter value $\beta(t+2)$ (two periods, i.e. 180 days after time t). From this result we can conclude that our prediction method would be accurate enough to estimate the future changes of the track surface irregularities by forecasting parameter value $\beta(t+L)$ with $L \geq 0$, integer.

(2) Restoration model

Using the actual data obtained from a railway track section, we show an example of comparison between the before-tamping parameter β_b and the quantity of improvement $\Delta\beta$ ($= \beta_b - \beta_a$, where β_a indicates an after-tamping parameter) with the regression line and 95% confidence interval (95% CI) in Fig. 6. The regression line and confidence interval are given as follows.

$$\Delta\beta = 0.636\beta_b - 0.313 \tag{2.6}$$

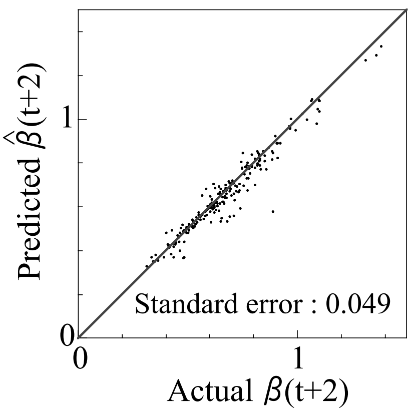

Fig. 5. Predicted and actual β (after 180 days).

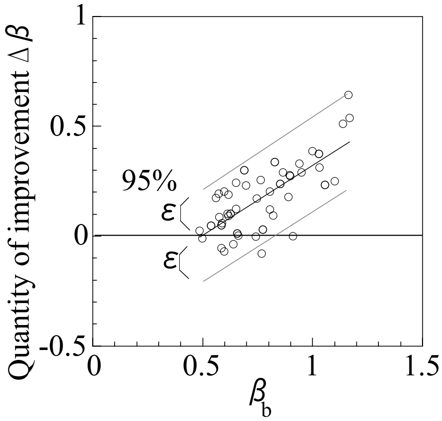

Fig. 6. Quantity of improvement.

where $0.49 \leq \beta_b \leq 1.17$ and 95% CI is given as 0.22.

From the above expression (2.6), the volume of improvement $\Delta\beta$ is found to increase linearly with β_b. Restoration process is assumed to be unchanged among all lots, thus we use the above restoration model shown in (2.6) in common for all lots in order to predict changes in surface irregularities.

3. Optimal Track Maintenance Scheduling Model

Before we solve the Optimal Track Maintenance Scheduling (OTMS) model, we show the Maintenance Unit Selection (MUS) model in which we try to select candidates of units for track maintenance activities. This selection process aims at finding units such that selected units would possibly bring the largest improvement when maintenance operation has been added. Thus units, each of which consists of N lots, are selected in order that they may contain such lots that would bring the largest maintenance effect. We show the formulation of the MUS model.

3.1. Structure of the MUS model

Major input data for the MUS model are the structure of the railway track network, candidates for the lots for MTT's operation, and historical data of surface

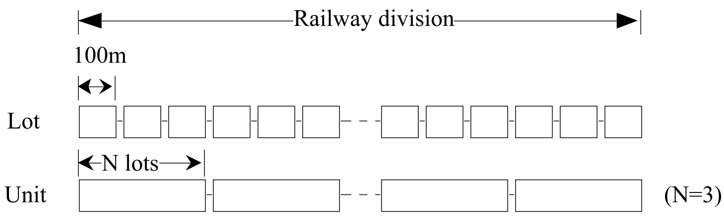

Fig. 7. Lot and unit.

irregularities measured in the latest two years for each lot. Each lot is 100m in length, and its degradation and restoration processes are predicted in the transition process model. As shown in Fig. 7, unit consists of certain number (N) of consecutive lots. Optimal operation schedule for MTT is obtained for each unit and for each term (10 days) of each month.

(1) Decision variables

v_i : binary variable, $i \in L$

where $L = \{1, 2, 3, \ldots, L^m\}$ and L^m indicates a total number of lots considered.

$$v_i = 1 : \text{unit of } N \text{ consecutive lots starting from lot } i \text{ is selected}$$
$$= 0 : \text{otherwise}$$

(2) Constraints

i) *Upper bounding constraints*

$$\sum_{i \in L} v_i \leq G^m \tag{3.1}$$

where G^m is an upper bound for the number of units selected. This constraint gives the upper bound (UB) for the number of units selected. This UB is obtained from the maximum number of days spent for the whole track maintenance operation.

ii) *Unit generation constraints*

$$\sum_{j=i}^{i+(N-1)} v_j \leq 1 \qquad i \in L \tag{3.2}$$

This constraint indicates that at most one unit can be selected in the N lots ranging between i and $i + (N - 1)$.

iii) *Additional unit generation constraints*

$$v_i = 0 \qquad i \in L_x \subseteq L \tag{3.3}$$

where L_x indicates a set of lots which we cannot select as a starting lot. This constraint indicates that we cannot select a unit starting from lots contained in the set L_x, thus the units need to be selected from outside L_x.

(3) Objective function

MUS model's criterion implies that we select units such that the total expected improvement obtained from adding track maintenance operation can be maximized.

$$\text{Maxmize} \qquad y = \sum_{i=1}^{L^m - N + 1} r_i v_i \tag{3.4}$$

Coefficients r_i's of the objective function indicate the amount of improvement for each lot after the maintenance operation. Thus they are given by the sum of $\Delta\beta_i$'s contained in the unit, which is proportional to the expected improvement in case that the unit starting from lot i is chosen for the maintenance operation.

$$r_i = \sum_{j=i}^{i+(N-1)} \Delta\beta_j \qquad i \in L \tag{3.5}$$

3.2. Structure of the OTMS model

The OTMS model aims at finding an optimal operation of MTT, which maximizes the total improvement of track surface irregularities under several related conditions. By solving this OTMS model, we can obtain an optimal operation of MTT including the decisions regarding such as which depot, which lot, and when we should choose to locate for MTT's operation (see [4, 5, 6, 7, 9, 10, 11, 12] for our previous works).

Major input data for the OTMS model are the structure of the railway track network, candidates for the location of depots for MTT's and units for MTT's operation. OTMS model selects depots where MTT should be located, then allocate the MTT to appropriate units for their tamping operation for each term (10 days) in the month. OTMS model's criterion is to maximize the total improvement of track surface irregularities measured during one year under several related conditions described in the next subsection. Total improvement of track surface irregularities is defined to be the sum of restorations for track surface irregularities, which is obtained from restoration transition model, for each lot when MTT operation is added. MTT operation is restricted e.g. by conditions such as MTT can operate tamping within a limited distance from the depot where the MTT was located in the corresponding term of the month.

Sets of months, terms, depots, units, and lots are denoted by

$$M = \{1, 2, 3, \ldots, M^m\,(= 12)\},\ K = \{1, 2, K^m\,(= 3)\},\ D = \{1, 2, 3, \ldots, D^m\},$$

and $U = \{1, 2, 3, \ldots, U^m\}$, respectively. We define the 0-1 type integral decision variables of the OTMS model as follows.

(1) Decision variables

z_{mkd} : binary variable, $m \in M,\ k \in K,\ d \in D$

$z_{mkd} = 1$: MTT is located at depot d in month m and term k

$= 0$: MTT is not located at depot d in month m and term k

w_{mkj} : binary variable, $m \in M,\ k \in K,\ j \in U$

$w_{mkj} = 1$: maintenance operation is executed at unit j in month m and term k

$= 0$: otherwise

(2) Constraints

The following constraints i)–v) are common to any railway divisions irrespective of the structure of the network system, while constraints vi)–viii) are more division specific depending upon the regulations, seasonal restrictions, strategic conditions and so on for each utility.

i) *MTT location constraints*

$$\sum_{d \in D} z_{mkd} \leq 1 \qquad m \in M, \quad k \in K \tag{3.6}$$

This constraint implies that the MTT can be located at most one of the depots in each 10-day term of each month. Namely, once MTT is located at any depot in month m, and term k, it cannot operate in any other depot.

ii) *Upper bounding constraints for the number of units to be tamped*

$$\sum_{j \in U} w_{mkj} \leq A_{mk} \qquad m \in M, \quad k \in K \tag{3.7}$$

where A_{mk} is the maximum number of units tamped in month m and term k. This constraint implies that the number of units to be tamped has an upper bound in each term of each month.

iii) *Upper bounding constraints for the frequency of tamping*

$$\sum_{m \in M} \sum_{k \in K} w_{mkj} \leq 1 \qquad j \in U \tag{3.8}$$

This constraint implies that each unit needs tamping operation at most once in a term and a month during the whole year.

iv) *Logical constraints for MTT location and operation*

$$w_{mkj} - z_{mkd} \leq 0 \qquad m \in M, \quad k \in K, \quad d \in D_j, \quad j \in U \tag{3.9}$$

where D_j is a set of depots which "cover" unit j. This constraint implies that the tamping operation of MTT can be executed only when the MTT is located to the depot such that each unit is "covered" by the MTT located at the depot in each month and each term.

v) *MTT movement constraints*

$$z_{mkd} + z_{m(k+1)d'} \leq 1 \qquad m \in M, \quad k \in K \backslash \{K^m\}, \quad d \in D, \quad d' \in D^d \tag{3.10}$$

$$z_{mK^m d} + z_{(m+1)1d'} \leq 1 \qquad m \in M \backslash \{M^m\}, \quad d \in D, \quad d' \in D^d \tag{3.11}$$

where D^d is a set of depots such that MTT can not be located in the next term from the depot d presently located. This constraint implies that MTT cannot move beyond the certain distance from the presently located depot in the next term. Thus, these constraints give all pairs of depots such that MTT cannot move in two consecutive terms.

vi) *MTT specific location constraints*

$$z_{mkd} = 1 \qquad m, k, d \in E \subseteq M \times K \times D \tag{3.12}$$

MTT needs to be located to certain depot during certain terms and months due to MTT specific scheduling and geographic restrictions. The following constraints force MTT to locate at certain depot in certain month and term given by the set of the combination $E \subseteq M \times K \times D$, respectively.

vii) *Unit specific scheduling constraint*

$$\sum_{j \in J} \sum_{(m,k) \in R_j} w_{mkj} = 0 \qquad j \in J \subseteq U \tag{3.13}$$

where J is a set of units such that this constraint is applied, and R_j gives the pair of month m and term k such that no tamping operation can be executed. For certain units such that tamping operation can not be applied due to regulatory, seasonal, and strategic restrictions, we give the above constraints.

viii) *Unit specific operation constraints*

$$\sum_{(m,k) \in J_j} w_{mkj} = 1 \qquad j \in J \subseteq U \tag{3.14}$$

For any lots contained in the railway track division, surface irregularities are not allowed to exceed the specified upper bound throughout the scheduling period. Thus, if we identify the lot's of which the surface irregularities exceed the specified upper bound during the scheduling period, then the units containing these lots need to be dealt with tamping operation, before the time limit. Denoting such sets of units by J, and corresponding pairs of month and term by $(m, k) \in J_j \subseteq M \times K$ for $j \in J$, we obtain the above unit specific operation constraint.

(3) Objective function

Two main purposes for tamping surface irregularities are securing running safety and obtaining good riding comfort when we get on vehicles. When we evaluate the mean value of β of surface irregularities in the whole scheduling period, it is desirable to take the influence of surface irregularities on both the vibration of vehicles and the conditions of train operation into consideration. Thus, the objective function of this scheduling model is defined by minimizing the mean value of β of surface irregularities weighted by the required bound of the surface irregularities in order to obtain the highest riding quality during the whole scheduling period. The mean value of β of surface irregularities during the whole scheduling period, which we want to minimize, is obtained from the following objective function. Namely, the objective function of the OTMS model is equivalent to maximizing the following expression.

$$\text{Maxmize} \qquad y = \sum_{m \in M} \sum_{k \in K} \sum_{j \in U} S_{mkj} w_{mkj} \tag{3.15}$$

where S_{mkj} indicates the amount of improvement of surface irregularities obtained from assigning MTT to the unit j in term k of month m. Coefficient S_{mkj} assumes that the improvement of surface irregularities is counted in the remaining periods after tamping operation is added in term k of month m. Thus the coefficient S_{mkj} is expressed by using the coefficient $\Delta\beta$'s in (3.5).

4. Numerical Experiments

We apply the above modeling approach represented by MUS and OTMS models to actual railway districts in Japan, denoted by RD I, RD II and RD III,

Table 2. Data descriptions on RD I, RD II and RD III.

	RD I	RD II	RD III
Total distance (km)	47	120	116
Track	double	double & single	double
Number of lots	966	1663	1270
Number of available days (days/year)	47	71	87
Available units for maintenance (units/day)	4	1	1

Table 3. Number of available days.

RD I

	term				term		
month	1	2	3	month	1	2	3
4	0	0	0	10	6	3	6
5	2	2	1	11	0	2	0
6	5	6	3	12	0	0	0
7	5	4	0	1	0	0	0
8	0	0	0	2	0	0	0
9	0	0	2	3	0	0	0

RD II

	term				term		
month	1	2	3	month	1	2	3
4	3	3	2	10	2	3	3
5	1	3	2	11	3	2	2
6	3	2	3	12	3	2	1
7	3	0	0	1	1	3	3
8	0	0	0	2	3	2	2
9	0	0	3	3	3	2	3

RD III

	term				term		
month	1	2	3	month	1	2	3
4	3	3	1	10	2	3	3
5	1	3	3	11	3	3	2
6	3	2	2	12	3	2	1
7	3	2	3	1	1	3	3
8	2	0	3	2	3	2	3
9	3	2	3	3	3	2	3

Table 4. Number of candidate units and lots per unit.

	RD I	RD II	RD III
Number of candidate units (G^m)	188	71	87
Number of lots per unit	3	10	7

respectively. These three railway districts RD I, RD II and RD III have different characteristics such that RD I located in the remote northern part has short distance, simple structure, least tonnage and least frequency. Then RD II located in the suburban area has long distance, complicated structure, and medium frequency, while RD III located in the metropolitan area has medium distance, most complicated structure and highest frequency. Detailed physical data for these railway districts are given in Table 2 while numbers of available days for each term in each month are given in Table 3. Numbers of candidate units and lots per unit for MUS and OTMS models are given in Table 4. Structures for these railway divisions RD I, RD II and RD III are illustrated in Fig. 8 (a), (b) and (c), respectively.

In Fig. 8 depot areas denoted by (i), (ii), ... show "coverage region" by MTT, indicating the region MTT can operate when it is located in the corresponding area. Hence MTT is located somewhere in the depot area during the corresponding term of the month when it is assigned to the corresponding area. Sizes of mathematical models MUS and OTMS are given in Table 5 with numbers of decision variables and constraints. MUS model is an all-integer type optimization model consisting of L^m binary variables corresponding to the number of lots. We find that the coefficient matrix corresponding to the set of constraints has the totally unimodular property as the constraint (3.1) has all nonzero coefficients one in a row while the constraint (3.2) has N consecutive coefficients one in each row. This means optimal solution for the MUS model can be obtained from a continuous type linear programming problem corresponding to the MUS model. Thus the MUS model can be solved within a second even for a large scale optimization model. We use

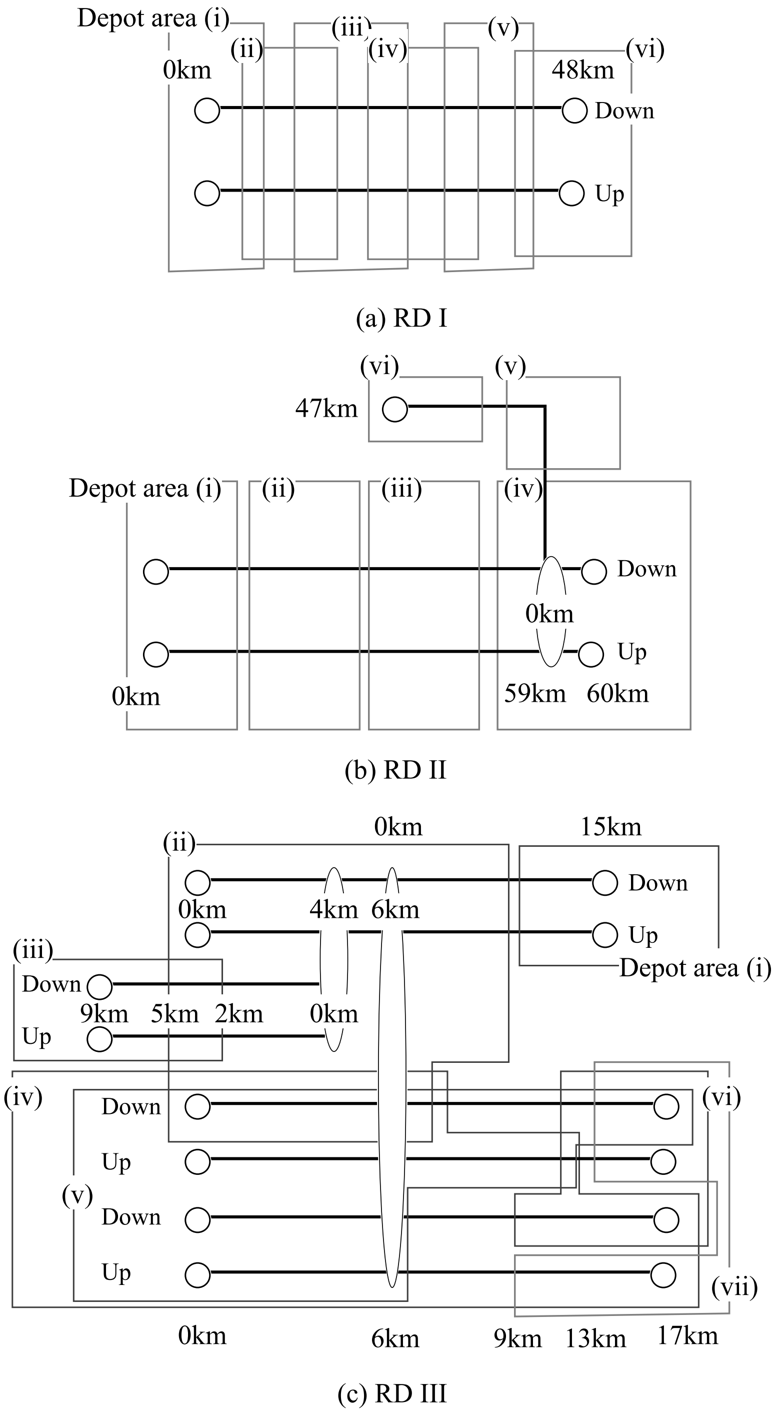

Fig. 8. Structure of RD I, RD II and RD III.

the optimization software package XPRESS-MP (Dash Associates Inc.) on the PC FUJITSU FMV700FL2 with CPU Pentium 4 of 1.7GHz and memory 512MB. Basic data for formulating and solving MUS and OTMS models are given as $M^m = 12$, $K^m = 3$, and $D^m = 6$ ($D^m = 7$ for RD III case). OTMS model generally becomes a large scale discrete optimization model as the number of binary decision variable is given by $|M|\times|K|(|D|+|U|) = M^m K^m(D^m+U^m)$, and $|U| = U^m$ usually exceeds a hundred. Also the number of constraints for the OTMS model is dominated mainly by the constraints (3.9) as it is expressed by the order $|M|\times|K|\times\max\{(|D|,|U|)\} = M^m K^m U^m$. Thus, obtaining an exact optimal solution for these large scale OTMS model with several thousands of binary decision variables and constraints becomes extremely difficult.

Table 5. Model sizes of MUS and OTMS.

Model	Items	RD I	RD II	RD III
MUS	Decision variables	966	1663	1270
	Constraints	965	1657	1229
OTMS	Decision variables	6984	2772	3384
	Constraints	7208	2985	3592

Table 6. Numerical results of the OTMS model.

	RD I	RD II	RD III
Number of selected units	188	66	79
Objective function value (LP solution)	1600.8	466.88	447.63
IP / LP ratio	0.999	0.985	0.996
Computation time (sec.)	30	1220	7200*

From optimal solutions of the MUS model for railway divisions RD I, RD II and RD III, we select 188, 71 and 87 units assigned for maintenance operations by MTT located in 6, 6 and 7 depot areas, respectively. Based upon the set of units obtained from optimal solutions of the MUS model, we apply the OTMS model in order to obtain an optimal maintenance schedule for the operation of MTT to the depot areas of RD I, RD II and RD III, respectively. We show the numerical results for these OTMS models for RD I, RD II and RD III in Table 6. In Table 6, LP solution indicates the objective function value for the exact optimal solution to the continuous type linear programming problem, thus an upper bound for the integer linear programming problem, while IP solution indicates the “best” discrete solutions obtained within the computation time shown in the table. As shown in Table 6, we could obtain exact optimal integer solutions for RD I and RD II in 30 and 1220 seconds, respectively, while 7200* indicates that we could not obtain an exact optimal integer solutions for RD III within 7200 seconds (two hours).

However, comparing the best feasible solutions obtained within two hours for the case RD III with a continuous linear programming objective function value which gives an upper bound for the original discrete optimization problem, the gap is found to be less than 1%. This means that our current best solution would be acceptable enough in the sense that the solution can be used practically as the given solution might be close enough to the exact optimal integer solution. The computational process approaching the optimal solution for these cases RD I, RD II and RD III are shown in Fig. 9 (a), (b) and (c), respectively. These figures show that in all cases good and possibly acceptable feasible integer solutions are found very quickly within 5 seconds, 10 seconds and 30 seconds even in case RD III, respectively. These best solutions are usually obtained within a reasonable amount of time and this tendency is quite common for solving discrete optimization problems.

We have tried several more cases in total so far in order to apply our MUS and OTMS models to the actual Japanese railway divisions. All models for these cases were large scale discrete optimization problems with several thousands of binary decision variables and constraints. In 3 out of 9 cases we could obtain an exact optimal solution in 30, 33 and 1220 seconds, respectively. For other 6 cases we could not find an exact optimal solution within 7200 seconds. In 4 out of 6 cases, we found that the gap between IP and LP solutions is found to be less than 1.5%. Remaining 2 cases had the gap 3.3% and 4.8%, respectively. From these numerical experiments results we can confirm that OTMS model has such properties as exact optimal solution's objective function value remains very close to the relaxed continuous type linear programming's one, thus most of the computation time will be spent only for searching an exact optimal solution with not so different ("improved") objective function values.

From this property and also from our numerical experiments, we believe that we can obtain an exact optimal solution with reasonable amount of time such as 20 to 30 minutes in case that the railway division network has rather simple structure, and also in case that we cannot find an exact optimal solution within e.g. two hours, the best feasible solution obtained would be acceptable as its corresponding objective function value ranges close to the upper bound given from the relaxed continuous linear programming problem. As our OTMS model has generally a large scale structure with several thousands of binary decision variables, it would be unavoidable we need a large amount of computation time, e.g. two hours for some cases.

We tried to apply our modeling approach to the actual railway system in Japan by assuming that we operate MTT facilities for track maintenance activities being based upon the schedule obtained from our model's optimal solution. Then we find that for RD I the standard deviation of the surface irregularity (σ) can be improved around 2.1% from 1.42mm in previous years to 1.39mm in the following year while the amount of maintenance operation activity (r) can be improved at least 16.1% from 49.6km and 58.4km in previous years to 41.6km in the following year. For RD II, σ can be improved around 2.5% from 2.07mm and 2.03mm in previous years to 1.98mm in the following years while r can be improved at least

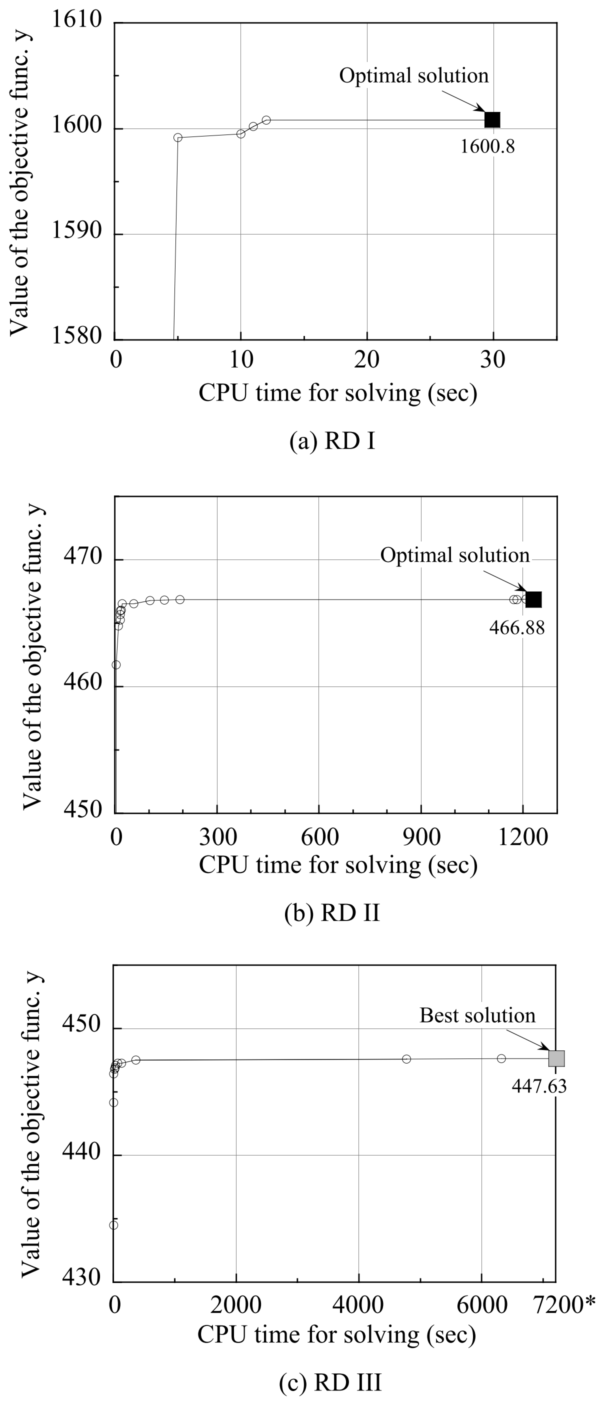

Fig. 9. Process for the solution of RD I, RD II and RD III.

9.8% from 70.6km to 63.7km and 53.2 km in the following years. For RD III we find that σ can be improved just 0.5% in two years from 2.28mm to 2.27mm while r can also be improved 40.3km to 40.1km during the same period. Thus, we find that the condition of surface irregularity corresponding to the model solution has been better than those recorded during previous years in both the standard deviation of the surface irregularity and the amount of maintenance operation activities. Therefore, we can conclude that the maintenance activities with MTT obtained from the model solution is efficient enough to be applied for the actual railway network system in Japan, and the model itself can be effective and useful for the practical use.

5. Conclusions and Future Problems

We built an all-integer linear programming model for obtaining an optimal tamping scheduling by an MTT. We summarize our results as follows.

(i) In order to solve the scheduling model on a wide-use PC, we proposed a simple procedure to obtain a feasible tamping schedule for problems of any scale in a reasonable time. We confirmed that the procedure is effective, in terms of the computation time and the accuracy of the optimal solution.

(ii) By applying the scheduling model to an actual railway network system, we confirmed that the model solution brought us a standard deviation of surface irregularities better than those recorded in the past.

(iii) Obtaining a practically feasible tamping schedule by solving our mathematical models, we found that obtained results were efficient and useful enough to improve our maintenance activities.

Based on the above results of our numerical experiments for the MUS and OTMS models, several JR companies have been using our models successfully in order to obtain an optimal tamping scheduling by an MTT.

Remaining problems follow that our current model does not take cost data explicitly into account yet. Both fixed costs and operating costs for MTT which are occupying large share among several major cost factors, need to be considered when we make a track maintenance schedule. Also some type of uncertainties including forecasting future deterioration and recovery processes for the tracks need to be incorporated into our MTT maintenance scheduling model as this may make the solution more practical and reliable for most Japanese railway companies. We have been working to modify our OTMS models so that the above remaining problems are to be solved efficiently.

Various types of train scheduling problems have been investigated by many researchers and practitioners and many results have been published (refer to e.g. [13, 14, 15, 16]). However, as far as the authors know, most publications have been focused on scheduling train itself and crew, rather than track maintenance facilities such as MTT, and very few have been dealing with this type of scheduling problem. We believe track maintenance scheduling problem is very important as the maintenance task has been extremely costly and the company can benefit greatly by saving the maintenance budget efficiently. Presently, we have been making our efforts to revise our track maintenance scheduling model by incorporating budgetary aspects of the railway company into our model.

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