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INTEGRAL EQUATIONS AND INVERSE BOUNDARY VALUE PROBLEMS

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Given the success of applying integral equations to direct boundary value problems both for the theoretical foundation and the numerical solution, it is no surprise that integral equations also play an important role in the analysis of the corresponding inverse problems. This special issue will cover recent developments in the use of integral equations for solving inverse boundary value problems arising from nondestructive evaluation using electro- and magnetostatic imaging and time-harmonic acoustic and electromagnetic wave scattering, i.e., inverse boundary value problems for the Laplace, Helmholtz and Maxwell equations. We expect the readership to consist of researchers and students interested in merging integral equations and inverse problems.

Roughly speaking, the methods for solving inverse boundary value problems can be classified into three groups. Iterative methods interpret the inverse boundary value problem as a nonlinear ill-posed operator equation and apply iterative schemes such as regularized Newton methods, Landweber iterations or conjugate gradient methods for its solution. Decomposition methods, in principle, separate the inverse boundary value problem into an ill-posed linear problem to reconstruct the solution of the partial differential equation from partial knowledge on it followed by a nonlinear problem that determines the boundary shape from the boundary condition. Finally, the third group consists of the more recently developed sampling and probe methods. These are based on indicator functions formulated in terms of solvability conditions for ill-posed linear integral equations that decide whether a point lies inside or outside the unknown object. For each of these three groups, this special issue contains two papers with the majority of them devoted to scattering theory.

Starting with iterative methods, the paper by H. Harbrecht and T. Hohage is concerned with the numerical solution of the inverse obstacle scattering problem to reconstruct the shape of a three dimensional sound-soft scatterer from the far field pattern of the scattered wave for scattering of plane waves. Their method is a regularized Newton iteration with the forward operator and the derivatives computed via a wavelet Galerkin method for the combined double- and single-layer potential approach. Due to the known performance of analogous methods in two dimensions it was to be expected that the regularized Newton scheme would also perform successfully in three dimensions. However, in three dimensions the efficient implementation of the boundary integral approximations and the surface parameterizations are quite challenging. In this sense, the paper is a valuable contribution to the area of inverse scattering by providing a detailed and solid description of the reconstruction method and illustrating its performance through illuminating numerical examples.

The paper by H. Eckel and R. Potthast makes an interesting contribution to the field of numerical solution methods for inverse boundary value problems. It is concerned with the inverse scattering problem to reconstruct the location and the shape of an unknown number of sound-soft obstacles from the far field pattern corresponding to the scattering of one incident plane wave. In order to avoid the shortcomings of the regularized Newton iterations, i.e., the need a priori to know the number of obstacles and the requirement for good initial guesses on the shape, the authors suggest to combine a classical Newton scheme with an evolutionary algorithm. They provide a detailed description of the proposed algorithm and illustrate its practicality through a number of two-dimensional numerical examples with the Newton steps as part of the combined algorithm implemented through the use of boundary integral equations.

Turning to decomposition methods, the contribution by O. Ivanyshyn and T. Johannson is a numerical study comparing the performance of two closely related iterative algorithms for solving the inverse scattering problem for sound-soft obstacles. In principle, both approaches can be interpreted as revisiting classical decomposition methods and are based on two nonlinear integral equations for the unknown shape and the unknown boundary flux of the total field that are derived from Huygen's principle. These two integral equations are equivalent to the inverse problem and can be viewed as the data and the field equation. The two methods considered in this paper rely on different linearizations of this system of integral equations and subsequent iteration. The authors provide a clear description of both algorithms and an extensive discussion of the differences in their performances based on a wide variety of two-dimensional examples.

The paper by R. Chapko and N. Vintonyak also is of a more computational nature. It is concerned with the inverse problem for the two-dimensional Laplace equation to recover the shape of a perfectly conducting inclusion within a semi-infinite region from overdetermined Cauchy data on part of the accessible exterior boundary. In principal, the method used in this paper may be considered as a decomposition method although it is named a hybrid method since it also contains some features of a Newton type iteration. In this way it combines the advantages of the decomposition method, i.e., low computational costs, and of Newton iterations, i.e., high quality reconstructions. The merit of the present paper consists of the extension of the theoretical foundation and numerical implementation of this type of decomposition method to the case of semi infinite two-dimensional domains.

The factorization method due to Kirsch may be considered as the most satisfying version of the sampling and probe methods in the sense that it leads to complete characterizations of the scattering object in terms of the data for the inverse problem. Unfortunately, for inverse electromagnetic scattering some parts of the analysis of the factorization method are still open. By considering the inverse scattering problem for an absorbing inhomogeneous medium embedded in a layered background, the contribution by A. Kirsch treats a configuration where the classical results on the factorization method with slight modifications are shown to be valid. The first part of the paper is concerned with a new integral equation method for the solution of the corresponding direct scattering problem via a Lippmann-Schwinger type integral equation in a Sobolev space setting. The second part is then devoted to the theoretical basis for the application of the factorization method to the present electromagnetic inverse problem.

Finally, F. Cakoni and H. Haddar consider the inverse scattering problem to determine the shape and the surface conductivity of a partially coated anisotropic dielectric embedded in a layered piecewise homogeneous background from electromagnetic Cauchy data. The method that is suggested is based on a recent modification of the linear sampling method via the use of the reciprocity gap functional. The main ingredient of the algorithm ultimately consists of solving ill-posed linear integral equations of the first kind. The particular advantage of the method is that neither explicit knowledge on the scattered field for the background medium nor a priori information on the coating and the index of refraction of the scatterer is required. The present paper provides the theoretical foundation of the method through some challenging and novel analysis. From experience obtained through numerical implementations in similar cases, the practicality of the method is to be expected.

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