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BOOK REVIEW

Differential Geometry of Curves and Surfaces, by Thomas Banchoff and Stephen Lovett, A K Peters Ltd., Natick 2010, xvi + 331 pp, ISBN 978-1-56881-456-8.

Differential Geometry of Manifolds, by Stephen Lovett, A K Peters Ltd., Natick 2010, xiii + 421 pp, ISBN 978-1-56881-457-5.

These are two books that together in a pair are intended to bring the reader through classical differential geometry of curves and surfaces into the modern differential geometry of manifolds. The first book, *Differential Geometry of Curves and Surfaces*, by Thomas Banchoff and Stephen Lovett, offers a complete guide for the study of classical theory of curves and surfaces and is intended as a textbook for a one semester course for undergraduates, assuming only experience in vector calculus and linear algebra. The readers are provided with computer graphics applets that can be used for computer labs, in-class illustrations, or simply as intuitive aids to support the material.

The second book, *Differential Geometry of Manifolds*, by Stephen Lovett, provides an introduction to the theory of differentiable manifolds - the natural generalization of regular curves and surfaces to higher dimensions. It is addressed to advanced undergraduate or beginning graduate readers and to be comprehensive and still only require the standard undergraduate math programs as prerequisites, three appendices provide the necessary background from topology, calculus of variations, and multilinear algebra. Neither book directly relies on the other but knowledge of the content of the first book is beneficial for the second one.

The first book contains eight chapters. In Chapters 1 through 4 the readers are provided with the local and global theory of plane and space curves. In Chapter 1 the authors present local properties of plane curves and in contrast to it in Chapter 2 they introduce global properties of plane curves. Chapter 3 is devoted to the local properties of space curves similarly to the theory of plane curves and Chapter 4 concerns global properties of space curves. In the local theory the authors introduce the fundamental notions of curvature and torsion, construct various associated objects such as the evolute, the osculating circle, the osculating sphere, and formulate the fundamental theorem of plane or space curves. The global theory presents how local properties relate to global properties of curves such as closedness, concavity, and knottedness. At the end of Chapter 4 the authors briefly discuss the concept of a knotted curve and two linked curves and present two main theorems: the Fary-Milnor Theorem, which shows how the property of knottedness imposes a condition on the total curvature of a curve, and the Gauss's formula for the linking number of two curves. The topics discussed in the first four chapters are illustrated by a lot of computer graphic applets. Many concepts and theorems introduced in the text are explained visually by interactive computer graphics, which help the reader to understand better the presented material.

Chapter 5 introduces the notion of a regular surface together with the notion of a tangent space, normal vector and the concept of orientability. Many examples and figures support the theory. Each section concludes with a large set of exercises that a suitable for self-study.

Chapter 6 is devoted to the local theory of surfaces, focusing on the first and the second fundamental form and the invariants they generate. The two fundamental geometric invariants - the Gaussian curvature and the mean curvature are introduced in this chapter. There is a special section studying the properties of surfaces with either Gaussian curvature everywhere zero (developable ruled surfaces) or mean curvature everywhere zero (minimal surfaces).

Chapter 7 concerns the fundamental equations of surfaces. After the readers are provided with the notation of tensors, the authors introduce the Christoffel symbols and present relations between the first and second fundamental forms - the Gauss equations and the Mainardi-Codazzi equations. They also present the famous Theorema Egregium of Gauss, which states that the Gaussian curvature depends only on the metric tensor. Another remarkable result in the theory of surfaces is provided in this chapter - the Fundamental Theorem of Surfaces, which proves that under appropriate conditions, the coefficients of the first and the second fundamental forms determine the surface up to a motion in the space.

The final chapter of the book presents the concept of *Intrinsic Geometry*. The chapter starts with the local theory of regular curves on regular surfaces introducing the Darboux frame and the basic invariants of a curve on a surface - the normal curvature, the geodesic curvature, and the geodesic torsion. A special attention in this chapter is paid to geodesics and geodesic coordinates. The surfaces of revolution are considered as an example on which the theory of geodesics is applied. The chapter ends with the profound Gauss-Bonnet Theorem, both in local and global forms, and applications of the Gauss-Bonnet Theorem to plane, spherical, and hyperbolic geometry.

The main advantages of the book are the careful introduction of the concepts, the good choice of the exercises, and the interactive computer graphics, which make the text well-suited for self-study. The reader could both follow the presentation in the textbook and use the software as supporting material. The access to online computer graphics applets that illustrate many concepts and theorems presented in the text provides the readers with an interesting and visually stimulating study of classical differential geometry.

The second book continues the development of differential geometry by studying manifolds. It contains six chapters and three appendices.

Chapter 1 presents the analysis of multivariable functions and supports the theory with a lot of examples and exercises. Chapter 2 introduces general coordinate systems and the concept of variable frames. It discusses the calculus of moving frames and matrix functions. The chapter ends with the introduction of classical tensor notation.

Having developed the technical machinery, Chapter 3 introduces the notion of a differentiable manifold and defines maps between manifolds. The author presents the definition of the tangent space on a manifold and gives a brief comment on the connection between the orientation of a manifold and the standard bases in coordinate systems. The chapter ends with establishing the notions of immersions, submersions, and submanifolds.

Chapter 4 develops the analysis on differentiable manifolds, introducing the formalism of vector bundles on a manifold, discussing vector and tensor fields, and developing the calculus of differential forms. The last topic in the chapter covers the theory of integration on manifolds and presents Stokes' theorem - a central result in the theory of integration on manifolds.

Chapter 5 is devoted to Riemannian geometry. The author introduces the notions of metrics, connections, geodesics, parallel transport, and the curvature tensor. Special attention is paid to the Ricci curvature tensor and the Einstein tensor, which is of central importance in the general relativity.

In Chapter 6 the theory of differentiable manifolds is applied to four areas in physics - Hamiltonian mechanics, electromagnetism, string theory, and general relativity.

At the end of the book three appendices provide the basic notions and some main theorems from topology, calculus of variations, and multilinear algebra.

Both books, *Differential Geometry of Curves and Surfaces* and *Differential Geometry of Manifolds*, will certainly be very useful for many students. A distinguishing feature of the books is that many of the basic notions, properties and results are illustrated by a great number of examples and figures. Each section includes numerous interesting exercises, which make these books ideal for self-study too.

These books give a nice addition to the existing literature [1-8] in the field of differential geometry of curves, surfaces, and manifolds. I strongly recommend them to anyone wishing to enter into the beautiful world of the differential geometry.

References

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Velichka Milousheva
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Block 8
1113 Sofia, BULGARIA
E-mail address:vmil@math.bas.bg