



## BOOK REVIEW

*Complex, Contact and Symmetric Manifolds*, In Honor of L. Vanhecke, Oldřich Kowalski, Emilio Musso and Domenico Perrone, Editors, Birkhäuser, Basel, 2005, viii + 277 pp., ISBN 0-8176-3850-4

This volume contains lectures delivered at the International Conference “Curvature in Geometry” held in Lecce, Italy, 11-14 June, 2003 in honor of Professor Lieven Vanhecke.

The name of Professor Vanhecke is well-known to those who are interested in differential geometry. His contributions to (pseudo-)Riemannian geometry are widely recognized. The scientific work of Professor Vanhecke is devoted to problems in many different fields of the differential geometry – Hermitian and Kähler manifolds, almost complex and almost contact manifolds, volumes of geodesic spheres and tubes, homogeneous and curvature homogeneous spaces, harmonic spaces, foliations, geometry of the tangent and the unit tangent bundle, minimal and harmonic vector fields, etc. Accordingly, the collection in the volume covers a variety of topics and might be of interest to geometers with different taste.

Contents (with short summaries). *Curvature of Contact Metric Manifolds* by David Blair. This is a survey of results and open questions concerning the curvature of a Riemannian metric associated to a contact form. *A Case for Curvature: the Unit Tangent Bundle* by Eric Boeckx. Curvature properties of the Sasaki metric on the unit tangent bundle are discussed in the paper. *Convex Hypersurfaces in Hadamard Manifolds* by A. Borisenko. An extremal property of Lobachevsky space among simply connected Riemannian manifolds of nonpositive curvature is established in this article. *Contact Metric Geometry of the Unit Tangent Sphere Bundle* by G. Calvaruso. This is a review on the relations between the geometry of a Riemannian manifold and the natural almost contact metric structure on its unit tangent sphere bundle. *Topological–Antitopological Fusion Equations, Pluriharmonic Maps and Special Kähler Manifolds* by Vicente Cortés and Lars Schäfer. Motivated by the equations of topological–antitopological fusion, the authors introduce the notion of a  $\mathcal{U}^*$ -bundle and give a characterization of the tan-

gent bundle of special complex and special Kähler manifolds as particular types of  $\mathcal{U}^*$ -bundles.  *$\mathbb{Z}_2$  and  $\mathbb{Z}$ -Deformation Theory for Holomorphic and Symplectic Manifolds* by Paolo de Bartolomeis. New  $\mathbb{Z}_2$ -construction for generalized moduli spaces of holomorphic and symplectic structures are presented within the general framework of deformation theory. *Commutative Condition on the Second Fundamental Form of CR-submanifolds of Maximal CR-dimension of a Kähler manifold* by Mirjana Djorić. The induced almost contact metric structure on  $m$ -dimensional real submanifolds with  $(m - 1)$ -dimensional maximal holomorphic tangent subspace in a Kähler manifold is studied. *The Geography of Non-Formal Manifolds* by Marisa Fernández and Vicente Muñoz. The authors show that there exist non-formal compact oriented manifolds of dimension  $n$  and first Betti number  $b_1 = b \geq 0$ , if and only if  $n \geq 3$  and  $b \geq 2$ , or  $n \geq (7 - 2b)$  and  $0 \leq b \leq 2$ . *Total Scalar Curvature of Geodesic Spheres and of Boundaries of Geodesic Disks* by J. C. Díaz-Ramos, E. García-Río, and L. Hervella. The first terms in the corresponding power series expansions for the total scalar curvature and  $L^2$ -norms of the scalar curvature, the Ricci tensor and the curvature tensor of boundaries of geodesic disks in Riemannian manifolds are obtained. *Curvature Homogeneous Pseudo-Riemannian Manifolds which are not Locally Homogeneous* by Corey Dunn and Peter Gilkey. The authors construct pseudo-Riemannian manifolds of split signature  $(p, p)$ ,  $p \geq 3$ , which are curvature homogeneous, but not locally homogeneous. *On Hermitian Geometry of Complex Surfaces* by A. Fujiki and M. Pontecorvo. The authors discuss the problem of which compact complex surfaces admit locally conformal Kähler metrics and present a new twistor construction of anti-self-dual Hermitian metrics (which are automatically l.c.K.) on class VII surfaces. *Unit Vector Fields that are Critical Points of the Volume and of the Energy: Characterization and Examples* by Olga Gil-Medrano. This is a survey describing examples and general results for harmonic and minimal vector fields. *On 3D-Riemannian Manifolds with Prescribed Ricci Eigenvalues* by Oldřich Kowalski and Zdeněk Vlášek. The authors address the problem of how many isometry classes of germs of Riemannian metrics exist for prescribed distinct Ricci eigenvalues given by real-analytic functions. *Two Problems in Real and Complex Integral Geometry* by A. M. Naveira. Inequalities of isoperimetric type and complex quermass integrals are considered and two open problems about them are posed in the paper. *Notes on the Goldberg Conjecture in Dimension Four* by Takashi Oguro and Kouei Sekigawa. This is a brief survey on recent progress in dimension four concerning the Goldberg conjecture that any compact almost Kähler Einstein manifold is Kähler. *Curved Flats, Exterior Differential Systems, and Conservation Laws* by Chuu-Lian Terng and Erxiao Wang. The main goals of

the paper are to review some results of the first named author about curved flats in symmetric spaces and to use techniques from soliton theory to construct infinitely many integral submanifolds and conservation laws for a certain exterior differential system defined by R. Bryant. *Symmetric Submanifolds of Riemannian Symmetric Spaces and Symmetric R-spaces* by K. Tsukada. The author explains the similarity between the theories of Riemannian symmetric spaces and symmetric submanifolds and the ideas of classification of symmetric submanifolds in the framework of Grassmann geometry, and finally states some recent classification results. *Complex Forms of Quaternionic Symmetric Spaces* by Joseph A. Wolf. A complete classification of the complex forms of quaternionic symmetric spaces is given in the paper.

All papers are written with the necessary introductory and contextual material and present research trends in modern geometry. This makes them quite appropriate for the non-specialists and students.

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