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EXACT CONTROLLABILITY OF A THERMOELASTIC SYSTEM WITH CONTROL IN THE THERMAL COMPONENT ONLY

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Abstract. In this work we give a result of exact controllability for a thermoelastic system in which the control term is placed solely in the thermal equation. With such an indirect control input, one is able to control exactly the displacement of the plate, as well as the temperature. This exact controllability occurs in arbitrarily small time. In the case that the moment of inertia parameter for the plate is absent (i.e., $\gamma = 0$ below), then one is provided here with a result of exact controllability for a thermoelastic system which is modelled by the generator of an *analytic* semigroup. The proof here depends upon a multiplier method so as to attain the associated observability inequality. The particular multiplier invoked is of an operator theoretic nature, and has been used previously by the author in deriving stability results for this partial differential equation model.

1. Introduction.

1.1. Statement of the problem. Let Ω be a bounded open subset of $\mathbb{R}^2$ with sufficiently smooth boundary Γ, and let terminal time $T > 0$. In this work, we will study the exact controllability problem for the following thermoelastic system, with the control function $u \in L^2(0,T;H^{-1}(\Omega))$:

$$\begin{cases} \omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta = 0 \\ \theta_t - \Delta\theta + \sigma\theta - \alpha\Delta\omega_t = u \end{cases} \quad \text{on } (0,T)\times\Omega; \tag{1}$$

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$$\omega = \frac{\partial \omega}{\partial \nu} = 0 \text{ on } (0,\infty)\times\Gamma; \quad \theta = 0 \text{ on } (0,\infty)\times\Gamma;$$
$$\omega(t=0)=\omega_0,\ \omega_t(t=0)=\omega_1,\ \theta(t=0)=\theta_0 \text{ on } \Omega.$$

Here, the coupling parameter $\alpha > 0$; the nonnegative constant γ is proportional to the thickness of the plate and assumed to be small with $0 \le \gamma \le M$; the constant σ is also nonnegative. There are other physical constants associated with system, but they have been set here to unity for the sake of simplicity. As usual, $[\nu_1, \nu_2]$ is the unit normal outward to the boundary. The PDE model (1), without the given interior control, is derived in [11], and mathematically describes a Kirchoff plate subjected to a thermal damping. The displacement of the plate is represented by the function ω, and the temperature denoted by the function θ.

Defining the space $H^1_{0,\gamma}(\Omega)$ to be

$$H^1_{0,\gamma}(\Omega) = \begin{cases} H^1_0(\Omega) & \text{if } \gamma > 0 \\ L^2(\Omega) & \text{if } \gamma = 0, \end{cases} \tag{2}$$

one can show wellposedness of the uncontrolled thermoelastic system (i.e. $u = 0$ in (1)) for initial data $[\omega_0, \omega_1, \theta_0] \in H^2_0(\Omega) \times H^1_{0,\gamma}(\Omega) \times L^2(\Omega)$ (see Proposition 2.1 below). In general, for given $u \in L^2(0, T; H^{-1}(\Omega))$, the corresponding solution $[\omega, \omega_t, \theta]$ is *a priori* continuous in time into a space strictly larger space than $H^2_0(\Omega) \times H^1_{0,\gamma}(\Omega) \times L^2(\Omega)$ (see (16) below). We note here at the outset the dichotomy presented by the parameter γ: With $\gamma > 0$, the system (1) is *hyperbolic–like*; when $\gamma = 0$, the system is modelled by the generator of an *analytic semigroup*, and so corresponds to parabolic–like dynamics (see [17] and [14]). With the basic space of wellposedness established, we are concerned with the following question of exact controllability on a given time interval $[0, T]$: For data $[\omega_0, \omega_1, \theta_0]$ (initial) and $\left[\omega^T_0, \omega^T_1, \theta^T_0\right]$ (terminal) in $H^2_0(\Omega) \times H^1_{0,\gamma}(\Omega) \times L^2(\Omega)$, is there a suitable control $u \in L^2(0, T; H^{-1}(\Omega))$ such that the corresponding solution $[\omega, \omega_t, \theta]$ to (1) satisfies at terminal time T,

$$[\omega(T), \omega_t(T), \theta(T)] = \left[\omega^T_0, \omega^T_1, \theta^T_0\right] \ ? \tag{3}$$

Controllability properties for this system have been much studied of late, under varying boundary conditions for the displacement, and with different choices of controls. The controllability of the system (1) is initially considered by J. Lagnese in [12], with control being implemented in the boundary conditions for ω (in this work, free boundary conditions are imposed, instead of the clamped ones in place here). With such a boundary–controlled thermoelastic system, a result of *partial exact controllability* is obtained for $\gamma > 0$ (the hyperbolic case); that is to say, the displacement ω is exactly controlled, provided the coupling parameter α is small enough. In a more recent work, valid for $\gamma > 0$ (the hyperbolic case), the present author and I. Lasiecka in [3] consider the very same thermoelastic system with free boundary conditions as in [12], and are able to remove the aforementioned size restriction on the parameter α, at the expense of inserting additional thermal control on an arbitrarily small subset of the boundary. The result ultimately is one of *exact controllability* for the displacement ω and *approximate controllability* for the temperature θ with α arbitrary. L. de Teresa and E. Zuazua in [6] derive the same sort of partial–approximate controllability result in the case that interior control is implemented in the Kirchoff component of (1). In each of these works, given that the control term is acting on the plate component of the dynamics, and that $\gamma > 0$, critical use is made of controllability results for the uncoupled Kirchoff plate so as to eventually treat the system (1) as a perturbation of a Kirchoff plate. Later still, S. Hansen and B. Zhang in [7] study a one–dimensional version of (1) under the influence of a control at one of the boundary conditions for ω. With such a single scalar control in place, they are able to obtain a result of *exact null controllability*; i.e., $[\omega, \omega_t, \theta]$ can be driven to zero at time T; this result holds for all $\gamma \geq 0$.

Here, we address the aforementioned question of exact controllability for *both* the displacement and the temperature, and in both the analytic and nonanalytic cases. Our main result in that direction is as follows:

Theorem 1.1. *For all $\gamma \geq 0$, the system (1) is exactly controllable in arbitrary time $T > 0$. That is to say, for any $T > 0$, and data $[\omega_0, \omega_1, \theta_0]$, $[\omega_0^T, \omega_1^T, \theta_0^T]$ in the space $H_0^2(\Omega) \times H_{0,\gamma}^1(\Omega) \times L^2(\Omega)$, one can find a control function $u \in L^2(0, T; H^{-1}(\Omega))$ such that the corresponding solution $[\omega, \omega_t, \theta]$ to (1) satisfies $[\omega(T), \omega_t(T), \theta(T)] = [\omega_0^T, \omega_1^T, \theta_0^T]$.*

The novelties inherent in this theorem are the following:

(1) Theorem 1.1 states that the displacement of the plate ω can be controlled exactly by the indirect means of placing the control input term in the thermal component. Note that since the control term is in the heat equation, the proof of exact controllability will *not* hinge on perturbation arguments which exploit known controllability results for Kirchoff plates in

the case that $\gamma > 0$, or Euler–Bernoulli beams in the case that $\gamma = 0$. The proof of Theorem 1.1 is necessarily direct.

(2) In the case that $\gamma = 0$, it has recently been demonstrated in [14] that the thermoelastic system (1), under all possible boundary conditions for ω, is abstractly modelled by the generator of an analytic semigroup (see (11) and (14) below). Therefore, Theorem 1.1 constitutes a result of exact controllability for an analytic system in the case that $\gamma = 0$. (It is well–known that exact controllability results for analytic systems are hard to come by. See [5] and [19] for statements of some sufficient conditions for the approximate and exact controllability of analytic systems.) In this respect, our work here complements that recently completed by I. Lasiecka and R. Triggiani in [15], which gives results of exact null controllability for the thermoelastic model (1) in the (analytic) case that $\gamma = 0$, under the influence of either mechanical or thermal control (as we said earlier, the aforementioned paper [7] also recovers the null controllability of a one–dimensional version of (1) for $\gamma = 0$).

The methodology employed in the proving of Theorem 1.1 is based upon the classical argument of showing the ontoness of the *control* $\to$ *terminal state* map $\mathcal{L}_T$ (see (13) below for the explicit description of $\mathcal{L}_T$). Establishing the surjectivity for $\mathcal{L}_T$ is in turn tantamount to deriving the following (observability) inequality for some $C_T > 0$:

$$\int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} \geq C_T \left\|[\phi_0, \phi_1, \psi_0]\right\|^2_{H_0^2(\Omega)\times H^1_{0,\gamma}(\Omega)\times L^2(\Omega)}\,, \tag{4}$$

where ψ is the thermal component of the solution $[\phi, \phi_t, \psi]$ to the following backwards thermoelastic system, adjoint with respect to (1):

$$\begin{cases} \phi_{tt} - \gamma\Delta\phi_{tt} + \Delta^2\phi + \alpha\Delta\psi = 0 \\ \psi_t + \Delta\psi - \sigma\psi - \alpha\Delta\phi_t = 0 \end{cases} \quad \text{on } (0,T)\times\Omega; \tag{5}$$
$$\phi = \frac{\partial\phi}{\partial\nu} = 0 \text{ on } (0,T)\times\Gamma; \quad \psi = 0 \text{ on } (0,T)\times\Gamma;$$
$$\phi(T) = \phi_0, \quad \phi_t(T) = \phi_1, \quad \psi(T) = \psi_0 \text{ on } \Omega.$$

A multiplier technique is invoked here to attain the inequality (4) (see [10] for a treatise of the multiplier method in PDE control theory), with the chosen multiplier being of an operator theoretic nature. In fact, the critical multiplier is $A_D^{-1}\theta$, where the operator A_D denotes the Laplacian with Dirichlet boundary conditions (see (6) below). This particular multiplier has also seen service in [1], [2] and [4], works which are concerned with ascertaining stability properties of linear and nonlinear variations of (1).

Remark 1.2. The result of Theorem 1.1 is also valid in the case that the space of controllability is $\left[L^2(\Omega)\right]^2$, and the control operator is taken to be the divergence of a vector field; i.e., for $\overline{u} \in \left[L^2(\Omega)\right]^2$, $\mathcal{B}\overline{u} = \frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}$. A control term of this sort has the physical interpretation of a radiative heat flux $(div(\overline{u}))$ acting through the volume of the plate.

2. Abstract operator formulation and analysis. Our proof of controllability (Theorem 1.1) hinges on establishing the observability inequality (4) for $T > 0$. To justify this statement, we will consider the PDE system (1) and its adjoint (5) as abstract evolution equations in a certain Hilbert space. To develop the associated operator models, we introduce the following definitions and notations.

(A.1) We define the following realizations of elliptic differential operators, $\mathring{\mathbf{A}}$: $L^2(\Omega) \supset D\left(\mathring{\mathbf{A}}\right) \to L^2(\Omega)$ and $A_D : L^2(\Omega) \supset D\left(A_D\right) \to L^2(\Omega)$:

$$\begin{aligned} \mathring{\mathbf{A}} &= \Delta^2,\ D(\mathring{\mathbf{A}}) = H^4(\Omega) \cap H_0^2(\Omega); \\ A_D &= -\Delta,\ D(A_D) = H^2(\Omega) \cap H_0^1(\Omega). \end{aligned} \tag{6}$$

(A.2) For $\gamma \geq 0$, we define the operator P_γ by

$$P_\gamma \equiv \mathbf{I} + \gamma A_D, \tag{7}$$

and define a space $H^1_{0,\gamma}(\Omega)$ equivalent to $H^1_0(\Omega)$ with its inner product being defined as

$$(\varpi, \varpi')_{H^1_{0,\gamma}(\Omega)} \equiv (\varpi, \varpi')_{L^2(\Omega)} + \gamma(\nabla\varpi, \nabla\varpi')_{L^2(\Omega)} \ \ \forall\, \varpi, \varpi' \in H^1_0(\Omega), \tag{8}$$

and with its dual denoted as $H^{-1}_{0,\gamma}(\Omega)$. Note that when $\gamma = 0$, $P_0 = \mathbf{I}$ and we set $H^1_{0,0}(\Omega) = H^{-1}_{0,0}(\Omega) = L^2(\Omega)$. The obvious $H^1_{0,\gamma}(\Omega)$–ellipticity of P_γ and Lax–Milgram give then that P_γ is boundedly invertible; i.e., $P_\gamma^{-1} \in \mathcal{L}(H^{-1}_{0,\gamma}(\Omega), H^1_{0,\gamma}(\Omega))$.

(A.3) We denote the Hilbert space $\mathbf{H}_\gamma$ to be

$$\mathbf{H}_\gamma \equiv H^2_0(\Omega) \times H^1_{0,\gamma}(\Omega) \times L^2(\Omega), \tag{9}$$

with the inner product

$$\left(\begin{bmatrix} \omega_0 \\ \omega_1 \\ \theta_0 \end{bmatrix}, \begin{bmatrix} \widetilde{\omega}_0 \\ \widetilde{\omega}_1 \\ \widetilde{\theta}_0 \end{bmatrix} \right)_{\mathbf{H}_\gamma} \tag{10}$$

$$= (\Delta\omega_0, \Delta\widetilde{\omega}_0)_{L^2(\Omega)} + (\omega_1, \widetilde{\omega}_1)_{L^2(\Omega)} + \gamma\,(\nabla\omega_1, \nabla\widetilde{\omega}_1)_{L^2(\Omega)} + \left(\theta_0, \widetilde{\theta}_0\right)_{L^2(\Omega)}.$$

(A.4) With the above definitions, we then set $\mathcal{A}_\gamma : \mathbf{H}_\gamma \supset D(\mathcal{A}_\gamma) \to \mathbf{H}_\gamma$ to be

$$\mathcal{A}_\gamma \equiv \begin{pmatrix} 0 & \mathbf{I} & 0 \\ -P_\gamma^{-1}\mathring{\mathbf{A}} & 0 & \alpha P_\gamma^{-1} A_D \\ 0 & -\alpha A_D & -A_D - \sigma\mathbf{I} \end{pmatrix}$$
$$\text{with } D(\mathcal{A}_\gamma) = \Big\{ [\omega_0, \omega_1, \theta_0] \in H_0^2(\Omega) \times H_0^2(\Omega) \times D(A_D) \text{ and such that } \mathring{\mathbf{A}}\omega_0 \in H_{0,\gamma}^{-1}(\Omega) \Big\}. \tag{11}$$

(A.5) We define a (control) operator $\mathcal{B} \in \mathcal{L}\big(H^{-1}(\Omega),\, H_0^2(\Omega) \times L^2(\Omega) \times H^{-1}(\Omega)\big)$ by having for $u \in H^{-1}(\Omega)$,

$$\mathcal{B}u = \begin{bmatrix} 0 \\ 0 \\ u \end{bmatrix}. \tag{12}$$

With this, we define the map $\mathcal{L}_T : L^2(0,T; H^{-1}(\Omega)) \to \mathbf{H}_\gamma$ by having for all $u \in L^2(0,T; H^{-1}(\Omega))$,

$$\mathcal{L}_T u \equiv \int_0^T e^{\mathcal{A}_\gamma (T-t)} \mathcal{B}u(t)dt. \tag{13}$$

Taking initial data $[\omega_0, \omega_1, \theta_0] \in \mathbf{H}_\gamma$ and control $u \in L^2(0,T; H^{-1}(\Omega))$, then the coupled system (1) becomes the operator theoretic model

$$\frac{d}{dt}\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = \mathcal{A}_\gamma \begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} + \mathcal{B}\overline{u}, \quad \begin{bmatrix} \omega(0) \\ \omega_t(0) \\ \theta(0) \end{bmatrix} = \begin{bmatrix} \omega_0 \\ \omega_1 \\ \theta_0 \end{bmatrix}, \tag{14}$$

which has an *a priori* meaning in $\left[D(\mathcal{A}_\gamma^*)\right]' \supset \mathbf{H}_\gamma$ only.

Similar to what was done in [11] and in [1] for the thermoelastic system (1) with higher order boundary conditions in place, one can show the existence of an associated semigroup $\left\{e^{\mathcal{A}_\gamma t}\right\}_{t\geq 0}$. In particular, we have

Proposition 2.1. (*wellposedness*) *With the parameter* $\gamma \geq 0$, $\mathcal{A}_\gamma$ *as defined in* (11) *generates a* C_0-*semigroup of contractions* $\left\{e^{\mathcal{A}_\gamma t}\right\}_{t\geq 0}$ *on the energy space* $\boldsymbol{H}_\gamma$.

With these dynamics in hand, the solution $[\omega, \omega_t, \theta]$ to (1) may be written explicitly as

$$\begin{bmatrix} \omega(t) \\ \omega_t(t) \\ \theta(t) \end{bmatrix} = e^{\mathcal{A}_\gamma t} \begin{bmatrix} \omega_0 \\ \omega_1 \\ \theta_0 \end{bmatrix} + \int_0^t e^{\mathcal{A}_\gamma (t-s)} \mathcal{B} u(s) ds. \tag{15}$$

A fortiori, $\mathcal{A}_\gamma^{-1}\mathcal{B} \in \mathcal{L}(H^{-1}(\Omega), \mathbf{H}_\gamma)$, or equivalently

$$\mathcal{B} \in \mathcal{L}\big(H^{-1}(\Omega), \left[D(\mathcal{A}_\gamma^*)\right]'\big);$$

the input to state map above thus gives that

$$[\omega, \omega_t, \theta] \in C\big([0,T]; \left[D(\mathcal{A}_\gamma^*)\right]'\big). \tag{16}$$

Given the representation (15) for the solution $[\omega, \omega_t, \theta]$, proving the exact controllability at given time $T > 0$ (Theorem 1.1) is then equivalent to showing the surjectivity of the operator $\mathcal{L}_T$, where $\mathcal{L}_T$ is defined in (13) (see e.g., [18] and [21]). Note that $\mathcal{L}_T$ is *a priori* well–defined as an element of $\mathcal{L}\big(L^2(0,T; H^{-1}(\Omega)), \left[D\big(\mathcal{A}_\gamma^*\big)\right]'\big)$ only; accordingly the control operator $\mathcal{L}_T$ as a mapping into the state space $\mathbf{H}_\gamma$ initially makes sense only as an unbounded operator with some given domain of definition. One can subsequently compute the adjoint $\mathcal{L}_T^* \in \mathcal{L}\left(D\left(\mathcal{A}_\gamma^*\right), L^2(0,T; H_0^1(\Omega))\right)$ as the classical expression (see e.g., [21])

$$\mathcal{L}_T^* \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} = \mathcal{B}^* e^{\mathcal{A}_\gamma^*(T-\cdot)} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix}. \tag{17}$$

We will have need of the "PDE form" of the adjoint $\mathcal{L}_T^*$, as the desired observability inequality (4) is tied–up with it. To this end, one can straightforwardly compute the adjoint $\mathcal{A}_\gamma^*$ as

$$\mathcal{A}_\gamma^* \;=\; \begin{pmatrix} 0 & -\mathbf{I} & 0 \\ P_\gamma^{-1}\mathring{\mathbf{A}} & 0 & -\alpha P_\gamma^{-1} A_D \\ 0 & \alpha A_D & -A_D - \sigma \mathbf{I} \end{pmatrix},$$

with $D(\mathcal{A}_\gamma^*) = D(\mathcal{A}_\gamma)$. Moreover, using the form of the control operator given in (12), one has the adjoint $\mathcal{B}^* \in \mathcal{L}\left(D(\mathcal{A}_\gamma), H_0^1(\Omega)\right)$ taking the form

$$\mathcal{B}^* \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} = \psi_0. \tag{18}$$

With the equality (17), the form of the adjoints in (18) and (18), and the definitions in (A.1)–(A.2) for the components of $\mathcal{A}_\gamma$ and $\mathcal{A}_\gamma^*$, one has then that the solution $[\phi, \phi_t, \psi] \in C([0,T]; \mathbf{H}_\gamma)$ to the PDE system (5) is given by

$$e^{\mathcal{A}_\gamma^*(T-t)} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} = \begin{bmatrix} \phi(t) \\ \phi_t(t) \\ \psi(t) \end{bmatrix}. \tag{19}$$

Moreover, we have explicitly

$$\mathcal{L}_T^* \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} (t) = \psi(t). \tag{20}$$

Remark 2.2. Note that for data $[\phi_0, \phi_1, \psi_0] \in D(\mathcal{A}_\gamma)$, the system (5) may be written abstractly as (see (18) and (19))

$$\begin{cases} P_\gamma \phi_{tt} = -\mathring{\mathbf{A}}\phi + \alpha A_D \psi & \text{in } H_{0,\gamma}^{-1}(\Omega); \\ \psi_t = -\alpha A_D \phi_t + (A_D + \sigma \mathbf{I})\psi & \text{in } L^2(\Omega); \\ [\phi(T), \phi(T), \psi(T)] = [\phi_0, \phi_1, \psi_0]. \end{cases} \tag{21}$$

Concerning this PDE, one can readily attain an energy relation which shows the inherent dissipation in the system. Indeed, let us make the following denotation for the *energy* of the system (to be used throughout)

$$E_\gamma(t) \equiv \frac{1}{2} \left\| e^{\mathcal{A}_\gamma^*(T-t)} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} \right\|_{\mathbf{H}_\gamma}^2, \text{ for } 0 \le t \le T, \tag{22}$$

so that in particular $E_\gamma(T) = \frac{1}{2}\|[\phi_0, \phi_1, \psi_0]\|_{\mathbf{H}_\gamma}^2$ and $E_\gamma(0) = \frac{1}{2}\|[\phi(0), \phi_t(0), \psi(0)]\|_{\mathbf{H}_\gamma}^2$. Therewith, one can multiply the first equation of (21) by ϕ_t, the second by ψ, integrate both equations from t to T, where $0 \le t \le T$, sum these expressions and perform computations similar to what was done in [11] and [1], so as to have:

Proposition 2.3. *The component* ψ *of the solution* $[\phi, \phi_t, \psi]$ *to the backward system* (5) *satisfies* $\psi \in L^2\left(0, \infty; H_0^1(\Omega)\right)$. *Indeed, we have the following relation valid for all data* $[\phi_0, \phi_1, \psi_0] \in \boldsymbol{H}_\gamma$ *and* $0 \leq t \leq T$:

$$E_\gamma(t) + \int_t^T \left[\|\nabla\psi(\tau)\|^2_{L^2(\Omega)} + \sigma\,\|\psi(\tau)\|^2_{L^2(\Omega)}\right] d\tau = E_\gamma(T). \tag{23}$$

Remark 2.4. From Proposition 2.3, we deduce that $\mathcal{L}_T^*$ is actually a bounded operator from $\mathbf{H}_\gamma$ into $L^2\left(0, T; H_0^1(\Omega)\right)$. By duality then, the control input operator $\mathcal{L}_T \in \mathcal{L}\left(L^2\left(0, T; H^{-1}(\Omega)\right), \mathbf{H}_\gamma\right)$.

We have gone on about $\mathcal{L}_T^*$, because to prove the ontoness of $\mathcal{L}_T$, it is enough, by a functional analytical principle (see e.g., [8], Lemma 3.8.18) and Remark 2.4, to show that there exists a constant $C_T > 0$ such that the following injectivity condition holds for all $[\phi_0, \phi_1, \psi_0] \in \mathbf{H}_\gamma$:

$$\left\| \mathcal{L}_T^* \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} \right\|_{L^2(0,T;H_0^1(\Omega))} \geq C_T \left\| \begin{bmatrix} \phi_0 \\ \phi_1 \\ \psi_0 \end{bmatrix} \right\|_{\mathbf{H}_\gamma}. \tag{24}$$

By the expression (20) and Poincaré's inequality, this in turn is equivalent to deriving the inequality (4) for arbitrary terminal data $[\phi_0, \phi_1, \psi_0] \in \mathbf{H}_\gamma$, where again ψ is the thermal component of the adjoint system (5). Our work in the next section is devoted to establishing just this inequality.

3. Proof of Theorem 1.1. As mentioned above, we are out to verify the inequality (4). Recall that using the semigroup generated by $\mathcal{A}_\gamma^*$, $[\phi, \phi_t, \psi]$ has the explicit form given in (19). Accordingly one can show by semigroup methods, similar to what is done in [1], [2] and [11], that for data $[\phi_0, \phi_1, \psi_0] \in D((\mathcal{A}_\gamma)^2)$, the corresponding solution $[\phi, \phi_t, \psi]$ to (5), besides residing in $C([0, T]; D((\mathcal{A}_\gamma)^2))$, enjoys the following regularity:

$$\phi \in C([0, T]; D(\mathbf{\mathring{A}})); \quad \phi_t \in C([0, T]; H^3(\Omega)); \quad \psi \in C([0, T]; H^3(\Omega)). \tag{25}$$

In view of this regularity for solutions corresponding to smooth initial data, a density argument will then allow the assumption throughout that $[\phi, \phi_t, \psi]$ has the regularity needed to justify the computations performed below.

Step 1. (A requisite trace result). A certain trace regularity result for the adjoint system (5) will be needed below, one which does not follow from the standard Sobolev trace theory, but is similar in nature to trace estimates for Euler–Bernoulli plates which were proved in [16], and for Kirchoff plates in [13]. This result is absolutely critical in obtaining the observability estimate (4). Multiplying the first equation of (5) by the quantity $h \cdot \nabla\phi$, where $h(x,y) \equiv [h_1(x,y), h_2(x,y)]$ is a $\left[C^2(\overline{\Omega})\right]^2$ vector field such that $h|_\Gamma = [\nu_1, \nu_2]$ on Γ, and subsequently integrating from 0 to T, one obtains the equation

$$\int_0^T \left(\phi_{tt} - \gamma\Delta\phi_{tt} + \Delta^2\phi + \alpha\Delta\psi, h \cdot \nabla\phi\right)_{L^2(\Omega)} dt = 0. \tag{26}$$

From here, we can proceed to estimate this equality, employing virtually the same computations as in [1] (see Lemma 2.3 therein) and [2] (see Lemma 2 therein) so as to eventually have

Lemma 3.1. *The component ϕ of the solution $[\phi, \phi_t, \psi]$ of (5) satisfies $\Delta\phi|_\Gamma \in L^2(0,T; L^2(\Gamma))$, with the accompanying estimate*

$$\int_0^T \|\Delta\phi\|^2_{L^2(\Gamma)} dt \leq C\left(\int_0^T E_\gamma(t)dt + \int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} dt + E_\gamma(T)\right), \tag{27}$$

where C does not depend on the parameter γ.

Step 2. (Proof of a lower–order tainted inequality). We prove by a multiplier method the following preliminary estimate.

Lemma 3.2. *For $\gamma \geq 0$ and $T > 0$, the solution $[\phi, \phi_t, \psi]$ to (5) obeys the following estimate:*

$$\begin{aligned} E_\gamma(T) \quad \leq \quad & C_T\left(\int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} dt + \|\phi\|^2_{C([0,T];H^1_{0,\gamma}(\Omega))}\right. \\ & \left. + \gamma\|\phi_t\|^2_{C([0,T];L^2(\Omega))} + \|\psi\|^2_{C([0,T];H^{-1}(\Omega))}\right). \end{aligned} \tag{28}$$

Proof of Lemma 3.2. We multiply the first equation in (5) by $A_D^{-1}\psi$ and integrate in time and space to obtain

$$\int_0^T \left(\phi_{tt} - \gamma\Delta\phi_{tt} + \Delta^2\phi + \alpha\Delta\psi, A_D^{-1}\psi\right)_{L^2(\Omega)} dt = 0, \tag{29}$$

and then proceed to estimate this quantity.

(B.1) *Dealing with* $\int_0^T \left(\phi_{tt} - \gamma\Delta\phi_{tt}, A_D^{-1}\psi\right)_{L^2(\Omega)} dt$: Using an integration by parts and the second differential equation of (21) yields

$$\begin{aligned}
&\int_0^T \left(\phi_{tt} - \gamma\Delta\phi_{tt}, A_D^{-1}\psi\right)_{L^2(\Omega)} dt \\
&= \left(\phi_t, A_D^{-1}\psi\right)_{L^2(\Omega)}\Big|_0^T + \gamma\left(\nabla\phi_t, \nabla A_D^{-1}\psi\right)_{[L^2(\Omega)]^2}\Big|_0^T \\
&- \int_0^T \left[\left(\phi_t, A_D^{-1}\psi_t\right)_{L^2(\Omega)} + \gamma\left(\nabla\phi_t, \nabla A_D^{-1}\psi_t\right)_{[L^2(\Omega)]^2}\right] dt
\end{aligned}$$

$$\begin{aligned}
&= \alpha\int_0^T \left[\|\phi_t\|_{L^2(\Omega)}^2 + \gamma\|\nabla\phi_t\|_{L^2(\Omega)}^2\right] dt \\
&- \int_0^T \left[\left(\phi_t, \left(\mathbf{I} + \sigma A_D^{-1}\right)\psi\right)_{L^2(\Omega)} + \gamma\left(\nabla\phi_t, \nabla\left(\mathbf{I} + \sigma A_D^{-1}\right)\psi\right)_{[L^2(\Omega)]^2}\right] dt \\
&+ \left(\phi_t, A_D^{-1}\psi\right)_{L^2(\Omega)}\Big|_0^T + \gamma\left(\nabla\phi_t, \nabla A_D^{-1}\psi\right)_{[L^2(\Omega)]^2}\Big|_0^T .
\end{aligned} \tag{30}$$

In regards to the last two terms of this relation, we have by Green's Theorem and the fact that $\phi_t \in H_0^1(\Omega)$ for $\gamma > 0$, that for any $t \in [0, T]$

$$\begin{aligned}
&\gamma\left(\nabla\phi_t(t), \nabla A_D^{-1}\psi(t)\right)_{[L^2(\Omega)]^2} = \gamma(\phi_t(t), \psi(t))_{L^2(\Omega)} \\
&\quad \le \gamma C_\epsilon \|\phi_t\|_{C([0,T];L^2(\Omega))}^2 + \gamma\frac{\epsilon}{8}\|\psi(t)\|_{L^2(\Omega)}^2 .
\end{aligned} \tag{31}$$

Moreover, using the fact that A_D is $H_0^1(\Omega)$–elliptic and the contraction of the semigroup $\{e^{\mathcal{A}_\gamma^* t}\}_{t\ge0}$, we have for $T \ge t \ge 0$

$$\left(\phi_t(t), A_D^{-1}\psi(t)\right)_{L^2(\Omega)} \le \frac{\epsilon}{8}\|\phi_t(t)\|_{H_{0,\gamma}^1(\Omega)}^2 + C_\epsilon\left\|A_D^{-1}\psi(t)\right\|_{L^2(\Omega)}^2 \tag{32}$$

$$\le \frac{\epsilon}{8}\|\phi_t(t)\|_{H_{0,\gamma}^1(\Omega)}^2 + C_\epsilon\|\psi(t)\|_{H^{-1}(\Omega)}^2 \le \frac{\epsilon}{4}E_\gamma(T) + C_\epsilon\|\psi\|_{C([0,T];H^{-1}(\Omega))}^2 .$$

Combining (30)–(32), we then have the estimate

$$\begin{aligned}
&\Big|\int_0^T \left(\phi_{tt} - \gamma\Delta\phi_{tt}, A_D^{-1}\psi\right)_{L^2(\Omega)} dt - \alpha\int_0^T [\|\phi_t\|_{L^2(\Omega)}^2 + \gamma\|\nabla\phi_t\|_{[L^2(\Omega)]^2}^2] dt\Big| \\
&\le C\int_0^T \left[\|\phi_t\|_{L^2(\Omega)}\|\nabla\psi\|_{L^2(\Omega)} + \gamma\|\nabla\phi_t\|_{L^2(\Omega)}\|\nabla\psi\|_{L^2(\Omega)}\right] dt
\end{aligned}$$

$$+\frac{\epsilon}{2}E_\gamma(T)+C_\epsilon\left(\gamma\left\|\phi_t\right\|^2_{C([0,T];L^2(\Omega))}+\|\psi\|^2_{C([0,T];H^{-1}(\Omega))}\right)$$
$$\leq C\left(\int_0^T\|\nabla\psi\|^2_{L^2(\Omega)}dt+\gamma\left\|\phi_t\right\|^2_{C([0,T];L^2(\Omega))}+\|\psi\|^2_{C([0,T];H^{-1}(\Omega))}\right)$$
$$+\frac{\epsilon}{2}\left(\int_0^T E_\gamma(t)dt+E_\gamma(T)\right), \tag{33}$$

where the constant C does not depend on γ, $0\leq\gamma\leq M$.
(B. 2) *Dealing with* $\int_0^T\left(\Delta^2\omega, A_D^{-1}\theta\right)dt$: Using Green's theorem and the fact that $A_D^{-1}\psi\big|_\Gamma=0$, we have

$$\int_0^T(\Delta^2\phi, A_D^{-1}\psi)_{L^2(\Omega)}dt=-\int_0^T(\Delta\phi,\psi)dt-\int_0^T\left(\Delta\phi,\frac{\partial A_D^{-1}\psi}{\partial\nu}\right)_{L^2(\Gamma)}dt. \tag{34}$$

Estimating the right hand side of (34) yields, after the use of trace theory and elliptic regularity,

$$\left|\int_0^T\left(\Delta^2\phi, A_D^{-1}\psi\right)_{L^2(\Omega)}dt\right|\leq C_0\int_0^T\|\Delta\phi\|_{L^2(\Omega)}\|\psi\|_{L^2(\Omega)}dt$$
$$+\frac{\epsilon}{4C}\int_0^T\|\Delta\phi\|^2_{L^2(\Gamma)}dt+C_\epsilon\int_0^T\|\nabla\psi\|^2_{L^2(\Omega)}dt$$

(where the inverted C is the same constant present in (27))

$$\leq C_0\int_0^T\|\Delta\phi\|_{L^2(\Omega)}\|\psi\|_{L^2(\Omega)}dt+C_\epsilon\int_0^T\|\nabla\psi\|^2_{L^2(\Omega)}dt$$
$$+\frac{\epsilon}{4}\left(\int_0^T E_\gamma(t)dt+E_\gamma(T)\right)\quad\text{(by Lemma 3.1)}$$
$$\leq C_\epsilon\int_0^T\|\nabla\psi\|^2_{L^2(\Omega)}dt+\frac{\epsilon}{2}\left(\int_0^T E_\gamma(t)dt+E_\gamma(T)\right), \tag{35}$$

after the use of the mean inequality.
(B.3) *Dealing with* $\int_0^T\left(\alpha\Delta\psi, A_D^{-1}\psi\right)_{L^2(\Omega)}dt$: Easily we have by (6)

$$\int_0^T\left(\alpha\Delta\psi, A_D^{-1}\psi\right)_{L^2(\Omega)}dt=-\alpha\int_0^T\left(A_D\psi, A_D^{-1}\psi\right)_{L^2(\Omega)}dt \tag{36}$$
$$=-\alpha\int_0^T\|\psi\|^2_{L^2(\Omega)}dt.$$

(B.4) Combining (29), (33), (35) and (36) thus results in the following: For arbitrary $\epsilon > 0$ small enough there exists a constant $C > 0$ (independent of γ) such that the solution $[\omega, \omega_t, \theta]$ of (1) satisfies

$$\begin{aligned} & \alpha \int_0^T \left[\|\phi_t\|^2_{L^2(\Omega)} + \gamma \|\nabla \phi_t\|^2_{[L^2(\Omega)]^2} \right] dt \\ \leq \quad & C \left(\int_0^T \|\nabla \psi\|^2_{L^2(\Omega)} dt + \gamma \|\phi_t\|^2_{C([0,T];L^2(\Omega))} + \|\psi\|^2_{C([0,T];H^{-1}(\Omega))} \right) \\ & \epsilon \left(\int_0^T E_\gamma(t) dt + E_\gamma(T) \right), \end{aligned} \tag{37}$$

where the noncrucial dependence of C upon ϵ has not been noted.

(B.5) The Conclusion of the Proof of Lemma 3.2: To majorize the norm of the component ϕ, we multiply the first equation of (5) by ϕ, integrate from 0 to T and integrate by parts to thereby obtain the relation

$$\begin{aligned} & (\phi_t, \phi)_{H^1_{0,\gamma}(\Omega)} \Big|_0^T - \int_0^T \left[\|\phi_t\|^2_{L^2(\Omega)} + \gamma \|\nabla \phi_t\|^2_{[L^2(\Omega)]^2} \right] dt \\ & = - \int_0^T \|\Delta \phi\|^2_{L^2(\Omega)} dt + \alpha \int_0^T (\nabla \psi, \nabla \phi)_{L^2(\Omega)} dt. \end{aligned} \tag{38}$$

Concerning the first term in this relation, we have, by using the definition of the inner product for $H^1_{0,\gamma}(\Omega)$ in (8) and the contraction of the underlying semigroup, that for $\forall\ t \in [0,T]$

$$\begin{aligned} (\phi_t(t), \phi(t))_{H^1_{0,\gamma}(\Omega)} & \leq \frac{\epsilon}{2} \|\phi_t(t)\|^2_{H^1_{0,\gamma}(\Omega)} + C_\epsilon \|\phi(t)\|^2_{H^1_{0,\gamma}(\Omega)} \\ & \leq \epsilon E_\gamma(T) + C_\epsilon \|\phi(t)\|^2_{H^1_0(\Omega)}. \end{aligned} \tag{39}$$

Combining (38) and (39), we eventually arrive at the following estimate for $\epsilon > 0$ small enough:

$$\begin{aligned} & \int_0^T \|\Delta \phi\|^2_{L^2(\Omega)} dt \leq \int_0^T \left[\|\phi_t\|^2_{L^2(\Omega)} + \gamma \|\nabla \phi_t\|^2_{L^2(\Omega)} \right] dt \\ & + C_T \left(\int_0^T \|\nabla \psi\|^2_{L^2(\Omega)} dt + \|\phi\|^2_{C([0,T];H^1_0(\Omega))} \right) + \epsilon E_\gamma(T), \end{aligned} \tag{40}$$

where the noncrucial dependence of C upon ϵ has not been noted.

Thus, if ϵ is small enough, we then have, upon combining (37) and (40) (and further recalling the definition of the energy $E_\gamma(t)$ in (22)) the existence of a constant C such that

$$\begin{aligned} & \int_0^T \left[\|\Delta\phi\|^2_{L^2(\Omega)} + \|\phi_t\|^2_{L^2(\Omega)} + \gamma \|\nabla\phi_t\|^2_{L^2(\Omega)} + \|\psi\|^2_{L^2(\Omega)} \right] dt \\ \leq \; & \frac{\epsilon(1+\alpha)}{\alpha-\epsilon} E_\gamma(T) + C_T \left(\int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} dt \right. \\ & \left. + \|\phi\|^2_{C([0,T];H_0^1(\Omega))} + \gamma \|\phi_t\|^2_{C([0,T];L^2(\Omega))} + \|\psi\|^2_{C([0,T];H^{-1}(\Omega))} \right). \end{aligned} \tag{41}$$

From here, we apply the relation (23) and its inherent property that $E_\gamma(t) \geq E_\gamma(0)\ \forall\ t \in [0,T]$ (recall that $[\phi, \phi_t, \psi]$ solves the *backward* problem (5)) to obtain

$$\begin{aligned} & T\left(E_\gamma(T) - \int_0^T \left[\|\nabla\psi(t)\|^2_{L^2(\Omega)} + \sigma \int_0^T \|\psi(t)\|^2_{L^2(\Omega)} \right] dt \right) = TE_\gamma(0) \\ & \leq \int_0^T E_\gamma(t) dt \leq \frac{\epsilon(1+\alpha)}{\alpha-\epsilon} E_\gamma(T) + C_T \left(\int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} dt \right. \\ & \left. + \|\phi\|^2_{C([0,T];H_0^1(\Omega))} + \gamma \|\phi_t\|^2_{C([0,T];L^2(\Omega))} + \|\psi\|^2_{C([0,T];H^{-1}(\Omega))} \right). \end{aligned} \tag{42}$$

Taking $\epsilon > 0$ small enough in (42), we then have the following preliminary inequality valid for all $\gamma \geq 0$ and $T > 0$:

$$\begin{aligned} E_\gamma(T) \leq C_T \Big(& \int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} dt + \|\phi\|^2_{C([0,T];H_0^1(\Omega))} \\ & + \gamma \|\phi_t\|^2_{C([0,T];L^2(\Omega))} + \|\psi\|^2_{C([0,T];H^{-1}(\Omega))} \Big), \end{aligned} \tag{43}$$

which concludes the proof of Lemma 3.2. ■

Step 3. Removal of the polluting terms in (43)). One sees that (43) is "almost" the desired inequality (4). To finish the proof of Theorem 1.1, we remove the corrupting lower order terms in (43) by invoking the following proposition which is to be proved through a (by now) classical compactness/uniqueness argument. For the sake of completion, we give the argument here.

Proposition 3.3. *The inequality* (43) *implies the existence of a constant* C_T *such that the corresponding solution* $[\phi, \phi_t, \psi]$ *of* (5) *satisfies*

$$\|\phi\|^2_{C([0,T];H_0^1(\Omega))} + \gamma \|\phi_t\|^2_{C([0,T];L^2(\Omega))} + \|\psi\|^2_{C([0,T];H^{-1}(\Omega))} \leq C_T \int_0^T \|\nabla\psi\|^2_{L^2(\Omega)} \, dt. \tag{44}$$

Proof. As usual, the proof is by contradiction. If the proposition is false, there then exists a sequence $\{[\phi_0^{(n)}, \phi_1^{(n)}, \psi_0^{(n)}]\}_{n=1}^{\infty} \subseteq \mathbf{H}_\gamma$, and a corresponding solution sequence $\{[\phi^{(n)}, \phi_t^{(n)}, \psi^{(n)}]\}_{n=1}^{\infty}$ to (5) which satisfies

$$\left\|\phi^{(n)}\right\|^2_{C([0,T];H_0^1(\Omega))} + \gamma\left\|\phi_t^{(n)}\right\|^2_{C([0,T];L^2(\Omega))} + \left\|\psi^{(n)}\right\|^2_{C([0,T];H^{-1}(\Omega))} = 1 \;\forall\, n; \tag{45}$$

$$\int_0^T \left\|\nabla\psi^{(n)}\right\|^2_{L^2(\Omega)} dt \to 0 \quad \text{as } n \to \infty. \tag{46}$$

The inequality (43) and (45)–(46) then imply that there exists a positive constant C such that

$$E_\gamma^{(n)}(T) \leq C, \text{ uniformly in } n. \tag{47}$$

Consequently, there is a subsequence in $\mathbf{H}_\gamma$, still denoted here as $\{[\phi_0^{(n)}, \phi_1^{(n)}, \psi_0^{(n)}]\}_{n=1}^{\infty}$, and $[\widetilde{\phi}_0, \widetilde{\phi}_1, \widetilde{\psi}_0] \in \mathbf{H}_\gamma$ such that

$$[\phi_0^{(n)}, \phi_1^{(n)}, \psi_0^{(n)}] \to [\widetilde{\phi}_0, \widetilde{\phi}_1, \widetilde{\psi}_0] \text{ weakly in } \mathbf{H}_\gamma. \tag{48}$$

Moreover, if we denote $[\widetilde{\phi}, \widetilde{\phi}_t, \widetilde{\psi}]$ to be the solution to (5), corresponding to terminal data $[\widetilde{\phi}_0, \widetilde{\phi}_1, \widetilde{\psi}_0]$, then *a fortiori*,

$$[\phi^{(n)}, \phi_t^{(n)}, \psi^{(n)}] \to [\widetilde{\phi}, \widetilde{\phi}_t, \widetilde{\psi}] \text{ in } L^\infty(0,T;\mathbf{H}_\gamma) \text{ weak star.} \tag{49}$$

Given the energy bound in (47) and the plate equation in (21), one can show readily, (in the same way as in [3] Proposition 4.3), the sequence $\{\phi_{tt}^{(n)}\}_{n=1}^{\infty}$ is bounded in $L^\infty(0,T;[D(\mathbf{\mathring{A}}^{\frac{1}{2}} P_\gamma^{-1})]')$, with the estimate

$$\|\phi_{tt}^{(n)}\|^2_{L^\infty(0,T;[D(\mathbf{\mathring{A}}^{\frac{1}{2}} P_\gamma^{-1})]')} \leq C_0 E_\gamma^{(n)}(T) \leq C_1, \tag{50}$$

where $\mathring{\mathbf{A}}^{\frac{1}{2}}$ is the square root of the elliptic operator defined in (6). Likewise, reading off the heat equation in (21), we have that $\{\psi_t^{(n)}\}_{n=1}^{\infty}$ is bounded in $L^{\infty}(0,T;[D(A_D)]')$, with the norm estimate

$$\|\psi_t^{(n)}\|^2_{L^{\infty}(0,T;[D(A_D)]')} \leq C_2 E_{\gamma}^{(n)}(T) \leq C_3. \tag{51}$$

This boundedness above of $\{[\phi_{tt}^{(n)}, \psi_t^{(n)}]\}$ and that of $\{[\phi^{(n)}, \phi_t^{(n)}, \psi^{(n)}]\}$ in $L^{\infty}(0,T;\mathbf{H}_{\gamma})$ (via the contraction of the semigroup $\{e^{\mathcal{A}_{\gamma}t}\}_{t\geq 0}$ and the estimate (47)) allow the use of Simon's compactness result in [20] so as to have

$$\begin{array}{lcl} \phi^{(n)} & \to & \widetilde{\phi} \ \text{ strongly in } C([0,T];H_0^1(\Omega)); \\ \phi_t^{(n)} & \to & \widetilde{\phi}_t \ \text{ strongly in } C([0,T];L^2(\Omega)) \ \text{ if } \gamma > 0; \\ \psi^{(n)} & \to & \widetilde{\psi} \ \text{ strongly in } C([0,T];H^{-1}(\Omega)). \end{array}$$

We can then pass to the limit in (45) so as to have

$$\|\widetilde{\phi}\|^2_{C([0,T];H_0^1(\Omega))} + \gamma\|\widetilde{\phi}_t\|^2_{C([0,T];L^2(\Omega))} + \|\widetilde{\psi}\|^2_{C([0,T];H^{-1}(\Omega))} = 1. \tag{52}$$

On the other hand, the convergence in (46) yields the deduction that $\widetilde{\psi} = 0$. This consequence coupled with the heat equation in (21) (with terminal data $[\widetilde{\phi}_0, \widetilde{\phi}_1, \widetilde{\psi}_0]$) implies that $\widetilde{\phi}_t = 0$. In turn, the fact that $\widetilde{\phi}_t$ is zero coupled with the plate equation in (21) gives that $\widetilde{\phi} = 0$. So $[\widetilde{\phi}, \widetilde{\phi}_t, \widetilde{\psi}] = [0,0,0]$ which contradicts the equality in (52). □

Remark 3.4. In Proposition 3.3, We note how the imposition of interior control allows for the relatively simple use of ellipticity so as to obtain the uniqueness part of the argument in Proposition 3.3. However, the carrying out of the analogous compactness–uniqueness argument in the work [3], a paper which deals with a boundary–controlled thermoelastic system, requires the new result in [9] of analytic continuation for thermoelastic systems.

The combination of the estimates (43) and (44) establish the observability inequality (4), thereby completing the proof of Theorem (1.1).

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