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ERRATA TO "THE CAUCHY PROBLEM FOR THE SEMI-LINEAR QUINTIC SCHRÖDINGER EQUATION IN 1D", DIFFERENTIAL INTEGRAL EQUATIONS, 18 (2005), NO. 8, 947–960.

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In the paper [4] we considered the quintic nonlinear Schrödinger equation (NLS) in one dimension (1d)

$$\begin{aligned} &iu_t + u_{xx} - |u|^4 u = 0 \\ &u(x,0) = u_0(x) \in H^s(\mathbb{R}), t \in \mathbb{R}. \end{aligned} \tag{0.1}$$

The main result of the paper was that the initial-value problem (IVP) is globally well posed for initial data in $H^s(\mathbb{R})$ for any $s > \frac{4}{9}$. A crucial part of the proof is the validation of Proposition 1 in [4]. But this proposition is false as the following counterexample, pointed out to us by Vedran Sohinger, shows. More precisely consider the multiplier

$$M_6(\xi_1, \xi_2, ..., \xi_6) = \frac{m_1^2\xi_1^2 - m_2^2\xi_2^2 + m_3^2\xi_3^2 - m_4^2\xi_4^2 + m_5^2\xi_5^2 - m_6^2\xi_6^2}{\xi_1^2 - \xi_2^2 + \xi_3^2 - \xi_4^2 + \xi_5^2 - \xi_6^2}$$

defined on $\Gamma_6 = \{(\xi_1, \xi_2, \ldots, \xi_6) \in \mathbb{R}^6 : \xi_1 + \xi_2 + \ldots + \xi_6 = 0\}$. The reader can consult [4] for the formula that gives the multiplier m_i, $i = 1, 2, ..., 6$. Then if we choose $(\xi_1, \xi_2, \ldots, \xi_6) = K(5, -3, 6, -2, 1, -7)$ where K a very large number, we have $\xi_1 + \xi_2 + \ldots + \xi_6 = 0$ and $\xi_1^2 - \xi_2^2 + \xi_3^2 - \xi_4^2 + \xi_5^2 - \xi_6^2 = 0$. From the definition of m for very high frequencies, and for a given Sobolev regularity index $0 < s < 1$, we can see that M_6 fails to be bounded on Γ_6. Therefore the result of [4] is not correct. In a different paper, [1], the author in collaboration with D. De Silva, N. Pavlović, and G. Staffilani, used the bound of Proposition 1, to improve the global well posedness of the IVP for any $s > \frac{1}{3}$. As we show in [2], the method in [1] can be proved to be independent of the bound for M_6 and thus the result in [1] is correct as claimed. Therefore, the quintic nonlinear Schödingers equation on $\mathbb{R}$ is globally well posed for any $s > \frac{1}{3}$. Proposition 1 in [4] was also used in the

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1d part of [3] concerning the periodic NLS in low dimensions. Although the problem can be shown to be globally well posed in 1d below the energy norm via the methods that the authors used, the regularity index s, where global well posedness is proved, is greater than the index claimed there and thus their result is weaker.

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