

FAST COMMUNICATION

QUENCHING AND PROPAGATION OF COMBUSTION FRONTS IN
POROUS MEDIA*

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Abstract. In this short note we study the model of subsonic detonation introduced by Sivashinsky. The model is described by the system of reaction-diffusion equations involving temperature, pressure and concentration of deficient reactant. It is shown that initial data with small support lead to quenching (decay of solution). In contrast, initial data with support large enough lead to propagation with finite velocity.

Key words. Combustion fronts, porous media, subsonic detonation, quenching

AMS subject classifications. 35K57, 80A25

1. Introduction

As is well known there are basically two mechanisms controlling propagation of combustion waves in gaseous mixtures : molecular transport and adiabatic compression. The first mechanism is referred to as deflagrative combustion or deflagration and the second regime is called detonation. Gaseous detonation is a phenomenon with very complicated dynamics which has been studied extensively by physicists, mathematicians and engineers for many years. Despite many efforts the problem is far from complete resolution. Recently Sivashinsky proposed a theory of subsonic detonation that occurs in highly resistable porous media [3]. This theory provides a model which is realistic, rich and suitable for a mathematical study. In particular, the model is capable of describing transition from a slowly propagating deflagration wave to the fast detonation. This phenomena is known as a deflagration to detonation transition and is one of the most challenging issues in combustion theory. The model reads [11]:

$$\begin{aligned} \gamma T_t - (\gamma - 1)P_t &= \varepsilon \Delta T + Y\Omega(T), \\ P_t - T_t &= \Delta P, \\ Y_t &= \varepsilon \text{Le}^{-1} \Delta Y - Y\Omega(T). \end{aligned} \tag{1.1}$$

Here P , T and Y are the appropriately scaled pressure, temperature and concentration of the deficient reactant; $\gamma > 1$ is the specific heat ratio, ε is a ratio of pressure and molecular diffusivities, Le is a Lewis number and $Y\Omega(T)$ is the reaction rate. The first and third equations of the system (1.1) represent the partially linearized conservation equations for energy and deficient reactant, while the second equation is a linearized continuity equation taking into account the equations of state and momentum (Darcy law).

We consider the problem (1.1) in the whole space, that is $(t, \mathbf{x}) \in \mathbb{R}^+ \times \mathbb{R}^d$. We also assume that the nonlinearity $\Omega(T)$ is of the ignition type. More specifically,

$$\Omega(T) = 0 \quad \text{for } 0 \leq T \leq \Theta. \tag{1.2}$$

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Moreover, $\Omega(T)$ is an increasing Lipschitz continuous function for $T \geq \Theta$, except for a possible discontinuity at the ignition temperature $T = \Theta$. In addition we assume that

$$\Omega(T) \leq KT \quad (1.3)$$

where K is some constant. An example of nonlinearity $\Omega(T)$ most commonly used in applications is the so-called Arrhenius law [13]:

$$\Omega(T) = \begin{cases} \exp(-Z/T) & \text{if } T \geq \Theta \\ 0 & \text{if } T < \Theta \end{cases} \quad (1.4)$$

where Z is the Zeldovich number (scaled activation energy).

One of the central problems in detonation theory is the initiation of detonation [12]. Subsonic detonation similar to the conventional (supersonic) one may be initiated by localized energy deposition, localized temperature elevation (hot spot). In a framework of the model (1.1) this corresponds to the following initial conditions:

$$T(0, \mathbf{x}) = T_0(\mathbf{x}) \geq 0, \quad Y(0, \mathbf{x}) = 1, \quad P(0, \mathbf{x}) = 0. \quad (1.5)$$

That is, concentration of deficient reactant and pressure are assumed to be constant while temperature is a localized function.

Most relevant for applications is the case when ε is small. For realistic materials ε varies in the range $\varepsilon \sim 10^{-2} - 10^{-5}$. Therefore we will assume throughout of this paper that $\varepsilon < 1$. In this case, the system (1.1) exhibits interesting dynamics. According to the results of numerical simulations there are three general scenarios of the long time behavior of solutions for system (1.1) with initial conditions (1.5). Let $\omega_\Theta = \{\mathbf{x} \in \mathbb{R}^d : T_0(\mathbf{x}) > \Theta\}$ be a support of the initial data for temperature strictly larger than the ignition temperature and set $\text{vol}(\omega_\Theta) = \text{meas}(\omega_\Theta)$. If $l_\Theta = \text{vol}(\omega_\Theta) \|T_0\|_{L^\infty}$ is large, the detonation wave forms almost immediately. This wave propagates with the velocity of order one. In the case when l_Θ has some intermediate value, the detonation wave forms after a long induction time as a result of an abrupt transition from a slowly spreading deflagration wave driven by thermal diffusivity. That is, initially propagation with characteristic velocity of order $\sqrt{\varepsilon}$ is observed for a time period of order of $1/\sqrt{\varepsilon}$, then the solution experiences rapid changes and jumps to the detonation regime which was described before. Finally when l_Θ is small quenching (decay of solution) occurs. That is $(T, Y, P)(\mathbf{x}, t) \rightarrow (0, 1, 0)$ as $t \rightarrow \infty$. The first two regimes have been extensively studied both rigorously and numerically in recent years see review [11] for details. In particular, it was shown that immediate initiation is impossible if the support of the initial data is small [9]; existence, uniqueness and stability of traveling waves have been analyzed in [9], [7], [8], [4]. Metastability of the deflagration regime has been explored in [2], [5]. However, a rigorous study of quenching has not yet been performed.

The present paper is concerned with the quenching and thermo-diffusive propagation of the disturbances in a highly resistable porous media. In particular, we show that if the initial data have a small support, quenching occurs (Section 2) and initial data with large enough support lead to propagation with a speed at least of the order $\sqrt{\varepsilon}$ (Section 3). Similar problems in the case of a single reaction diffusion equation were studied in the famous works by Aronson and Weinberger [1] and Kanel [10].

2. Quenching

In this section we show that if the support of the initial data for temperature is small, then quenching occurs.

As a first step we make a change of variables. We represent the temperature field as a linear combination of pressure and some new variable R

$$T(t, \mathbf{x}) = \lambda P(t, \mathbf{x}) + (1 - \lambda)R(t, \mathbf{x}) \tag{2.1}$$

with λ being a positive solution of the following equation

$$(1 - \lambda)(\gamma + \varepsilon\lambda) = 1. \tag{2.2}$$

After simple algebraic manipulations with Eqs.(1.1)₁ and (1.1)₂ and scaling $\Delta \rightarrow (1 - \lambda)\Delta$ we have

$$\begin{aligned} R_t &= \delta\Delta R + Y\Omega(T), \\ P_t - R_t &= \Delta P, \\ Y_t &= \mu\Delta Y - Y\Omega(T), \end{aligned} \tag{2.3}$$

with $\delta = \varepsilon(1 - \lambda)^2$, $\mu = \varepsilon(1 - \lambda)Le^{-1}$ and T defined by (2.1). Next we add Eqs.(2.3)₁ and (2.3)₂, in a way (2.3)₂ + $\frac{1}{1-\delta}$ (2.3)₁. Then the system (2.3) can be written in an equivalent form

$$\begin{aligned} R_t &= \delta\Delta R + Y\Omega(T), \\ S_t &= \Delta S + \frac{1}{1-\delta}Y\Omega(T), \\ Y_t &= \mu\Delta Y - Y\Omega(T), \end{aligned} \tag{2.4}$$

where $S(t, \mathbf{x})$ is defined as follows

$$S(t, \mathbf{x}) = P(t, \mathbf{x}) + \frac{\delta}{1-\delta}R(t, \mathbf{x}). \tag{2.5}$$

In other words, we have made a linear transform of the variables $(T, P) \rightarrow (R, S)$. As is clearly seen from Eqs.(2.1) and (2.4) this transform is defined as follows

$$T(t, \mathbf{x}) = \lambda S(t, \mathbf{x}) + \left((1 - \lambda) - \frac{\lambda\delta}{1-\delta} \right) R(t, \mathbf{x}) \tag{2.6}$$

$$P(t, \mathbf{x}) = S(t, \mathbf{x}) - \frac{\delta}{1-\delta}R(t, \mathbf{x}). \tag{2.7}$$

Initial values for S and R are easily determined as

$$R(0, \mathbf{x}) = \frac{1}{(1 - \lambda)}T(0, \mathbf{x}), \quad S(0, \mathbf{x}) = \frac{\delta}{(1 - \delta)}R(0, \mathbf{x}) = \frac{\delta}{(1 - \delta)(1 - \lambda)}T(0, \mathbf{x}). \tag{2.8}$$

So the problem (1.1) is equivalent to the following

$$\begin{aligned} R_t &= \delta\Delta R + Y\Omega(T), \\ S_t &= \Delta S + \frac{1}{1-\delta}Y\Omega(T), \\ Y_t &= \mu\Delta Y - Y\Omega(T), \\ T(t, \mathbf{x}) &= \lambda S(t, \mathbf{x}) + \left((1 - \lambda) - \frac{\lambda\delta}{1-\delta} \right) R(t, \mathbf{x}), \end{aligned} \tag{2.9}$$

with initial conditions:

$$R(0, \mathbf{x}) = \frac{1}{(1-\lambda)}T_0(\mathbf{x}), \quad S(0, \mathbf{x}) = \frac{\delta}{(1-\delta)(1-\lambda)}T_0(\mathbf{x}), \quad Y(0, \mathbf{x}) = 1. \quad (2.10)$$

The system (2.9) resembles that arising in conventional combustion (deflagration in a free space). The main difference is that the nonlinear term Ω is a function of two variables R and S whereas in conventional combustion the nonlinear term is a function of a single variable.

The strategy of proving that an initial data with small support is quenched is as follows. First we will show that the function R is bounded by S uniformly in (t, \mathbf{x}) (Lemma 2.1). This will enable us to construct an equation which describes super-solutions of S . This equation is formally identical to the one studied in conventional combustion. Then we apply the technique suggested in [6] to obtain the result.

LEMMA 2.1. *Let (R, S) be solution of the system (2.9), (2.10).*

Then,

$$S(t, \mathbf{x}) \geq \frac{\delta^{d/2+1}}{1-\delta}R(t, \mathbf{x}) \quad \text{for all } t, \mathbf{x}. \quad (2.11)$$

Proof. Formally solution of Eqs.(2.9)_{1,2},(2.10) can be written in the form

$$\begin{aligned} R(t, \mathbf{x}) &= G_\delta(t) \star R_0 + \int_0^t G_\delta(t-s) \star Y\Omega(T) \\ S(t, \mathbf{x}) &= G_1(t) \star S_0 + \frac{1}{1-\delta} \int_0^t G_1(t-s) \star Y\Omega(T) \end{aligned} \quad (2.12)$$

where

$$G_\delta(t, \mathbf{x}) = \frac{1}{(4\delta\pi t)^{d/2}} e^{-|\mathbf{x}|^2/4\delta t} \quad (2.13)$$

is the heat kernel in \mathbb{R}^d . Taking into account the initial conditions we also have

$$\begin{aligned} R(t, \mathbf{x}) &= \frac{1}{1-\lambda}G_\delta(t) \star T_0 + \int_0^t G_\delta(t-s) \star Y\Omega(T) = I_R + II_R \\ S(t, \mathbf{x}) &= \frac{\delta}{(1-\delta)(1-\lambda)}G_1(t) \star T_0 + \frac{1}{1-\delta} \int_0^t G_1(t-s) \star Y\Omega(T) = I_S + II_S. \end{aligned} \quad (2.14)$$

Next we define

$$H_\alpha(t, \mathbf{x}) = G_1(t, \mathbf{x}) - \alpha G_\delta(t, \mathbf{x}) = G_1(t, \mathbf{x}) \left(1 - \frac{\alpha}{\delta^{d/2}} e^{-(1-\delta)|\mathbf{x}|^2/4\delta t}\right). \quad (2.15)$$

The function $H_\alpha(t, \mathbf{x}) \geq 0$ for all (t, \mathbf{x}) whenever $\delta \leq 1$ and $\alpha \leq \delta^{d/2}$. Therefore

$$I_S - pI_R = \frac{1}{1-\lambda} \frac{\delta}{1-\delta} H_{p(1-\delta)/\delta} \star T_0 \geq 0 \quad p \leq \frac{\delta^{d/2+1}}{1-\delta}. \quad (2.16)$$

Similarly

$$II_S - qII_R = \frac{1}{1-\delta} \int_0^t H_{q(1-\delta)}(t-s) \star Y\Omega(T) ds \geq 0 \quad q \leq \frac{\delta^{d/2}}{1-\delta}. \quad (2.17)$$

Inequalities (2.16), (2.17) and initial condition (2.8) imply the statement of the lemma. \square

In the following lemma we obtain a useful bound on S .

LEMMA 2.2. *Let $\omega = \{\mathbf{x} \in \mathbb{R}^d : T_0(\mathbf{x}) > 0\}$ and set $\text{vol}(\omega) = \text{meas}(\omega)$. Then,*

$$S(t, \mathbf{x}) \leq \frac{\delta \|T_0\|_{L^\infty} \text{vol}(\omega) e^{\sigma t}}{(4\pi t)^{d/2} (1-\lambda)(1-\delta)} \quad \text{with} \quad \sigma = \frac{KC_0}{\delta^{d/2+1}(1-\delta)}. \tag{2.18}$$

Proof. Due to lemma 2.1, $T(t, \mathbf{x}) \leq \frac{C_0}{\delta^{d/2+1}} S(t, \mathbf{x})$ with $C_0 = (1-\lambda)(1-\delta) - \delta\lambda(1-\delta^{d/2}) < 1$. Next since $\Omega(T) \leq KT$ we have $\Omega(T) \leq \frac{KC_0}{\delta^{d/2+1}} S$. Therefore the solution of the equation

$$\bar{S}_t = \Delta \bar{S} + \frac{KC_0}{\delta^{d/2+1}(1-\delta)} \bar{S}, \quad \bar{S}(0, \mathbf{x}) = S(0, \mathbf{x}) \tag{2.19}$$

will be a supersolution for S , that is $\bar{S} \geq S$ for all (t, \mathbf{x}) . We are looking for the solution of Eq.(2.19) in the form $\bar{S}(t, \mathbf{x}) = \Phi(t, \mathbf{x})e^{\sigma t}$, where $\sigma = \frac{KC_0}{\delta^{d/2+1}(1-\delta)}$ and Φ is the solution of the following equation

$$\bar{\Phi}_t = \Delta \bar{\Phi}, \quad \bar{\Phi}(0, \mathbf{x}) = S(0, \mathbf{x}). \tag{2.20}$$

Using the heat kernel we have

$$\bar{\Phi}(t, \mathbf{x}) = \frac{\delta}{(1-\lambda)(1-\delta)} G_1(t) \star T_0 \leq \frac{\delta \|T_0\|_{L^\infty} \text{vol}(\omega)}{(4\pi t)^{d/2} (1-\lambda)(1-\delta)}, \tag{2.21}$$

which implies the statement of the lemma. \square

THEOREM 2.3. *Let $\omega = \{\mathbf{x} \in \mathbb{R}^d : T_0(\mathbf{x}) > 0\}$ and set $\text{vol}(\omega) = \text{meas}(\omega)$. Assume that*

$$\|T_0\|_{L^\infty} \text{vol}(\omega) \leq \left(\frac{2\pi d}{KC_0 e} \right)^{d/2} \frac{(1-\lambda)(1-\delta)}{C_0} \Theta \delta^{\frac{d}{2}(\frac{d}{2}+2)}. \tag{2.22}$$

Then,

$$T(t, \mathbf{x}), P(t, \mathbf{x}) \rightarrow 0 \quad \text{and} \quad Y(t, \mathbf{x}) \rightarrow 1 \quad \text{as} \quad t \rightarrow \infty \quad \text{uniformly in } \mathbf{x}. \tag{2.23}$$

Proof. Since $T(t, \mathbf{x}) \leq \frac{C_0}{\delta^{d/2+1}} S(t, \mathbf{x})$, the solution of the problem

$$\bar{S}_t = \Delta \bar{S} + \frac{1}{1-\delta} \Omega \left(\frac{C_0}{\delta^{d/2+1}} \bar{S} \right) \tag{2.24}$$

will be a supersolution for S . Since Ω is of the ignition type all we have to show is that $\bar{S} = S_{ign} \leq \frac{\Theta \delta^{d/2+1}}{C_0}$ for all \mathbf{x} at some instance of time $t = t^*$. If this happens, then $\bar{S} \leq S_{ign}$ for all \mathbf{x} and $t > t^*$ since S_{ign} will be a supersolution of the problem (2.24) and S will satisfy a heat equation

$$\bar{S}_t = \Delta \bar{S} \tag{2.25}$$

for $t > t^*$. Therefore, $S \rightarrow 0$ as $t \rightarrow \infty$ uniformly in \mathbf{x} . Moreover, by lemma 2.1, R is bounded from above by S and thus $R \rightarrow 0$ as $t \rightarrow \infty$ uniformly in \mathbf{x} . Since P and T are linear combinations of R and S , previous observations imply that $T \rightarrow 0$, $P \rightarrow 0$ as $t \rightarrow \infty$ uniformly in \mathbf{x} . Using these facts and equation for Y , we also deduce that $Y \rightarrow 1$ as $t \rightarrow \infty$ uniformly in \mathbf{x} .

Now let us show that under assumptions of the theorem this indeed happens. We set

$$F(t) = \frac{\delta \|T_0\|_{L^\infty} \text{vol}(\omega)}{(4\pi t)^{d/2} (1-\lambda)(1-\delta)} e^{\sigma t}, \quad \text{with} \quad \sigma = \frac{KC_0}{\delta^{d/2+1}(1-\delta)}. \tag{2.26}$$

The minimum of this expression is achieved at time $t^* = d/2\sigma$ and

$$F(t^*) = \left(\frac{KC_0 e}{2\pi d}\right)^{d/2} \frac{\text{vol}(\omega) \|T_0\|_{L^\infty}}{\delta^{\frac{d}{2}(\frac{d}{2}+1)-1}} \tag{2.27}$$

$S \leq F$ for all t , and in particular at $t = t^*$. Moreover, at t^* , $F(t^*) \leq S_{ign}$ as long as

$$\|T_0\|_{L^\infty} \text{vol}(\omega) \leq \left(\frac{2\pi d}{KC_0 e}\right)^{d/2} \frac{(1-\lambda)(1-\delta)}{C_0} \Theta \delta^{\frac{d}{2}(\frac{d}{2}+2)} \tag{2.28}$$

which implies the statement of the theorem. □

Proof of theorem 2.3 provides a transparent physical picture of how quenching occurs. There are two stages of the evolution involved. The first stage is associated with a fast reaction in a localized region of space and intense transport of heat from the reaction zone. Indeed, initially pressure is constant and the amount of fuel is abundant while the region of high temperature is extremely localized in space. The reaction raises pressure and temperature in a localized reaction zone and leads to consumption of fuel. At the same time diffusion prevents high gradient of temperature and eliminates sharp peaks by rapid transport of hot gas away from the reaction zone. Thus after some (relatively short) time a reaction zone is surrounded by the large spot of preheated gas. However, under assumptions of the theorem, the temperature of this preheated gas is still smaller than the ignition temperature. This prevents initiation of the reaction in a larger region. In other words, at the instance of time which is estimated from above by t^* , the temperature drops below the ignition temperature and the reaction stops in the entire space. Moreover, since t^* is finite, just a finite amount of fuel is consumed by this time. At this moment the second stage of evolution starts. At this stage the dynamics of the system are governed exclusively by diffusion. Diffusion tends to smooth out the pressure and temperature fields that approach zero as time progresses. At the same time diffusion brings the fuel from infinity in order to make the concentration field uniform and equal to unity at large times.

3. Diffusive propagation

In this section we show that initial data with large support lead to the propagation of disturbances with a finite speed. The result is based on comparison principle and classical result of Aronson and Weinberger [1].

We will use the following strategy to prove the result. First we restrict ourself to the case when $Le \geq 1/(1-\lambda)$. Similar restrictions arises in contest of study of deflagration in a free space. This restriction allows us to estimate concentration Y from below in terms of R uniformly in (t, \mathbf{x}) (Lemma 3.1). The latter estimate makes it possible to construct a single reaction diffusion equation which describes a subsolution

for the function R . Solution of this equation propagates with the finite speed due to results of [1]. This, in turn, implies that temperature T also propagates with finite speed since T can be estimated in terms of R from below.

We start with the bound for the concentration Y in terms of R .

LEMMA 3.1. *Let Y, R be a solution of the problem (2.9), (2.10). Assume that $Le \geq 1/(1-\lambda)$. Then,*

$$Y(t, \mathbf{x}) \geq 1 - ((1-\lambda)Le)^{d/2} R(t, \mathbf{x}) \quad \text{for all } t, \mathbf{x}. \tag{3.1}$$

Proof. The proof is similar to the one of the lemma 2.1. We formally write the solution of the problem (2.9) as

$$\begin{aligned} R(t, \mathbf{x}) &= G_\delta \star R_0 + \int_0^t G_\delta(t-s) \star Y\Omega(T) \\ Y(t, \mathbf{x}) &= G_\mu \star Y_0 - \int_0^t G_\mu(t-s) \star Y\Omega(T) \end{aligned} \tag{3.2}$$

where the heat kernel G_δ is defined by (2.13). Next adding Eqs.(3.2) we have

$$Y = G_\mu \star Y_0 + \alpha G_\delta \star R_0 - \alpha R + \int_0^t (\alpha G_\delta(t-s) - G_\mu(t-s)) \star Y\Omega(T) \tag{3.3}$$

with α being a positive constant. Define

$$Z_\alpha(t, \mathbf{x}) = \alpha G_\delta - G_\mu \tag{3.4}$$

After straightforward computations we have

$$Z_\alpha(t, \mathbf{x}) = \frac{\alpha}{(4\pi t\delta)^{d/2}} e^{-|\mathbf{x}|^2/4\delta t} \left(1 - \frac{1}{\alpha} \left(\frac{\delta}{\mu} \right)^{d/2} e^{-|\mathbf{x}|^2(\delta-\mu)/4t\mu\delta} \right). \tag{3.5}$$

We then observe that $Z_\alpha(t, r) \geq 0$ whenever $\alpha \geq \left(\frac{\delta}{\mu}\right)^{d/2}$. Thus,

$$\int_0^t (\alpha G_\delta(t-s) - G_\mu(t-s)) \star Y\Omega(T) = \int_0^t Z_\alpha(t-s) \star Y\Omega(T) ds \geq 0 \quad \text{for } \alpha \geq \left(\frac{\delta}{\mu}\right)^{d/2}. \tag{3.6}$$

Therefore

$$Y = G_\mu \star Y_0 + \alpha G_\delta \star R_0 - \alpha R, \quad \alpha \geq \left(\frac{\delta}{\mu}\right)^{d/2}. \tag{3.7}$$

Taking into account that

$$G_\mu \star Y_0 = G_\mu \star 1 = 1, \quad \alpha G_\delta \star R_0 \geq 0 \tag{3.8}$$

we have

$$Y \geq 1 - \left(\frac{\delta}{\mu}\right)^{d/2} R = 1 - ((1-\lambda)Le)^{d/2} R. \tag{3.9}$$

□

Using the result of the previous lemma, we now can construct a subsolution for R . Indeed by lemma 3.1, $Y \geq 1 - ((1 - \lambda)\text{Le})^{d/2} R$. Moreover, by lemma 2.1, we have $S \geq \frac{\delta^{d/2+1}}{1-\delta} R$. Thus by (2.6) $T > \left((1 - \lambda) - \frac{\lambda\delta}{1-\delta} \right) R$. These observations lead to the construction of a subsolution for R which satisfies the following equation

$$\underline{R}_t = \delta \Delta \underline{R} + (1 - ((1 - \lambda)\text{Le})^{d/2} \underline{R}) \underline{\Omega} \left(\left(1 - \frac{\lambda}{1-\delta} \right) \underline{R} \right) \tag{3.10}$$

with initial condition

$$\underline{R}(0, \mathbf{x}) = \frac{1}{(1 - \lambda)} T_0(\mathbf{x}). \tag{3.11}$$

Here $\underline{\Omega}(s)$ is a C^1 function satisfying the following conditions: $\underline{\Omega}(s) = 0$ for $s < \Theta$, $\underline{\Omega}(s)$ is an increasing function for $s > \Theta$, $\underline{\Omega}(s) \leq \Omega(s)$ and $|\Omega(s) - \underline{\Omega}(s)| \leq \kappa$ for κ sufficiently small. In other words, $\underline{\Omega}$ is a C^1 approximation of Ω . We also assume that

$$\Theta < \frac{1}{(\text{Le}(1 - \lambda))^{d/2}} \left(1 - \frac{\lambda}{1-\delta} \right). \tag{3.12}$$

It is important to note that equation (3.10) admits a planar traveling wave solution traveling in the direction of ν . That is, solutions of the form $\underline{R}(\eta)$, with $\eta = \mathbf{x} \cdot \nu - ct$ where ν is any unit vector in \mathbb{R}^d . These traveling waves approach 0 and $((1 - \lambda)\text{Le})^{-d/2}$ as $\eta \rightarrow \pm\infty$. Moreover, there is a unique value of $c = c^*$ for which such type of solution exists [1]. In addition, standard scaling arguments show that the velocity of the traveling wave c^* is of order of $\sqrt{\varepsilon}$. In [1] (Theorem 6.2 p.70) it was shown that if initial data $R(0, \mathbf{x}) \geq 0$ has sufficiently large support, then propagation with the speed c^* is observed. More precisely if

$$R(0, \mathbf{x}) = \begin{cases} ((1 - \lambda)\text{Le})^{-d/2} & \text{for } |\mathbf{x}| \leq C\varepsilon^{-1/2} \\ \phi(|\mathbf{x}|) & \text{for } C\varepsilon^{-1/2} < |\mathbf{x}| \leq 1 + C\varepsilon^{-1/2} \\ 0 & \text{for } |\mathbf{x}| > 1 + C\varepsilon^{-1/2} \end{cases} \tag{3.13}$$

for some C independent of ε and $\phi(z) \in C^0$ such that $\phi(C\varepsilon^{-1/2}) = ((1 - \lambda)\text{Le})^{-d/2}$ and $\phi(C\varepsilon^{-1/2} + 1) = 0$. Then,

$$\lim_{t \rightarrow \infty} \min_{|\mathbf{z} - \mathbf{y}| \leq ct} \underline{R}(t, \mathbf{z}) = ((1 - \lambda)\text{Le})^{-d/2} \tag{3.14}$$

for any $\mathbf{y} \in \mathbb{R}^d$ and $c \in (0, c^*)$. These results lead to the following theorem.

THEOREM 3.2. *Assume that*

$$T(0, \mathbf{x}) = \begin{cases} 1 & \text{for } |\mathbf{x}| \leq C\varepsilon^{-1/2} \\ \sigma(|\mathbf{x}|) & \text{for } C\varepsilon^{-1/2} < |\mathbf{x}| \leq 1 + C\varepsilon^{-1/2} \\ 0 & \text{for } |\mathbf{x}| > 1 + C\varepsilon^{-1/2} \end{cases} \tag{3.15}$$

where C is some constant independent of ε and $\sigma(z) \in C^0$ with $\sigma(C\varepsilon^{-1/2}) = 1$, $\sigma(C\varepsilon^{-1/2} + 1) = 0$. Assume also that

$$\Theta < \frac{1}{(\text{Le}(1 - \lambda))^{d/2}} \left(1 - \frac{\lambda}{1-\delta} \right). \tag{3.16}$$

Then,

$$\lim_{t \rightarrow \infty} \min_{|\mathbf{z}-\mathbf{y}| \leq ct} T(t, \mathbf{z}) > \frac{1}{(\text{Le}(1-\lambda))^{d/2}} \left(1 - \frac{\lambda}{1-\delta}\right) \quad (3.17)$$

for any $c \in (0, c^*)$ where $c^* \sim \sqrt{\varepsilon}$ is the speed of the traveling wave solution of the problem (3.10).

Proof. Under the assumptions of the theorem, results of [1] are applicable. In particular an estimate (3.14) holds. Taking into account that $T > \left(1 - \frac{\lambda}{1-\delta}\right) \underline{R}$ we have (3.17). \square

REMARK 3.1. We do not know whether an estimate (3.17) is optimal. One may expect that the temperature in a burned gas region will approach unity in the case when a stable propagation with constant velocity is observed. However, it is known that flames may suffer instabilities for $\text{Le} > 1$, which leads to oscillations even in dimension one and may sufficiently reduce the minimal value of the temperature in the burned gas.

REMARK 3.2. Theorems 2.3 and 3.2 provide sufficient conditions for quenching and propagation to take place. In order for quenching to occur, the support of the temperature should be of order $\varepsilon^{d(d/2+2)/2}$, whereas to ensure propagation support of the initial data for temperature should be order $\varepsilon^{-1/2}$. At this point it is not known what happens if the support of the initial data has an intermediate scaling between these two. We note that even for a single reaction diffusion equation this problem was solved only recently [14].

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