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Classical States and the BRST Charge

David McMullan*

Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin 4, Ireland

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Abstract. The role played by the BRST-charge in isolating the physical states in a classical first-class constrained system is analysed. Contrary to popular belief, the cohomological argument used to characterize the physical observables in such a system does not extend to the classical states. It is shown that, in order to recover the physical states, the BRST-charge must be augmented with a new charge, of ghost number minus one, constructed out of a set of gauge fixing conditions for the original constraints. The relevance of this construction to the quantum theory is discussed.

1. Introduction

In recent years the use of ghost variables has been extended from a diagrammatic trick to maintain unitarity in one-loop calculations [1], to a general procedure for isolating the physical observables in both quantum and classical first-class constrained systems [2–7]. Although assigning ghost variables a classical role may, at first sight, seem rather surprising, their use in classical dynamics can be given a precise mathematical meaning which, in turn, supplies an important theoretical underpinning to their applications in the quantum theory.

The aim of this paper is to use such a classical analysis to investigate the role of ghost variables in directly isolating the physical states of a constrained system (all constraints in this paper will be first-class). This is motivated by the observations [8–10] that in the quantum theory the *natural* definition of physical states (ghost number zero states that are BRST invariant but not the BRST transform of another state) does not yield a satisfactory result (indeed, such states generically have zero norm). We shall show that this apparent complication should come as no surprise since it is also there in the classical theory. We shall also see that there is a straightforward solution to this problem of isolating the classical physical states that can be applied directly to the quantum theory.

* email: zwills@irlearn.bitnet

We are interested in the situation where we have a phase space P (finite dimensional, of dimension $2n$, and usually a cotangent bundle with associated canonical symplectic structure) and a set, ϕ_α, of k ($\leqq n$) independent, smooth functions on P. These are the constraints; being first-class implies that $\{\phi_\alpha, \phi_\beta\} = C^\gamma_{\alpha\beta}\phi_\gamma$, for some structure functions $C^\gamma_{\alpha\beta}$. For simplicity we restrict attention to systems where the constraints can be identified with the generators of some Lie-group G acting on P, hence the structure functions are actually the structure constants of the group. The first-class nature of the constraints implies that the constraint surface $K \subset P$ is not itself a phase space: the true degrees of freedom being the quotient K/G, a phase space of dimension $2(n-k)$. The smooth functions on K/G, $C^\infty(K/G)$, are the physical observables for this system.

The aim of any constrained formalism is then to isolate the physical dynamics from the dynamics on some extended – although more accessible – phase space.

In Dirac's approach [11] to this problem the physical observables are identified with the sub-algebra of $C^\infty(P)$ consisting of equivalence classes of weakly invariant functions on P, where two functions are said to be (weakly) equivalent if they are equal when restricted to K.

In the approach initiated by Batalin, Fradkin and Vilkovisky (BFV) [12, 13], the physical observables are identified as a subalgebra of a graded extension to $C^\infty(P)$ – the new variables being the ghost and conjugate ghost variables. It will be useful to recall the main steps in this construction.

The graded extension to $C^\infty(P)$ needed in the BFV approach can be identified with $A(P) := C^\infty(P) \otimes \Lambda(g \oplus g^*)$, where g (g^*) is the Lie-algebra (dual) of G and $\Lambda(g \oplus g^*)$ denotes the exterior algebra over these vector spaces. The ghost variables, η^α, and their conjugates, ρ_α, are then the generators of this exterior algebra, and the natural pairing between g and g^* (along with the Poisson algebra on $C^\infty(P)$) allows us to define a Poisson bracket on this graded algebra such that $\{\eta^\alpha, \rho_\beta\} = \{\rho_\beta, \eta^\alpha\} = -\delta^\alpha_\beta$.

There are various ways to grade the functions in $A(P)$, the most important of which is with respect to ghost number i.e., the number of ghosts minus the number of conjugate ghosts occurring in the functions. Given a function $\mathscr{F} \in A(P)$, of ghost number r, we define $\delta\mathscr{F}$, a function of ghost number $r+1$, by

$$\delta\mathscr{F} = \{\mathscr{Q}, \mathscr{F}\}\,, \tag{1.1}$$

where $\mathscr{Q}$ is the BRST-charge given by

$$\mathscr{Q} = \phi_\alpha\eta^\alpha + \frac{1}{2}C^\gamma_{\alpha\beta}\eta^\alpha\eta^\beta\rho_\gamma\,. \tag{1.2}$$

Since $\{\mathscr{Q}, \mathscr{Q}\} = 0$, we have that $\delta^2 \equiv 0$ and hence we can construct the cohomology groups $H^r(\delta)$ – those functions of ghost number r that are BRST closed but not exact. Then $H^0(\delta)$ can be identified with the physical observables $C^\infty(K/G)$, and hence we have a constrained formalism.

As it stands, both of the above approaches (Dirac and BFV) seem to have only given us half of the information we would require in order to have a complete description of the physical dynamics; we would also like to recover the physical phase space, K/G, of the system. As we will discuss in more detail in Sect. 2, the reason why we are usually content just to describe the physical observables is that it contains the subalgebra of observables that vanish at infinity, $C_\infty(K/G)$. This can be extended into a commutative C^*-algebra and hence the phase space K/G can be recovered as the pure states on this algebra. So, in principle, all we need is a way to pick out all the physical observables in order to describe the physical dynamics. In practice, though, this is not a satisfactory procedure since we are usually only interested in one observable, the Hamiltonian, and thus would like a more direct route to the physical states of the system.

Within Dirac's approach there is a straightforward way to isolate (at least locally) the physical states from all the states on P. This is done by introducing a set of gauge fixing conditions χ^α, whose zero set gives a slice for the action of G on the constrained surface K. Hence the physical states can be locally described by the $2k$-conditions $\phi_\alpha = 0$ and $\chi^\alpha = 0$ on P.

Things are not quite so clear in the BFV approach. If we argue in analogy with what is suggested in the quantum theory, we should start with all states on the graded phase space and let the BRST-charge act on them in some natural manner. Then we would expect the physical states to emerge as the BRST-invariant, ghost number zero states that are not the BRST-transform of some other states. Clearly there are various steps in this proposal that need to be elaborated on. In particular, we need to make clear what is a state on a graded phase space, and then determine how the BRST-charge should act on it.

The bulk of this paper will concern itself with providing a sensible definition of states when one is dealing with a graded manifold. We will then see, through simple examples, that the above proposal does *not* work. Although this is an unexpected result, it really should not come as too much of a surprise since, at heart, the BFV and Dirac approaches have a lot in common. Thus it would be surprising if the BFV description of physical states could be done without the use of gauge fixing, as this was central to Dirac's method. What we shall see is that in order to directly isolate the physical states within the BFV formalism, gauge fixing is needed to construct a (symplectic) dual, $\bar{\mathcal{Q}}$, to the BRST-charge. The physical states will then be the $\mathcal{Q}$ and $\bar{\mathcal{Q}}$ invariant states on the graded phase space.

The plan of this paper is as follows: After this introduction, in Sect. 2, states and graded states, on a graded manifold will be defined. This will be achieved by carefully translating the definition of $A(P)$ – the graded extension of the algebra of functions on P, into an algebra of functions on a superspace, in the sense of Rogers [14]. We will then show how our definition of pure states on this superspace recover the body manifold, and how the graded pure states recover the superspace itself; thus giving us confidence that we have a reasonable definition of states. In Sect. 3 we shall show, through simple examples, that the physical states are not picked out using the BRST-charge $\mathcal{Q}$. Then, in Sect. 4 we shall present a method for isolating the physical states from the graded states using the BRST-charge $\mathcal{Q}$ and an additional charge $\bar{\mathcal{Q}}$. In the conclusions we will discuss the relevance of this classical construction to the quantum theory.

2. States and Graded States

In this section we shall start by reviewing the relationship between functions on a manifold and the manifold through the use of pure states. This construction will then be extended to the graded case and we shall give a definition of states and graded states on a supermanifold. We will show that the states on such a manifold can be identified with the body of the supermanifold, while the graded states recover the supermanifold itself; thus giving us confidence that these are sensible definitions.

Given a smooth manifold M, $C_\infty(M)$ is the set of smooth functions on M that vanish at infinity. On $C_\infty(M)$ we can define a norm $\| \ \|_\infty$, where $\|f\|_\infty = \sup\{|f(x)|: x \in M\}$. The completion of $C_\infty(M)$ with respect to this norm is the abelian C^*-algebra, $C_0(M)$, of continuous functions which vanish at infinity. On such a Banach algebra there are various ways to characterise the states and, in particular, the pure states (see, for example, ref. 15). The most convenient for us is in terms of the characters of $C_0(M)$. A character is a non-zero linear map, ω, of $C_0(M)$ into $\mathbb{R}$ such that

$$\omega(fg) = \omega(f)\omega(g) \tag{2.1}$$

for all functions f and g in $C_0(M)$. The spectrum of the C^*-algebra $C_0(M)$, which we denote by spec$(C_0(M))$, is defined to be the set of all characters. It is straightforward to see that the spectrum is just the original manifold M. Indeed, we can use the Riesz representation theorem to write

$$\omega(f) = \int_M f d\mu_\omega ,$$

where $d\mu_\omega$ is a regular Borel measure on M. Being a character then implies that

$$\int_M fg d\mu_\omega = \int_M f d\mu_\omega \int_M g d\mu_\omega .$$

As this must be true for all f and g, we conclude that the measure can have support at only one point of M, i.e., $\omega(f) = f(a)$, for some $a \in M$. Thus spec$(C_0(M))$ contains all points of M, as claimed.

Note that in (2.1) we can take the functions f and g to be smooth since, as we have already stated, $C_\infty(M)$ is dense in $C_0(M)$ and the character is continuous.

Following the discussion in the introduction, we now want to investigate how this type of argument can be extended to the situation where the commutative normed algebra $C_\infty(M)$ is replaced by a graded commutative algebra of the form

$$A^s_\infty(M) := C_\infty(M) \otimes \Lambda(\mathbb{R}^s) . \tag{2.2}$$

The problem we face is that, since this does not look like the algebra of functions on some space, it is not clear what we should mean by a character on this algebra. To proceed we need to first of all show how the graded algebra $A^s_\infty(M)$ can be viewed as the algebra of functions on a supermanifold $\mathscr{M}$. Heuristically, a supermanifold of dimension (m, s), is a space in which the local coordinates can be decomposed into m even coordinates and s odd ones. To make this precise we follow the approach taken by Rogers [14], which we now summarize.

Given a positive integer L, B_L is the Grassmann algebra defined over the reals with generators $1, \beta_1, \ldots, \beta_L$ and relations

$$1\beta_i = \beta_i 1 = \beta_i \qquad i = 1, \ldots, L ,$$

$$\beta_i\beta_j = -\beta_j\beta_i \qquad i, j = 1, \ldots, L .$$

An economic way to represent the elements of B_L is to follow Kostant [16] and let M_L denote the set of sequences which includes the empty sequence, denoted by 0, and the finite sequence of positive integers $\mu = (\mu_1, \ldots, \mu_r)$ with $1 \leqq \mu_1 < \cdots < \mu_r \leqq L$. Then, for each μ in M_L, we define

$$\beta_\mu = \beta_{\mu_1} \cdots \beta_{\mu_r} ,$$

and

$$\beta_0 = 1 \ .$$

A typical element b of B_L can then be written as

$$b = \sum_{\mu \in M_L} b^\mu \beta_\mu \ ,$$

where the coefficients b^μ are real numbers.

On B_L a norm is defined by

$$\| b \| = \sum_{\mu \in M_L} |b^\mu| \ , \tag{2.3}$$

which makes it a Banach algebra. In fact, B_L is a $\mathbb{Z}_2$-graded algebra: $B_L = (B_L)_0 \oplus (B_L)_1$, where $(B_L)_0$ is the even part and $(B_L)_1$ the odd part. $B_L^{m,s}$ is then defined to be the Cartesian product of m copies of $(B_L)_0$ and s copies of $(B_L)_1$. A typical element of $B_L^{m,s}$ can be written as $(x^1, \ldots, x^m; \theta^1, \ldots, \theta^s)$, or simply $(x; \theta)$, where the x coordinates are even and the θ coordinates are odd.

There are various classes of superdifferentiable functions $\mathscr{F}: B_L^{m,s} \to B_L$. Since both $B_L^{m,s}$ and B_L are Banach algebras we can define $C^\infty(B_L^{m,s}, B_L)$ (which we also write as $C^\infty(B_L^{m,s})$ when it is clear what target space we are dealing with) to be the smooth functions between these Banach spaces. This class of functions, though, is too large for our application to ghost variables. What we need is a class of functions on $B_L^{m,s}$ that is insensitive to the replacement of $\mathbb{R}^m$ by $B_L^{m,0}$. In order to define such functions we need some additional definitions:

The augmentation (body) map $\varepsilon: B_L \to \mathbb{R}$ is defined by $\varepsilon(b) = b^0$. Acting on $B_L^{m,s}$ we have $\varepsilon_{m,s}: B_L^{m,s} \to \mathbb{R}^m$ with

$$\varepsilon_{m,s}(x^1, \ldots, x^m; \theta^1, \ldots, \theta^s) := (\varepsilon(x^1), \ldots, \varepsilon(x^m)) \ .$$

We write $x_B = \varepsilon_{m,s}(x; \theta)$. Complementary to the augmentation map is the (soul-) mapping $s: B_L \to B_L$ given by $s(b) = b - \varepsilon(b)1$. Since $B_L \simeq \mathbb{R} \oplus N$, where N is the subspace of B_L consisting of nilpotent elements; the mapping s simply picks out the nilpotent part of b.

If $f \in C^\infty(\mathbb{R}^m)$, we define $z(f): B_L^{m,0} \to B_L$ by

$$z(f)(x^1, \ldots, x^m) = \sum_{\substack{i_1 = 0 \\ \vdots \\ i_m = 0}}^{L} \frac{1}{i_1! \ldots i_m!} (\partial_1^{i_1} \ldots \partial_m^{i_m} f(\varepsilon(x^1) \ldots \varepsilon(x^m))) s(x^1)^{i_1} \ldots s(x^m)^{i_m} \ . \tag{2.4}$$

The mapping z has various nice properties that follow from its similarity with the Taylor series; in particular, it preserves products of functions i.e.,

$$z(fg) = z(f) z(g) \ . \tag{2.5}$$

Following [14], we now define, for $L > s$, two important sub-algebras of $C^\infty(B_L^{m,s})$. The $H^\infty(B_L^{m,s})$ functions are the smooth functions $\mathscr{F}$ on $B_L^{m,s}$ for which there exists $f_\mu \in C^\infty(\mathbb{R}^m)$ such that

$$\mathscr{F}(x; \theta) = \sum_{\mu \in M_s} z(f_\mu)(x) \theta^\mu \ . \tag{2.6}$$

The $H_\infty(B_L^{m,s})$ functions are defined in a similar way with the f_μ's now elements of $C_\infty(\mathbb{R}^m)$. The important observation for us is that the algebra of H_∞ functions is, in fact, a normed algebra over the reals with norm

$$\|\mathscr{F}\| := \sum_{\mu \in M_s} \|f_\mu\|_\infty . \tag{2.7}$$

We denote the completion of this algebra by $H_0(B_L^{m,s})$, which is now a Banach algebra.

In [14] it was shown that $H^\infty(B_L^{m,s}) \simeq C^\infty(\mathbb{R}^m) \otimes \Lambda(\mathbb{R}^s)$, so clearly $H_\infty(B_L^{m,s}) \simeq C_\infty(\mathbb{R}^m) \otimes \Lambda(\mathbb{R}^s)$. What we shall now show is that, as the notation suggests, $H_0(B_L^{m,s}) \simeq C_0(\mathbb{R}^m) \otimes \Lambda(\mathbb{R}^s)$. In the proof of this last isomorphism we shall exploit the Banach algebra structure on H_0 functions to define pure states. In doing this we want to follow as close as possible the account given earlier. However, there are two technical obstacles to directly defining pure states as characters on $H_0(B_L^{m,s})$. The first is that we have not yet determined what the elements of $H_0(B_L^{m,s})$ look like – at present they are simply the completion of $H_\infty(B_L^{m,s})$. But, as was noted after the discussion following (2.1), it is sufficient to define the action of the pure states on a dense sub-algebra of the Banach algebra; this suggests that we identify the pure states as characters on the sub-algebra of H_∞ functions. This, though, still leaves us with the second technical problem; that is, we want the action of ω on $\mathscr{F} \in H_\infty(B_L^{m,s})$ to be given by

$$\omega(\mathscr{F}) = \sum_{\mu \in M_s} \omega(z(f_\mu))\omega(\theta^\mu) .$$

But we note that θ^μ is not an element of $H_\infty(B_L^{m,s})$, rather it belongs to $C^\infty(B_L^{m,s}, B_L)$. So we want the characters to be defined on the whole of $C^\infty(B_L^{m,s}, B_L)$.

This discussion motivates the following definitions.

Definition 2.1. *A character on $C^\infty(B_L^{m,s})$ is a non-zero linear map, ω, of $C^\infty(B_L^{m,s})$ into $\mathbb{R}$ such that*

$$\omega(\mathscr{F}\mathscr{G}) = \omega(\mathscr{F})\omega(\mathscr{G}) \tag{2.8}$$

for all $\mathscr{F}$ and $\mathscr{G}$ in $C^\infty(B_L^{m,s})$.

Definition 2.2. *A pure state on the graded Banach algebra $H_0(B_L^{m,s})$ is to be identified with the restriction of a character of $C^\infty(B_L^{m,s})$ to $H_\infty(B_L^{m,s})$. The spectrum of $H_0(B_L^{m,s})$, which we also denote by* $\operatorname{spec}(H_0(B_L^{m,s}))$, *is the set of all such pure states.*

Note that these two definitions combine to identify the pure states on $H_0(B_L^{m,s})$ with the characters on this algebra.

From these definitions we can deduce the following result:

Proposition 2.1. $\operatorname{spec}(H_0(B_L^{m,s})) = \mathbb{R}^m$.

Proof. The pure states ω are homomorphisms from a graded algebra into $\mathbb{R}$ – which has no grading. So clearly its action on odd elements is restricted. Indeed

$$\begin{aligned}\omega(\theta^i\theta^j) &= \omega(\theta^i)\omega(\theta^j)\\ &= \omega(\theta^j)\omega(\theta^i)\\ &= \omega(\theta^j\theta^i)\\ &= -\,\omega(\theta^i\theta^j) .\end{aligned}$$

Hence $\omega(\theta^i\theta^j) = 0$, which in turn implies $\omega(\theta^i) = 0$ since $\mathbb{R}$ has no nilpotent elements. Therefore, acting on $H_\infty(B_L^{m,s})$ we must have

$$\omega(\mathscr{F}) = \omega(z(f_0)) .$$

Then condition (2.8) becomes

$$\omega(z(f_0)z(g_0)) = \omega(z(f_0))\omega(z(g_0))$$

for all f_0 and g_0 in $C_\infty(\mathbb{R}^m)$. Using (2.5) this tells us that $\omega \circ z$ is a character, and hence pure state on $C_0(\mathbb{R}^m)$. Thus $\mathrm{spec}(H_0(B_L^{m,s})) = \mathbb{R}^m$ by our earlier argument.

The algebra $H_0(B_L^{m,0})$ is, in fact, an abelian C^*-algebra (this is easy to see since, on the dense subspace of H_∞-functions, $\|\mathscr{F}^2\| = \|f_0^2\|_\infty = \|f_0\|_\infty^2 = \|\mathscr{F}\|^2$). Now any abelian C^*-algebra is isomorphic to $C_0(M)$, where M is the spectrum of the algebra. Thus, by our previous result, $H_0(B_L^{m,0}) \simeq C_0(\mathbb{R}^m)$. The mapping taking us from an element of $H_0(B_L^{m,0})$ to a continuous function on $\mathbb{R}^m$ being the Gelfand transform $\mathscr{F} \to \hat{\mathscr{F}}$ where

$$\hat{\mathscr{F}}(\omega) := \omega(\mathscr{F}) \tag{2.9}$$

for all $\omega \in \mathrm{spec}(H_0(B_L^{m,0}))$.

When $s \neq 0$, the Gelfand mapping (2.9) is clearly not an isomorphism. The kernel of this map is the radical of $H_0(B_L^{m,s})$; hence we see that the Banach algebra $H_0(B_L^{m,s})$ is not semi-simple. However, it is clear from (2.6) that $H_0(B_L^{m,s}) \simeq H_0(B_L^{m,0}) \otimes \Lambda(\mathbb{R}^s)$. Thus we can extend the Gelfand mapping to the whole of $H_0(B_L^{m,s})$ by requiring it to be the identity on the Grassmann algebra $\Lambda(\mathbb{R}^s)$. So we have shown the following result:

Proposition 2.2. $H_0(B_L^{m,s}) \simeq C_0(\mathbb{R}^m) \otimes \Lambda(\mathbb{R}^s)$.

The previous two results show that Definition 2.2 gives a sensible class of pure states on $H_0(B_L^{m,s})$. However, just as the pure states allowed us to recover the manifold M from the algebra of functions $C_0(M)$, we would also like to be able to recover the superspace $B_L^{m,s}$ from some generalised states on the graded algebra $H_0(B_L^{m,s})$. This motivates the following definitions (recall the discussion preceding Definitions (2.1) and (2.2)):

Definition 2.3. *A graded character on $C^\infty(B_L^{m,s})$ is a non-zero linear map, ω_g, of $C^\infty(B_L^{m,s})$ into B_L such that*

$$\omega_g(\mathscr{F}\mathscr{G}) = \omega_g(\mathscr{F})\omega_g(\mathscr{G}) \tag{2.10}$$

for all $\mathscr{F}$ and $\mathscr{G}$ in $C^\infty(B_L^{m,s})$.

Definition 2.4. *A graded pure state on the graded Banach algebra $H_0(B_L^{m,s})$ is to be identified with the restriction of a graded character of $C^\infty(B_L^{m,s})$ to $H_\infty(B_L^{m,s})$. The graded spectrum of $H_0(B_L^{m,s})$, which we denote by g-$\mathrm{spec}(H_0(B_L^{m,s}))$, is the set of all such graded pure states.*

It is easy to see that $B_L^{m,s} \subset g$-$\mathrm{spec}(H_0(B_L^{m,s}))$: indeed for all $a \in B_L^{m,s}$, $\delta_a\mathscr{F} := \mathscr{F}(a)$ is a graded character and hence a graded pure state of $H_0(B_L^{m,s})$. To show that all graded states are of this form we argue as follows: If we call $\varepsilon \circ \omega_g$ the body of the graded state, then it is clear that the body is a state, and hence concentrated at some point $a_B \in \mathbb{R}^m$. So all that needs to be determined is the action of ω_g on the nilpotent parts of the H_∞ functions. Since ω_g is a character, all we need is its action on $s(x)$ and θ. We must have $\omega_g(s(x)) = (a_s)_0$ and $\omega_g(\theta) = (a_s)_1$

for some even (odd) element $(a_s)_0$ ($(a_s)_1$) of B_L. Hence $\omega_g = \delta_a$ where $\varepsilon(a) = a_B$ and $s(a) = (a_s)_0 + (a_s)_1$. Thus g-spec$(H_0(B_L^{m,s})) \subset B_L^{m,s}$ and hence we have proven

Proposition 2.3. g-spec$(H_0(B_L^{m,s})) = B_L^{m,s}$.

Various operations can be defined on the H^∞ functions in much the same way as for C^∞ functions. In particular, the even and odd derivatives are defined by

$$\frac{\partial}{\partial x^i}\mathscr{F}(x;\theta) = \sum_{\mu\in M_s} z(\partial_i f_\mu)(x)\theta^\mu \tag{2.11}$$

and

$$\frac{\partial}{\partial \theta^\alpha}\mathscr{F}(x;\theta) = \sum_{\mu\in M_s} z(f_\mu)\theta^{\mu/\alpha}, \tag{2.12}$$

where $\theta^{\mu/\alpha} = (-1)^{t-1}\dots\theta^{\mu_{t-1}}\theta^{\mu_{t+1}}\dots\theta^{\mu_r}$ if $\alpha = \mu_t$ for some t, $1 \leqq t \leqq r$, and $\theta^{\mu/\alpha} = 0$ otherwise.

Supermanifolds over B_L are then topological spaces that locally look like $B_L^{m,s}$. So a H^∞ supermanifold $\mathscr{M}$ will have a chart (U, ψ) of open sets U_α and homeomorphisms $\psi_\alpha: U \to B_L^{m,s}$ such that $\psi_\beta \circ \psi_\alpha^{-1}$ is a H^∞ mapping of $\psi_\alpha(U_\alpha \cap U_\beta)$ onto $\psi_\beta(U_\alpha \cap U_\beta)$. This definition is, however, too general for our applications. Instead we follow DeWitt [17] and use a coarser topology on $B_L^{m,s}$ – the DeWitt topology. Now a subset V of $B_L^{m,s}$ is open if and only if $V = \varepsilon_{m,s}^{-1}(W)$ for some open set W of $\mathbb{R}^m$. Then a DeWitt H^∞ supermanifold is a H^∞ supermanifold such that for each element of the chart, $\psi_\alpha(U_\alpha)$ is open in $B_L^{m,s}$ in the DeWitt topology.

Given a DeWitt H^∞ supermanifold $\mathscr{M}$ then an equivalence relation $\sim$ can be defined on $\mathscr{M}$ by $p_1 \sim p_2$ if there is an open set U_α from the chart such that $p_1 \in U_\alpha$ and $p_2 \in U_\alpha$ and also

$$\varepsilon_{m,s}(\psi_\alpha(p_1)) = \varepsilon_{m,s}(\psi_\alpha(p_2)).$$

Then $M \equiv B(\mathscr{M}) = \mathscr{M}/\sim$ is a m dimensional real C^∞ manifold called the body of $\mathscr{M}$. If $\mathscr{F} \in H^\infty(\mathscr{M})$ then on an open set U_α we have

$$\mathscr{F}|_{U_\alpha} = \sum_{\mu\in M_s} z(f_\mu)\theta^\mu,$$

where $f_\mu \in C^\infty(\varepsilon_{m,s}(U_\alpha))$. Thus, with the obvious extension of notation, on a DeWitt H^∞ supermanifold $\mathscr{M}$ the elements of $H^\infty(\mathscr{M})$, or $H_\infty(M)$, can be represented by

$$\mathscr{F}(x;\theta) = \sum_{\mu\in M_s} z(f_\mu)\theta^\mu, \tag{2.13}$$

where $f_\mu \in C^\infty(B(\mathscr{M}))$, or $f_\mu \in C_\infty(B(\mathscr{M}))$.

Definitions 2.1–2.4 can then be directly extended to the Banach algebra $H_0(\mathscr{M})$. Also the arguments used in Propositions 2.1–2.3 can be extended to this algebra since the DeWitt topology implies that in patching results together the only complications come from the structure of the body manifold – where Riesz's theorem already characterizes the pure states. Thus we have the following result

Theorem 2.1. *If $\mathscr{M}$ is a DeWitt H^∞ supermanifold with body M then*
(1) spec$(H_0(\mathscr{M})) = M$;
(2) $H^\infty(\mathscr{M}) \simeq C^\infty(M) \otimes \Lambda(\mathbb{R}^s)$;
(3) g-spec$(H_0(\mathscr{M})) = \mathscr{M}$.

The proof of (2) can be found in [14]. Again we note that if $\mathscr{M}$ is $(m, 0)$-dimensional then $H_0(\mathscr{M})$ is a C^*-algebra and the Gelfand theorem in conjunction with (1) tells us that $H_0(\mathscr{M}) \simeq C_0(M)$. Then, as before, we can use this to directly prove the version of (2) appropriate to H_0 functions.

3. States and the BRST Charge

In this section we start by constructing the BFV formalism on a super phase space defined over the phase space P. The action of the BRST-charge on the graded pure states will then be investigated and it will be shown that a cohomological characterization of the physical states is not possible.

The extended phase space P upon which the constraints are defined is a smooth $2n$ dimensional manifold. Thus P can be identified as the body manifold of an $(2n, 2k)$ dimensional DeWitt H^∞ supermanifold $\mathscr{P}$. Recall that if $f \in C^\infty(P)$ then $z(f) \in H^\infty(\mathscr{P})$. On such functions we define the Poisson bracket $\{,\}$ by

$$\{z(f), z(g)\} = z(\{f, g\}) . \tag{3.1}$$

In particular, for the constraints ϕ_α on P we get constraint $z(\phi_\alpha)$ on $\mathscr{P}$ which are still first class since

$$\begin{aligned}\{z(\phi_\alpha), z(\phi_\beta)\} &= z(\{\phi_\alpha, \phi_\beta\}) \\ &= z(C^\gamma_{\alpha\beta}\phi_\gamma) \\ &= z(C^\gamma_{\alpha\beta})z(\phi_\gamma) .\end{aligned}$$

If the structure functions are actually constants then this last expression is $C^\gamma_{\alpha\beta} z(\phi_\gamma)$.

Generic elements of $H^\infty(\mathscr{P})$ can be written as in (2.13) with $s = 2k$. We now relax our notation and write the element of $H^\infty(\mathscr{P})$ corresponding to $f \in C^\infty(P)$ by the same symbol; thus, as long as there is no confusion, we write $z(f) \equiv f$. In keeping with the BFV formalism, we also divide the odd coordinates into two subsets: the ghosts η^α and conjugate ghosts ρ_α ($\alpha = 1 \dots k$). The Poisson bracket defined above is then extended to the whole of $H^\infty(\mathscr{P})$ by requiring that the only new non-vanishing bracket is $\{\eta^\alpha, \rho_\beta\} = -\delta^\alpha_\beta$, and that it acts as a (graded) Poisson bracket should. (For a more geometric account of this see [18].)

From (1.2) we see that the BRST-charge $\mathscr{Q}$ is a H^∞ function of ghost number one on $\mathscr{P}$, which satisfies the Poisson algebra; $\{\mathscr{Q}, \mathscr{Q}\} = 0$. The BRST-operator δ is then defined to act on $H^\infty(\mathscr{P})$ by

$$\delta\mathscr{F} = \{\mathscr{Q}, \mathscr{F}\} . \tag{3.2}$$

Using the super-Jacobi identity, and the abelian nature of the odd charge $\mathscr{Q}$, it follows that $\delta^2 = 0$. The physical observables for this system can then be identified with the zeroth cohomology group associated with this operator.

Given a (graded pure) state ω_g on $\mathscr{P}$ we define $\delta\omega_g$, the BRST transform of ω_g, by

$$\delta\omega_g(\mathscr{F}) = -(-1)^{\mathscr{F}}\omega_g(\delta\mathscr{F}) \tag{3.3}$$

for all homogeneously graded functions $\mathscr{F}$ where this makes sense. Then it is clear that acting on the state ω_g we have $\delta^2 \equiv 0$.

We note, though, that $\delta\omega_g$ is *not* a state. Indeed, for homogeneous $\mathscr{F}$ and $\mathscr{G}$ we have

$$\delta\omega_g(\mathscr{F}\mathscr{G}) = (-1)^{\mathscr{G}}\delta\omega_g(\mathscr{F})\omega_g(\mathscr{G}) + \omega_g(\mathscr{F})\delta\omega_g(\mathscr{G}) ,$$

so $\delta\omega_g$ is not a character on $H_\infty(\mathscr{P})$. In this context it is probably best to think of ω_g as a (point) distribution on $\mathscr{P}$, then $\delta\omega_g$ is another distribution on $\mathscr{P}$ with the stated properties.

We now need to determine what are sensible conditions to impose on ω_g in order for it to be a physical state ω_g^{phy}. Clearly functions of the form $\delta\mathscr{F}$ are unphysical for all functions on $\mathscr{P}$. Thus we require ω_g^{phy} to satisfy $\omega_g^{\mathrm{phy}}(\delta\mathscr{F}) = 0$, for all such $\mathscr{F}$. That is

$$\delta\omega_g^{\mathrm{phy}} = 0\,. \tag{3.4}$$

Now if $\omega_g = \delta\omega_g^{-1}$, for some acceptable distribution ω_g^{-1}, then $\delta\omega_g = 0$. But this implies that $\omega_g(\mathscr{F}) = \omega_g^{-1}(\delta\mathscr{F})$ would be zero on all physical observables $\mathscr{F}$. Such states should not be thought of as physical.

This argument suggests that the physical states are defined as those states on $\mathscr{P}$ that are BRST-invariant, but not the BRST transform of some other allowed distribution. Note that ghost number is not used in the definition. Also we have been vague about what types of distributions are allowed. To make this more precise, and to investigate whether this does indeed recover the physical states, it is best to study in detail a simple example.

The paradigm example of a constrained theory is the system with extended phase space $P = \mathbb{R}^{2n}$ and pure momenta constraints $p_\alpha = 0$. If we use the canonical coordinate system (q^A, p_A), $A = 1, \ldots, n$, on P, where the constraints are just the first k momenta, then the true degrees of freedom are parameterized by the coordinate functions (q^{k+i}, p_{k+i}), $i = 1, \ldots, n-k$.

The BRST-charge is thus

$$\mathscr{Q} = p_\alpha \eta^\alpha\,, \tag{3.5}$$

and acting on functions

$$\delta = -p_\alpha \frac{\partial}{\partial \rho_\alpha} - \eta^\alpha \frac{\partial}{\partial q^\alpha}\,. \tag{3.6}$$

The pure states on $\mathbb{R}^{2n}$ can be usefully represented by delta functions. So the pure state concentrated at the point $(\bar{q}^A, \bar{p}_A)$ in $\mathbb{R}^{2n}$ can be written as

$$\begin{aligned}\omega_{(\bar{q},\bar{p})}(f) &= \int f(q^A, p_A)\delta(q^A - \bar{q}^A)\delta(p_A - \bar{p}_A)dq^A dp_A\\ &= f(\bar{q}^A, \bar{p}_A)\,.\end{aligned}$$

It is straightforward to see how gauge fixing within the Dirac formalism extracts the physical states from these. Indeed, taking the gauge fixing condition to be $q^\alpha = 0$, we require the physical states to be such that $\omega^{\mathrm{phy}}(p_\alpha) = \omega^{\mathrm{phy}}(q^\alpha) = 0$. Which implies the correct result that the physical states are associated to the delta functions on $\mathbb{R}^{2n}$ of the form

$$\delta(q^\alpha)\delta(p_\alpha)\delta(q^{k+i} - \bar{q}^{k+i})\delta(p_{k+i} - \bar{p}_{k+i})\,.$$

It will be useful to develop a similar representation for the graded pure states on $\mathscr{P}$. In order to do this we need to discuss how to integrate on the superspace $\mathscr{P}$.

Over the odd variables integration is purely formal and we use the Berezin rules $\int d\theta = 0$; $\int \theta d\theta = 1$. Thus on $B_L^{m,s}$ we take $d\theta^\mu = i^{s(s-1)/2} d\theta^1 \ldots d\theta^s$ and use the odd delta function

$$\delta(\theta^\mu - \bar{\theta}^\mu) := i^{s(s-1)/2}(\theta^1 - \bar{\theta}^1)\ldots(\theta^s - \bar{\theta}^s)\,. \tag{3.7}$$

Over the even variables more care is needed to define integration since some remnant of the measure theoretic aspects on integration on the body manifold $\mathbb{R}^m$ should survive. Thus we follow DeWitt [17] and define $\int f(x)dx$ in terms of the body measure dx_B and a section $\sigma\colon \mathbb{R}^m \to B_L^{m,0}$ such that $\varepsilon \circ \sigma$ is the identity mapping on $\mathbb{R}^m$. Given these, and a smooth test function f on $\mathbb{R}^m$, the integral $\int z(f)(x)dx$ is defined to be equal to

$$\int_{\mathbb{R}^m} z(f)(\sigma(x_B))d\sigma(x_B)\,. \tag{3.8}$$

Clearly this expression is independent of section σ used.

Similarly, we can define $\delta(x)$ on $B_L^{m,s}$ by pulling back the delta function from $\mathbb{R}^m$ using (2.4), i.e., $\delta(x) := z(\delta)(x)$. Extending this construction we define the even delta function to be

$$\begin{aligned}\delta(x^j - \bar{x}^j) = \sum_{\substack{i_1=0\\ \vdots \\ i_m=0}}^{L} &\frac{1}{i_1! \dots i_m!}(\partial_1^{i_1} \dots \partial_m^{i_m}\delta(x_B^j - \bar{x}_B^j))\\ &\times s(x^1 - \bar{x}^1)^{i_1} \dots s(x^m - \bar{x}^m)^{i_m}\,,\end{aligned} \tag{3.9}$$

where, as usual, $\delta(x_B^j - \bar{x}_B^j)$ is actually the product of delta function $\prod_{j=1}^m \delta(x_B^j - \bar{x}_B^j)$.

Then for $f \in C^\infty(\mathbb{R}^m)$

$$\int z(f)(x)\delta(x - \bar{x})dx := \int_{\mathbb{R}^m} z(f)(\sigma(x_B))\delta(\sigma(x_B) - \bar{x})d\sigma(x_B) \tag{3.10}$$

is also independent of section σ. Exploiting this we take $\sigma(x_B) = x_B + s(\bar{x})$ to get

$$\begin{aligned}\int z(f)(x)\delta(x - \bar{x})dx &= \int_{\mathbb{R}^m} z(f)(x_B + s(\bar{x}))\delta(x_B - \varepsilon(\bar{x}))dx_B\\ &= z(f)(\bar{x})\,.\end{aligned}$$

So the even delta function behaves as it should.

Hence, on the super phase space $\mathscr{P}$, the graded pure state ω_g concentrated at the point $(\bar{x}; \bar{\theta}) = (\bar{q}^A, \bar{p}_A; \bar{\eta}^\alpha, \bar{\rho}_\alpha)$ can be represented by the expression

$$\begin{aligned}\omega_g(\mathscr{F}) = \int &\mathscr{F}(q^A, p_A, \eta^\alpha, \rho_\alpha)\delta(q^A - \bar{q}^A)\delta(p_A - \bar{p}_A)\\ &\times \delta(\eta^\alpha - \bar{\eta}^\alpha)\delta(\rho_\alpha - \bar{\rho}_\alpha)dq^A dp_A d\eta^\alpha d\rho_\alpha\,.\end{aligned} \tag{3.11}$$

Therefore, using (3.3) and (3.6), we see that the action of the BRST-operator δ on states is simply given by the action of δ on the delta functions representing the state.

The condition $\delta\omega_g = 0$ then implies that $\bar{p}_\alpha = 0$ and $\bar{\eta}^\alpha = 0$, i.e., ω_g corresponds to the distribution

$$\delta(q^\alpha - \bar{q}^\alpha)\delta(p_\alpha)\delta(\eta^\alpha)\delta(\rho_\alpha - \bar{\rho}_\alpha)\omega^{\mathrm{phy}}\,, \tag{3.12}$$

where ω^{phy} is the physical state represented by

$$\omega^{\mathrm{phy}} = \delta(q^{i+k} - \bar{q}^{i+k})\delta(p_{i+k} - \bar{p}_{i+k})\,.$$

However, such a state can always be written in the form $\delta\omega_g^{-1}$, where ω_g^{-1} corresponds, for example, to the distribution

$$-i^{k(k-1)/2}\eta^2 \dots \eta^k \theta(q^1 - \bar{q}^1)\prod_{j=2}^{k}\delta(q^j - \bar{q}^j)\delta(p_\alpha)\delta(\rho_\alpha - \bar{\rho}_\alpha)\omega^{\mathrm{phy}}\,. \tag{3.13}$$

Hence, allowing such distributions implies that the BRST cohomology is trivial and hence that there are no physical states within the BFV approach.

It is clear, though, that for this distribution, $\delta\omega_g^{-1}(\mathscr{F})$ is not identically equal to zero for all physical observables $\mathscr{F}$, since, in general, this is equal to $\omega_g^{-1}(\delta\mathscr{F})$ and a surface term arising from the step function in (3.13).

Thus we find that the BRST-charge only isolates states of the form (3.12) – which include the physical states. What is lacking is a natural way to impose the additional conditions that $\bar{q}^\alpha = 0$ and $\bar{p}_\alpha = 0$.

4. Gauge Fixing and Physical States

In this section we introduce explicit gauge fixing into the BFV formalism through the introduction of a dual charge $\bar{\mathscr{Q}}$ of ghost number minus one. It is shown how this allows us to directly isolate the physical states of the system.

We saw in our analysis of the simple constrained system on $\mathbb{R}^{2n}$ that in the Dirac approach a set of gauge fixing conditions, $q^\alpha = 0$, was needed to reduce to the physical states. Geometrically, the gauge fixing conditions determine a surface in the phase space P that, on its intersection with the constraint surface K, slices the orbits of the "gauge group" $\mathbb{R}^k$. There is a clear duality in this set-up. One could just as well have started with the first class constraints $q^\alpha = 0$ and then impose the gauge fixing conditions $p_\alpha = 0$ to reduce to the true degrees of freedom.

Motivated by this duality, and extending it to the ghost variables, we define a dual BRST-charge, $\bar{\mathscr{Q}}$, for this system (in this gauge) by

$$\bar{\mathscr{Q}} = q^\alpha \rho_\alpha \, . \tag{4.1}$$

This is a H^∞ function on $\mathscr{P}$ that has ghost number minus one and is abelian.

Repeating the discussion presented in Sect. 3, we get a dual BRST operator $\bar{\delta}$ whose action on states is given by

$$\bar{\delta} = -\, q^\alpha \frac{\partial}{\partial \eta^\alpha} + \rho_\alpha \frac{\partial}{\partial p_\alpha} \, . \tag{4.2}$$

Now, in addition to the condition (3.4), we require that the physical pure graded states should satisfy

$$\bar{\delta}\omega_g^{\text{phy}} = 0 \, . \tag{4.3}$$

Then, following the argument leading up to (3.12) we deduce that on $\mathscr{P}$ the states satisfying both (3.4) and (4.3) correspond to delta functions of the form

$$\delta(q^\alpha)\delta(p_\alpha)\delta(\eta^\alpha)\delta(\rho_\alpha)\omega^{\text{phy}} \, . \tag{4.5}$$

These graded pure states are actually pure states and hence have a spectrum given by the body of the superspace $B_L^{2(n-k),0}$. Hence they are the correct physical states on $\mathscr{P}$.

There are two directions in which this argument needs to be extended: first we would like to see how to construct $\bar{\delta}$ for more general gauge fixing conditions and, secondly, we would also like to be able to deal with more general, first-class, constraints. In both of these generalizations we must ensure that the resulting physical states are equivalent to the ones described in (4.5).

The appropriate concept of equivalence in this phase space formalism is that induced through canonical transformations. Thus we initially need to show how to construct an even canonical transformation on $\mathscr{P}$ such that the BRST-charge (3.5) is preserved while the symplectic dual (4.1) is transformed into an H^∞ function of the form

$$\bar{\mathscr{Q}} = \chi^\alpha \rho_\alpha + \cdots \tag{4.6}$$

with the χ^α's a more general set of gauge fixing conditions.

In (4.6) we cannot expect to the gauge fixing conditions to be arbitrary since $\bar{\mathscr{Q}}$ must still be abelian. However, given this caveat, the ability to do such a transformation is discussed in detail in [18] and will not be repeated here. Similarly, the rescaling of the BRST-charge can be achieved through canonical transformation, as discussed in [18].

The final result from this analysis is as follows

Theorem 4.1. *The physical states associated to the first-class constraints $\phi_\alpha = 0$ on the extended phase space P can be identified with those graded pure states on the super phase space $\mathscr{P}$ (of dimension $(2n, 2k)$ and body P) which satisfy the conditions*

$$\delta\omega_g^{\mathrm{phy}} = \bar{\delta}\omega_g^{\mathrm{phy}} = 0 \, ,$$

where δ and $\bar{\delta}$ are the Hamiltonian vector fields corresponding to the BRST-charge and dual charge introduced above.

5. Conclusions

In Sect. 2 a definition was given for pure and graded pure states on a supermanifold. These definitions were motivated by the analysis of pure states on an ordinary manifold. The usefulness of these definitions was shown by the results that the spectrum of pure states could be identified with the body manifold, while the graded spectrum of graded pure states recovered the original supermanifold.

This analysis was then applied in Sect. 3 to the super phase space approach to constrained systems, developed by Batalin, Fradkin and Vilkovisky. The main conclusion from this analysis was that the BRST-charge could not be used to give a direct cohomological description of the physical states of the system. In Sect. 4, though, it was shown how gauge fixing could be used to supplement the BFV formalism; allowing us to construct a dual to the BRST-charge. The physical states could then locally be identified with those graded pure states that were both BRST and dual-BRST invariant.

It is clear that in this presentation the dual-BRST charge has been introduced in a purely pragmatic way – it solved the problem. What one would like to see is a more geometric account of why it is needed. This should have some overlap with the geometric discussion of the physical observables to be found in [19].

It should be noted that the dual to the BRST-charge used here is quite distinct from the anti-BRST charge used in the literature (see for example [20]). Indeed for the simple abelian system discussed in Sects. 3 and 4, the anti-BRST charge would just be given by $p_\alpha \rho_\alpha$. Requiring states to be both BRST and anti-BRST invariant would then set $\bar{p}_\alpha = \bar{\eta}^\alpha = \bar{\rho}_\alpha = 0$, but not fix the value of $\bar{q}^\alpha$.

As discussed in the introduction, in the quantum theory the use of the BRST-charge to directly isolate the physical states has been problematic. Many parallels can be drawn with the problems encountered there and the classical analysis presented here. In particular the cohomological argument is seen to break down in both situations and one is forced to include states that are formally trivial (being coboundaries) yet do not give trivial results on physical observables since the BRST-charge is not self-adjoint on this class of functions (see [9] for the quantum construction, the discussion following (3.13) in this paper gives the classical point of view where self-adjoint is taken to mean that the action of the BRST-charge on observables and states is the same).

In [21] it was shown that in the quantum theory the correct way to characterize the physical states was to impose the two conditions $\hat{\mathcal{Q}}|\psi\rangle = 0$ and $\hat{\bar{\mathcal{Q}}}|\psi\rangle = 0$. (Although in the analysis presented there the classical role of $\bar{\mathcal{Q}}$ in isolating states was not discussed). In [18] a path integral quantisation of these systems was performed with the result that unitarity could be shown to hold when the physical observables were of the form $H_{\mathrm{eff}} = H_{\mathrm{phys}} + \{\mathcal{Q}, \bar{\mathcal{Q}}\}$. Such an observable satisfies the conditions $\delta H_{\mathrm{eff}} = 0$ and $\bar{\delta} H_{\mathrm{eff}} = 0$; hence, we now see that it would also preserve the classical physical states isolated by the BRST-charge and dual charge. Thus the classical analysis presented here can be seen to supply support for the apparently ad-hoc prescriptions used in these two approaches to the quantum theory.

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