Instructions to Authors

Communications in
Mathematical
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The instructions should be read carefully before preparing the manuscript.

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Papers submitted for publication should preferably be written in **English.**

A **summary** for *Zentralblatt für Mathematik* should be attached. Manuscripts **(in duplicate)** must be in their final form and typed on one side of the paper only in double-line spacing with wide margins. The author should also keep a copy of the manuscript. An **abstract** must be included.

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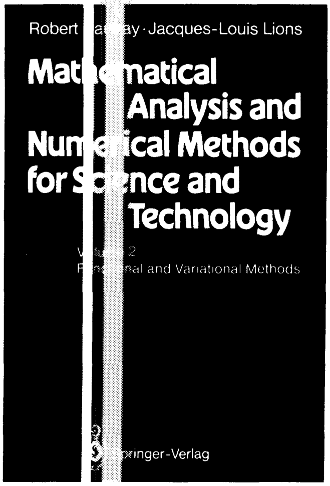

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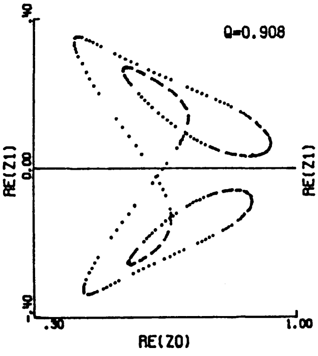
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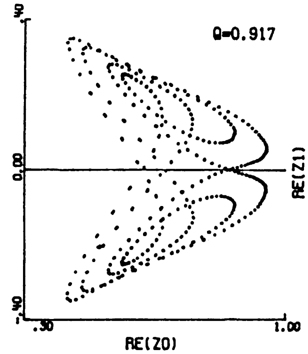
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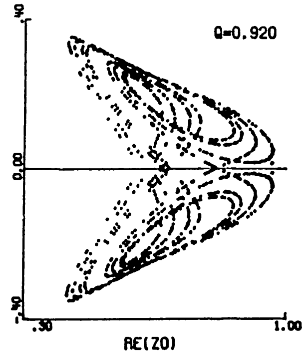
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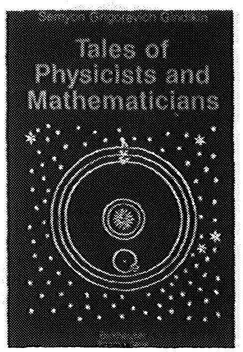
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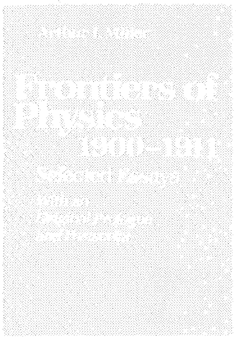
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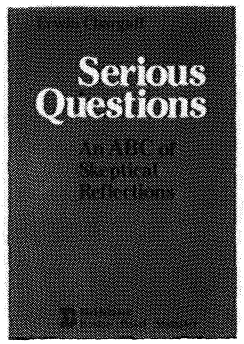
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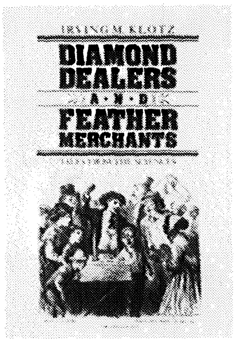
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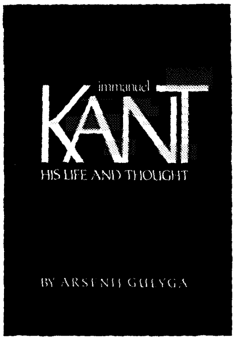
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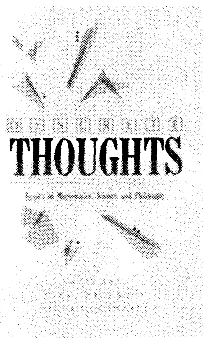
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written as

$$(D\mathscr{R}\varphi)(z) = -\frac{1}{\lambda}\varphi(L_+(\tau z)) - \frac{r}{\lambda} f'(L_+(\tau z))\, f(\tau z)^{r-1}\, \varphi(\tau z). \tag{3.6}$$

If we define the linear operator Q by

$$(Q\varphi)(z) = \frac{\varphi(z)}{f'(z)}, \tag{3.7}$$

then

$$QD\mathscr{R} = TQ,\ T = QD\mathscr{R}Q^{-1}. \tag{3.8}$$

The domains of these operators will be chosen to be certain spaces of functions holomorphic in complex neighborhoods of $[0,1]$ in such a way that these equations make sense. The spectrum of $D\mathscr{R}$ will then be the same as that of T.

It is natural to expect (and true) that T will preserve the class of functions holomorphic in the same domain as f. Indeed recall that U is holomorphic in $\Omega(\lambda)$ which it maps bijectively onto a certain bounded open subset Δ of $\mathbf{C}$, on which f is holomorphic. By (2.7), $\tau\Delta \subset \Delta$. The identity (2.15) shows that $L_+ \circ \tau$ is analytic on Δ and maps it into itself. But $L(\tau\Delta) = U(-\lambda\Omega(\lambda))$ is not relatively compact in Δ since their non-real points are the same. We will therefore use a sub-domain of Δ to make T analyticity-improving. A convenient choice is given by

$$\Delta_1 = U\left(\left\{z : |z| < \frac{1}{\lambda}\right\}\right), \qquad \Delta_0 = U(\{z : |z| < u(-1)\}). \tag{3.9}$$

Note that $\Delta_0 \Subset \Delta_1$ since

$$1 = u(-\lambda) < u(-1) < u(-1/\lambda) = x_0/\lambda.$$

Lemma 3.1. *If v is a real function on $[0,1]$ which extends to a holomorphic function on Δ_0, then Tv extends to a holomorphic function on Δ_1. For every $\Delta' \Subset \Delta_1$ one has*

$$\sup_{z\in\Delta'} |(Tv)(z)| \leqq \frac{1}{\tau} \sup_{z\in\Delta'} \left(1 + \left|\frac{1}{L'_+(\tau z)}\right|\right) \cdot \sup_{y\in\Delta_0} |v(y)|.$$

Proof. We shall use the following simple fact: if a Herglotz or anti-Herglotz function is holomorphic on a real segment (a,b) and maps it into the real segment (a',b'), then it maps the disk with diameter (a,b) into the disk with diameter (a',b'). (See e.g. [E2].)

We claim now that $\tau\Delta_1 \subset \Delta_0$ and $L_+(\tau\Delta_1) \subset \Delta_0$. Indeed, let $z = U(\zeta)$, for some $|\zeta| < 1/\lambda$. Then $\tau z = U(u(-\lambda\zeta))$, and $u(-\lambda\zeta)$ is contained in the disk with diameter $(0, u(-1))$; hence $\tau z \in \Delta_0$. On the other hand, by (2.15), we have $L_+(\tau z) = U(-\lambda\zeta)$ and this is also in Δ_0. The derivative of $L_+ \circ \tau$ tends to zero near y_0/τ. But its reciprocal is bounded in modulus in any $\Delta' \Subset \Delta_1$. This completes the proof of the lemma.

We denote by $\mathscr{B}$ the Banach space of holomorphic, bounded functions on Δ_0 which are real on $[0,1]$, equipped with the "sup" norm. It follows from Lemma 3.1 that $T\mathscr{B} \subset \mathscr{B}$ and T is a compact linear operator on $\mathscr{B}$ whose eigenvalues form an exponentially decaying sequence.

We use the following lemma to take advantage of the simple form of T:

Lemma 3.2. *The function L_+ is convex on $[0, y_0]$, and the function L_- is convex on $[y_0, 1]$.*

Proof. By our general assumptions, the function U is holomorphic and anti-Herglotzian in the cut plane $\Omega(\lambda)$, described by (2.3). As such it has positive Schwarzian derivative on the interval $(-\lambda^{-1}, \lambda^{-2})$, i.e. $\phi = U''/U'$ satisfies $2\phi'/\phi^2 - 1 \geqq 0$. Integrating this inequality gives

$$-\frac{2\lambda}{1+\lambda z} \leqq \frac{U''(z)}{U'(z)} \leqq \frac{2\lambda^2}{1-\lambda^2 z}. \tag{3.10}$$

By (2.14),

$$-\frac{S''_\pm(\zeta)}{S'_\pm(\zeta)} = \frac{1}{r\zeta}\left[r-1-z\frac{U''(z)}{U'(z)}\right] \quad \text{with} \quad z = \pm\zeta^{1/r}. \tag{3.11}$$

We now use the lower bound for r obtained in [E1]:

$$r > \frac{1+\lambda^2}{1-\lambda^2}. \tag{3.12}$$

For $z = \zeta^{1/r} > 0$ we find:

$$-\frac{S''_+(\zeta)}{S'_+(\zeta)} > \frac{1}{r\zeta}\left[\frac{1+\lambda^2}{1-\lambda^2} - \frac{1+\lambda^2 z}{1-\lambda^2 z}\right]. \tag{3.13}$$

This is positive for $z < 1$. For $z = -\zeta^{1/r} \leqq 0$, we get:

$$-\frac{S''_-(\zeta)}{S'_-(\zeta)} > \frac{1}{r\zeta}\left[\frac{1+\lambda^2}{1-\lambda^2} - \frac{1-\lambda z}{1+\lambda z}\right]. \tag{3.14}$$

This is positive for $-\lambda \leqq z \leqq 0$. Thus:

$$-\frac{S''_+(\zeta)}{S'_+(\zeta)} > 0 \;\; \forall \zeta \in [0,1], \qquad -\frac{S''_-(\zeta)}{S'_-(\zeta)} > 0 \;\; \forall \zeta \in [0,\tau]. \tag{3.15}$$

To see that the inequalities (3.15) remain *strict* even in the limit $r \to \infty$, we rewrite r in (3.11) as $\log(1/\tau)/\log(1/\lambda)$ and, using again the bounds (3.10), and $\log(1/\lambda) < 1/\lambda - 1$, we find:

$$-\frac{S''_+(\zeta)}{S'_+(\zeta)} > \frac{1}{\zeta}\left[1 - \frac{1+\lambda^2}{\lambda(1+\lambda)\log(1/\tau)}\right] \quad \forall \zeta \in (0,1], \tag{3.16}$$

and exactly the same inequality for S''_-/S'_- on $(0, \tau]$. This proves that L_+ and L_- are convex on $[0, y_0]$ and $[y_0, 1]$ respectively. This completes the proof of Lemma 3.2.

Corollary 3.3. *For all $z \in [0,1]$, we have $L_+(\tau z) > \tau z$ and $L'_+(\tau z) < -1$.*

Proof. By the monotonicity and convexity of L_+ it suffices to prove this for $z = 1$. Applying the functional equation (2.11) and its derivative at $z = 0$ gives

$$L(1) = -\tau, \qquad L'(1) = -1. \tag{3.17}$$

Reapplying them at $z=1$ gives

$$L(L(\tau))=\tau^2, \qquad L'(L(\tau))\,L'(\tau)=1\,. \tag{3.18}$$

It follows that $L(\tau)<y_0$, and also $L(\tau)>\tau$. Otherwise $L(L(\tau)/\tau)$ would be in $[-1,1]$, contradicting

$$L(L(\tau)/\tau)=-L(\tau^2)/\tau<-L(\tau y_0)/\tau=-y_0/\tau<-1\,. \tag{3.19}$$

The convexity of L_+ implies $-L'(\tau)>-L'(L(\tau))$ and hence $-L'(\tau)>1$ by (3.18).

From the convexity of $L_\pm$ we can now derive, following an idea of [CE], the existence of invariant cones for the operator T. However, the cones we define here do not coincide with the cones defined there because of the use of $v=\delta f/f'$ instead of δg. (The cones of [CE] could not be shown to be invariant under the tangent map for r much above 2 because of the lack of concavity of g on $(x_0,1]$.)

Definition. Define Γ_1 as the set of real $\mathscr{C}^1$ functions v on $[0,1]$ for which

i) $v(z)\geqq 0$ for all $z\in[0,1]$,

ii) $v'(z)\leqq 0$ for all $z\in[0,1]$.

We also define $\Gamma=\Gamma_1\cap\mathscr{B}$. Γ is a closed cone with non-empty interior in $\mathscr{B}$.

Lemma 3.4. *The tangent map T maps Γ_1 into itself. Furthermore, T^2 maps any non-zero vector in Γ into the interior of Γ.*

Proof. Suppose $v\in\Gamma_1$. Then, since (by Corollary 3.3) for any $z\in[0,1]$, $L_+(\tau z)>\tau z$, and since v is decreasing,

$$\tau(Tv)(z)\geqq v(\tau z)\,[1+1/L'_+(\tau z)]\,. \tag{3.20}$$

This is non-negative since $L'_+(\tau z)<-1$ by Corollary 3.3. Furthermore

$$(Tv)'(z)=v'(L_+(\tau z))-\frac{v(L_+(\tau z))\,L''_+(\tau z)}{L'_+(\tau z)^2}+v'(\tau z)\,. \tag{3.21}$$

The point is now that all three terms of this formula are non-positive, so that Tv is indeed in Γ_1. The interior of Γ is clearly composed of those v for which the inequalities defining Γ are all strict. Suppose $v\in\Gamma$ is not 0. If $v(z)$ vanished for some $z\in[0,1)$, it would have to vanish on $[z,1]$, hence everywhere by analyticity, i.e. 1 is the only place in $[0,1]$ where v can vanish. But Tv cannot vanish even at 1 by (3.20). Furthermore the middle term in (3.21) cannot vanish in $(0,1]$, and can vanish at 0 only if $v(1)=0$. Hence T^2v is in the interior of Γ as claimed.

4. Inequalities and Numerical Bounds

Suppose $v_e\in\Gamma\backslash\{0\}$ and $Tv_e=\varrho v_e$. Then v_e is in the interior of Γ by Lemma 3.4, and

$$\varrho v_e(0)=\frac{v_e(1)}{\tau L'(0)}+\frac{v_e(0)}{\tau}>v_e(0)\left[\frac{1}{\tau}+\frac{1}{\tau L'(0)}\right]>v_e(0)\left(\frac{1}{\tau}-\frac{1}{\lambda}\right). \tag{4.1}$$

The last inequality uses $-\tau L'(0)>\lambda$ due to the convexity of L_+. The middle inequality is strict because v_e is in the interior of Γ, so that $v_e(1)<v_e(0)$. Finally,

since $v_e(1) > 0$, we get the inequality announced in the Introduction:

$$\frac{1}{\tau} - \frac{1}{\lambda} < \frac{1}{\tau} + \frac{1}{\tau L'(0)} < \varrho < \frac{1}{\tau}. \tag{4.2}$$

Applying the theorem of Krein and Rutman [KR] we obtain from Lemma 3.4:

Lemma 4.1. *As an operator on $\mathscr{B}$, T possesses an eigenvalue of largest modulus δ which is real and positive. The spectral subspace corresponding to this eigenvalue is one-dimensional and generated by an element of the interior of Γ which is (up to rescaling) the only eigenvector of T in Γ. This eigenvalue satisfies the bounds* (4.2). *The adjoint T^* of T has a unique eigenvector φ_e in the cone Γ^* dual to Γ (i.e., the set of continuous linear functionals on $\mathscr{B}$ which take positive values on all elements of Γ) and the corresponding eigenvalue is δ.*

At $r = \infty$, we can use the rigorous numerical bounds obtained in [EW1], written here just as ordinary numbers, not as intervals:

$$y_0 = 0.391132999351022542,\ \tau = 0.033381055,\ L'_+(0) = -67.42069.$$

This gives

$$\frac{1}{\tau} = 29.957112,\ \frac{1}{\tau} - 1 = 28.957112,\ \frac{1}{\tau}\left(1 + \frac{1}{L'(0)}\right) = 29.5128,$$

to be compared with the following numerical estimate of δ:

$$\delta = 29.5763.$$

This shows that the bounds (4.2) become rather satisfactory at $r = \infty$. They are poorer at, e.g. $r = 2$, where

$$\delta = 4.669201609, \quad \text{while} \quad \frac{1}{\tau} = 6.26454783121704,\ \frac{1}{\tau} - \frac{1}{\lambda} = 3.7616,$$

$$f'(0) = -1.52763299703630145,\ \frac{1}{\tau}\left(1 + \frac{1}{L'(0)}\right) = 4.2141.$$

Acknowledgements. This work was begun during the first author's stay at the IHES, where he profited from the usual warm hospitality. Further support was received from the Fonds National Suisse.

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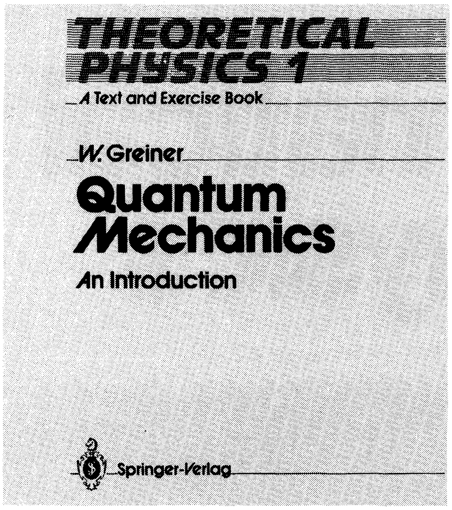
Communicated by A. Jaffe

Received June 20, 1989

Note added in proof. Using the upper bounds on τ given in [E1], it is easy to see that $(1/\tau - 1/\lambda) > 1$ (and hence $\delta > 1$) for all $r > 1$.



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Printers Brühlsche Universitätsdruckerei, Giessen
Printed in Germany

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