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2- and 3-Cochains in 4-Dimensional SU(2) Gauge Theory

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Abstract. Explicit formulae are derived for the 2- and 3-cochains $\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)$ and $\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,\ell)$ in SU(2) gauge theory in 4 dimensions. It turns out that $\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,\ell)$ is given by the volume of a spherical tetrahedron spanned by the gauge transformations relating the gauges i,j,k,l.

I. Introduction

Higher-order cocycles

$$\omega^{(n)} = \int \alpha^{3-n}\sigma_{\mu\ldots}\Omega^{(n)}_{\mu\ldots} \tag{1}$$

(here written for 4 space-time dimensions), where $\Omega^{(n)}_{\mu\ldots}$ is the n-cochain, play an important role in group representation theory, in the investigation of the structure of anomalies, Wess-Zumino effective actions and groups associated with a Kac-Moody algebra [1] as well as in the derivation of a closed expression for the topological charge [2]. It is therefore of great interest to know $\Omega^{(n)}_{\mu\ldots}$ explicitly. In this paper we shall consider the case of gauge group SU(2) in 4 dimensions and derive explicit expressions for $\Omega^{(2)}_{\mu\nu\varrho}$ and $\Omega^{(3)}_{\mu\nu\varrho\sigma}$.

The starting-point is the Chern-Pontryagin density

$$P = -\frac{1}{32\pi^2}\varepsilon_{\mu\nu\varrho\sigma}\operatorname{Tr}[F^i_{\mu\nu}F^i_{\varrho\sigma}],\; F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + [A^i_\mu, A^i_\nu], \tag{2}$$

where the index i specifies a particular gauge. The 4-dimensional integral of P is the topological charge, which is an invariant. The Chern-Pontryagin density can be written as a total divergence,

$$P = \partial_\mu \Omega^{(0)}_\mu(i), \tag{3}$$

where

$$\Omega^{(0)}_\mu(i) = -\frac{1}{8\pi^2}\varepsilon_{\mu\nu\varrho\sigma}\operatorname{Tr}[A^i_\nu(\partial_\varrho A^i_\sigma + \tfrac{2}{3}A^i_\varrho A^i_\sigma] \tag{4}$$

is the Chern-Simons density or 0-cochain. The latter is gauge variant. What interests us naturally is its gauge variation, which is given by the coboundary operation,

$$\begin{aligned}\Delta\Omega_\mu^{(0)}(i,j) &= \Omega_\mu^{(0)}(i) - \Omega_\mu^{(0)}(j) \\ &= -\frac{1}{24\pi^2}\varepsilon_{\mu\nu\varrho\sigma}\,\mathrm{Tr}[v_{ij}^{-1}\partial_\nu v_{ij}v_{ij}^{-1}\partial_\varrho v_{ij}v_{ij}^{-1}\partial_\sigma v_{ij}] \\ &\quad -\frac{1}{8\pi^2}\varepsilon_{\mu\nu\varrho\sigma}\partial_\nu\,\mathrm{Tr}[\partial_\varrho v_{ij}v_{ij}^{-1}A_\sigma^i]\,, \end{aligned} \tag{5}$$

where v_{ij} relates the gauges i and j,

$$A_\mu^j = v_{ij}^{-1}(A_\mu^i + \partial_\mu)v_{ij}\,. \tag{6}$$

$\Delta\Omega_\mu^{(0)}(i,j)$ can again be written as a divergence [3],

$$\Delta\Omega_\mu^{(0)}(i,j) = \partial_\nu\Omega_{\mu\nu}^{(1)}(i,j)\,, \tag{7}$$

where $\Omega_{\mu\nu}^{(1)}(i,j)$ is the 1-cochain given by

$$\begin{aligned}\Omega_{\mu\nu}^{(1)}(i,j) &= -\frac{1}{8\pi^2}(\alpha - \sin\alpha\cos\alpha)\varepsilon_{\mu\nu\varrho\sigma}\mathbf{e}_\alpha\cdot(\partial_\varrho\mathbf{e}_\alpha\times\partial_\sigma\mathbf{e}_\alpha) \\ &\quad -\frac{1}{8\pi^2}\varepsilon_{\mu\nu\varrho\sigma}\,\mathrm{Tr}[\partial_\varrho v_{ij}v_{ij}^{-1}A_\sigma^i]\,,\end{aligned} \tag{8}$$

and

$$v_{ij} = \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}) = \cos\alpha + i\sin\alpha\,\mathbf{e}_\alpha\cdot\boldsymbol{\tau}\,. \tag{9}$$

The expression for the 1-cochain has been extended to any semi-simple and compact Lie group in [4].

II. 2- and 3-Cochains

It is known that the descent (from the 0- to the 1-cochain, cf. Fig. 1a and b) continues, and we shall turn to the higher-order cochains now.

The gauge variation of $\Omega_{\mu\nu}^{(1)}(i,j)$ is given by the coboundary operation

$$\begin{aligned}\Delta\Omega_{\mu\nu}^{(1)}(i,j,k) &= \Omega_{\mu\nu}^{(1)}(i,j) - \Omega_{\mu\nu}^{(1)}(i,k) + \Omega_{\mu\nu}^{(1)}(j,k) \\ &= -\frac{1}{8\pi^2}\varepsilon_{\mu\nu\varrho\sigma}[(\alpha - \sin\alpha\cos\alpha)\mathbf{e}_\alpha\cdot(\partial_\varrho\mathbf{e}_\alpha\times\partial_\sigma\mathbf{e}_\alpha) \\ &\quad + (\beta - \sin\beta\cos\beta)\mathbf{e}_\beta\cdot(\partial_\varrho\mathbf{e}_\beta\times\partial_\sigma\mathbf{e}_\beta) \\ &\quad - (\gamma - \sin\gamma\cos\gamma)\mathbf{e}_\gamma\cdot(\partial_\varrho\mathbf{e}_\gamma\times\partial_\sigma\mathbf{e}_\gamma)] \\ &\quad -\frac{1}{4\pi^2}\varepsilon_{\mu\nu\varrho\sigma}[(\partial_\varrho\alpha\mathbf{e}_\alpha + \sin\alpha\cos\alpha\,\partial_\varrho\mathbf{e}_\alpha + \sin^2\alpha\mathbf{e}_\alpha\times\partial_\varrho\mathbf{e}_\alpha) \\ &\quad \cdot(\partial_\sigma\beta\mathbf{e}_\beta + \sin\beta\cos\beta\,\partial_\sigma\mathbf{e}_\beta - \sin^2\beta\mathbf{e}_\beta\times\partial_\sigma\mathbf{e}_\beta)]\,.\end{aligned} \tag{10}$$

Chain of descent | Geometry of Gauges

$$P = \partial_\mu \Omega_\mu^{(o)}(i)$$

$$\Delta\Omega_\mu^{(o)}(i,j) = \partial_\nu \Omega_{\mu\nu}^{(1)}(i,j)$$

$$\Delta\Omega_{\mu\nu}^{(1)}(i,j,k) = \partial_\rho \Omega_{\mu\nu\rho}^{(2)}(i,j,k)$$

$$\Delta\Omega_{\mu\nu\rho}^{(2)}(i,j,k,l) = \partial_\sigma \Omega_{\mu\nu\rho\sigma}^{(3)}(i,j,k,l)$$

$$\Delta\Omega_{\mu\nu\rho\sigma}^{(3)}(i,j,k,l,m) = \varepsilon_{\mu\nu\rho\sigma}\, n \,, \quad n \in \mathbb{Z}$$

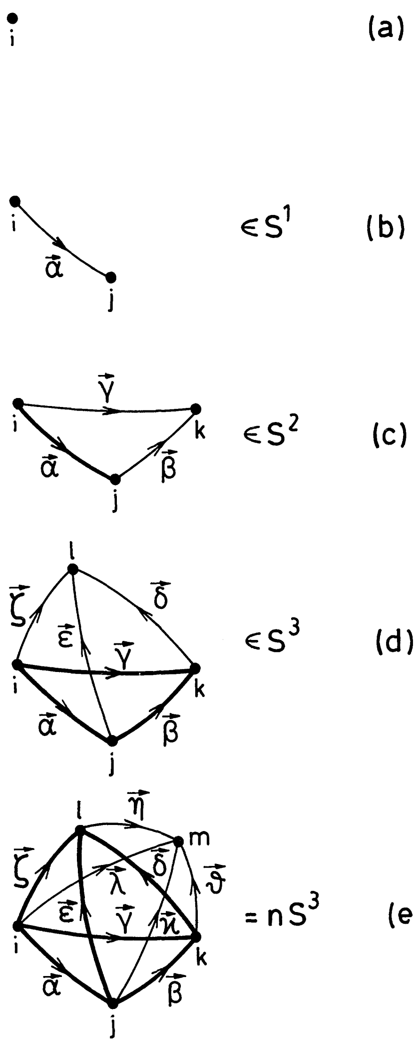

Fig. 1. Pictorial view of the cochain reduction from the Chern-Pontryagin density down to the "local winding number" n

In deriving (10) we have made use of the cocycle condition

$$v_{ij} v_{jk} = v_{ik} , \tag{11}$$

and written

$$v_{ij} = \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}), \qquad v_{jk} = \exp(i\boldsymbol{\beta}\cdot\boldsymbol{\tau}), \qquad v_{ik} = \exp(i\boldsymbol{\gamma}\cdot\boldsymbol{\tau}). \tag{12}$$

The cocycle condition (11) defines a spherical triangle by

$$\begin{aligned}\cos\gamma &= \cos\alpha\cos\beta - \sin\alpha\sin\beta\mathbf{e}_\alpha\cdot\mathbf{e}_\beta\,,\\ \sin\gamma\mathbf{e}_\gamma &= \sin\alpha\cos\beta\mathbf{e}_\alpha + \cos\alpha\sin\beta\mathbf{e}_\beta - \sin\alpha\sin\beta\mathbf{e}_\alpha\times\mathbf{e}_\beta\,,\end{aligned} \tag{13}$$

as indicated in Fig. 1c. $\Delta\Omega^{(1)}_{\mu\nu}(i,j,k)$ is again a total divergence,

$$\Delta\Omega^{(1)}_{\mu\nu}(i,j,k) = \partial_\varrho\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)\,, \tag{14}$$

where $\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)$ is the 2-cochain.

We find the expression [5]

$$\begin{aligned}&\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)\\ &= -\frac{1}{8\pi^2}\varepsilon_{\mu\nu\varrho\sigma}(1+2\cos\alpha\cos\beta\cos\gamma-\cos^2\alpha-\cos^2\beta-\cos^2\gamma)^{-1}\\ &\quad\cdot\{(\boldsymbol{\alpha}+\boldsymbol{\beta}-\boldsymbol{\gamma})\cdot(\sin\alpha\mathbf{e}_\alpha)[\partial_\sigma(\sin\beta\mathbf{e}_\beta)\cdot(\sin\gamma\mathbf{e}_\gamma)-\sin\beta\mathbf{e}_\beta\cdot\partial_\sigma(\sin\gamma\mathbf{e}_\gamma)]\\ &\quad+(\boldsymbol{\alpha}+\boldsymbol{\beta}-\boldsymbol{\gamma})\cdot(\sin\beta\mathbf{e}_\beta)[\partial_\sigma(\sin\gamma\mathbf{e}_\gamma)\cdot(\sin\alpha\mathbf{e}_\alpha)-\sin\gamma\mathbf{e}_\gamma\cdot\partial_\sigma(\sin\alpha\mathbf{e}_\alpha)]\\ &\quad+(\boldsymbol{\alpha}+\boldsymbol{\beta}-\boldsymbol{\gamma})\cdot(\sin\gamma\mathbf{e}_\gamma)[\partial_\sigma(\sin\alpha\mathbf{e}_\alpha)\cdot(\sin\beta\mathbf{e}_\beta)-\sin\alpha\mathbf{e}_\alpha\cdot\partial_\sigma(\sin\beta\mathbf{e}_\beta)]\}\,.\end{aligned} \tag{15}$$

The derivation of (15) is quite tedious and relegated to the appendix. It can be shown that for infinitesimal gauge transformations (15) reduces to the form given in [1].

The gauge variation of $\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)$ combines 4 spherical triangles to form a spherical tetrahedron as indicated in Fig. 1d. I.e.

$$\begin{aligned}\Delta\Omega^{(2)}_{\mu\nu\varrho}(i,j,k,l) &= \Omega^{(2)}_{\mu\nu\varrho}(i,j,k) - \Omega^{(2)}_{\mu\nu\varrho}(i,j,l)\\ &\quad+\Omega^{(2)}_{\mu\nu\varrho}(i,k,l) - \Omega^{(2)}_{\mu\nu\varrho}(j,k,l)\,.\end{aligned} \tag{16}$$

We show in the appendix that (16) can be written in the form

$$\Delta\Omega^{(2)}_{\mu\nu\varrho}(i,j,k,l) = \frac{1}{4\pi^2}\varepsilon_{\mu\nu\varrho\sigma}(\alpha\partial_\sigma A+\beta\partial_\sigma B+\gamma\partial_\sigma\Gamma+\delta\partial_\sigma\Delta+\varepsilon\partial_\sigma E+\zeta\partial_\sigma Z)\,, \tag{17}$$

where A, B, Γ, Δ, E, Z are the angles between two spherical triangles intersecting along the hinges $\alpha,\beta,\gamma,\delta,\varepsilon,\zeta$ (for the explicit expressions see the appendix).

We recognize that the term in brackets on the right-hand side of Eq. (17) is Schläfli's differential form [6] for the volume $V(i,j,k,l)$ of the spherical tetrahedron of Fig. 1d, i.e.

$$\tfrac{1}{2}(\alpha\partial_\sigma A+\beta\partial_\sigma B+\gamma\partial_\sigma\Gamma+\delta\partial_\sigma\Delta+\varepsilon\partial_\sigma E+\zeta\partial_\sigma Z) = \partial_\sigma V(i,j,k,l)\,. \tag{18}$$

This allows us to give an explicit expression for the 3-cochain $\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l)$ defined

$$\Delta\Omega^{(2)}_{\mu\nu\varrho}(i,j,k,l) = \partial_\sigma\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l)\,. \tag{19}$$

That is

$$\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l) = \frac{1}{2\pi^2}\varepsilon_{\mu\nu\varrho\sigma}V(i,j,k,l)\,. \tag{20}$$

The volume $V(i,j,k,l)$ can be constructed explicitly from the angles $A, B, \Gamma, \Delta, E, Z$ following [7].

The gauge variation of $\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l)$ combines 5 spherical tetrahedra (see Fig. 1e),

$$\begin{aligned}
&\Delta\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l,m)\\
&=\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l)-\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,m)\\
&\quad+\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,l,m)-\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,k,l,m)+\Omega^{(3)}_{\mu\nu\varrho\sigma}(j,k,l,m)\\
&=\frac{1}{2\pi^2}\varepsilon_{\mu\nu\varrho\sigma}[V(i,j,k,l)-V(i,j,k,m)\\
&\quad+V(i,j,l,m)-V(i,k,l,m)+V(j,k,l,m)]\,,
\end{aligned}\tag{21}$$

which wind around S^3, the group space of SU(2). The volume of S^3 is $2\pi^2$, so that we can write

$$\Delta\Omega^{(3)}_{\mu\nu\varrho\sigma}(i,j,k,l,m)=\varepsilon_{\mu\nu\varrho\sigma}n\,,\tag{22}$$

where

$$n\in\mathbb{Z}\,.\tag{23}$$

The latter is a consequence of the fact that the 5 spherical tetrahedra together are compact and so cover S^3.

III. Discussion

The result, that the 3-cochain is given by the volume of the spherical tetrahedron $V(i,j,k,l)$, is not really surprising. E.g. in 2-dimensional U(1) gauge theory the corresponding 1-cochain is a segment of S^1.

As will be discussed in a subsequent paper [2], Eq. (22) allows us to derive a local, fully algebraic expression for the topological charge in SU(2) and SU(3) gauge theory.

Appendix

We shall first derive Eq. (15). Noticing that γ in Eq. (10) can be expressed in terms of $\boldsymbol{\alpha}, \boldsymbol{\beta}$ by using the cocycle condition (13), the most general ansatz for the tensor structure of $\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)$ is

$$\begin{aligned}
\Omega^{(2)}_{\mu\nu\varrho}(i,j,k)=&-\frac{1}{8\pi^2}\varepsilon_{\mu\nu\varrho\sigma}[f_1\partial_\sigma\alpha+f_2\partial_\sigma\beta+f_3(\partial_\sigma\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)\\
&+f_4(\mathbf{e}_\alpha\cdot\partial_\sigma\mathbf{e}_\beta)+f_5\partial_\sigma\mathbf{e}_\alpha\cdot(\mathbf{e}_\alpha\times\mathbf{e}_\beta)+f_6\partial_\sigma\mathbf{e}_\beta\cdot(\mathbf{e}_\alpha\times\mathbf{e}_\beta)]
\end{aligned}\tag{A.1}$$

with

$$f_i\equiv f_i(\alpha,\beta,\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)\,.\tag{A.2}$$

Equation (14) is then equivalent to the following set of coupled partial differential equations:

$$\frac{\partial f_2}{\partial \alpha} - \frac{\partial f_1}{\partial \beta} = 2\mathbf{e}_\alpha \cdot \mathbf{e}_\beta - 2\sin\alpha\sin\beta\frac{\gamma - \sin\gamma\cos\gamma}{\sin^2\gamma}[1-(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)^2],$$

$$\frac{\partial f_4}{\partial \alpha} - \frac{\partial f_1}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = 2\sin\beta\cos\beta\frac{1-\gamma\cot\gamma}{\sin^2\gamma} + 2\cos\alpha\sin\beta\frac{\gamma-\sin\gamma\cos\gamma}{\sin^3\gamma},$$

$$\frac{\partial f_3}{\partial \beta} - \frac{\partial f_2}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = -2\sin\alpha\cos\alpha\frac{1-\gamma\cot\gamma}{\sin^2\gamma} - 2\sin\alpha\cos\beta\frac{\gamma-\sin\gamma\cos\gamma}{\sin^3\gamma},$$

$$\frac{\partial f_3}{\partial \alpha} - \frac{\partial f_1}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = 0, \quad \frac{\partial f_4}{\partial \beta} - \frac{\partial f_2}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = 0,$$

$$\frac{\partial(f_4 - f_3)}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = 2\sin^2\alpha\sin^2\beta\frac{1-\gamma\cot\gamma}{\sin^2\gamma}, f_4 - f_3 = 2\sin\alpha\sin\beta\frac{\gamma}{\sin\gamma},$$

$$\frac{\partial f_5}{\partial \alpha} = 2\sin\alpha\sin\beta\frac{\gamma-\sin\gamma\cos\gamma}{\sin^3\gamma}, \ \frac{\partial f_5}{\partial \beta} = 2\sin^2\alpha\frac{1-\gamma\cot\gamma}{\sin^2\gamma},$$

$$\frac{\partial f_6}{\partial \alpha} = -2\sin^2\beta\frac{1-\gamma\cot\gamma}{\sin^2\gamma}, \ \frac{\partial f_6}{\partial \beta} = -2\sin\alpha\sin\beta\frac{\gamma-\sin\gamma\cos\gamma}{\sin^3\gamma},$$

$$f_5 + (\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)\frac{\partial f_5}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} + \frac{\partial f_6}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = 2\sin^2\alpha\sin\beta\cos\beta\frac{1-\gamma\cot\gamma}{\sin^2\gamma},$$

$$f_6 + (\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)\frac{\partial f_6}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} + \frac{\partial f_5}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} = -2\sin\alpha\cos\alpha\sin^2\beta\frac{1-\gamma\cot\gamma}{\sin^2\gamma},$$

$$[1-(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)^2]\frac{\partial f_5}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} - 2(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)f_5 = 2\alpha - 2\sin\alpha\cos\alpha\frac{1-\gamma\cot\gamma}{\sin^2\gamma} - 2\sin\alpha\cos\beta\frac{\gamma-\sin\gamma\cos\gamma}{\sin^3\gamma},$$

$$[1-(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)^2]\frac{\partial f_6}{\partial(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)} - 2(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)f_6 = -2\beta + 2\sin\beta\cos\beta\frac{1-\gamma\cot\gamma}{\sin^2\gamma} + 2\cos\alpha\sin\beta\frac{\gamma-\sin\gamma\cos\gamma}{\sin^3\gamma}, \quad \text{(A.3)}$$

which can be solved giving

$$\begin{aligned} f_1 &= (\boldsymbol{\alpha}-\boldsymbol{\gamma})\cdot\mathbf{e}_\alpha, \\ f_2 &= -(\boldsymbol{\beta}-\boldsymbol{\gamma})\cdot\mathbf{e}_\beta, \\ f_3 &= -f_4 = -\gamma\frac{\sin\alpha\sin\beta}{\sin\gamma}, \\ f_5 &= 2\frac{(\boldsymbol{\alpha}+\boldsymbol{\beta}-\boldsymbol{\gamma})\cdot\mathbf{e}_\beta}{1-(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)^2}, \\ f_6 &= -2\frac{(\boldsymbol{\alpha}+\boldsymbol{\beta}-\boldsymbol{\gamma})\cdot\mathbf{e}_\alpha}{1-(\mathbf{e}_\alpha\cdot\mathbf{e}_\beta)^2}. \end{aligned} \quad \text{(A.4)}$$

Inserting (A.4) into (A.1) gives after a straightforward calculation Eq. (15).

We shall prove now Eq. (17). The angle A is given by (cf. Fig. 1d)

$$\tan A = -\frac{\mathbf{e}_\alpha \cdot (\mathbf{e}_\beta \times \mathbf{e}_\varepsilon)}{(\mathbf{e}_\alpha \times \mathbf{e}_\beta) \cdot (\mathbf{e}_\alpha \times \mathbf{e}_\varepsilon)}. \tag{A.5}$$

The other angles $B, \Gamma, \ldots$ follow from (A.5) by permutation. From (A.5) we derive

$$\begin{aligned}\partial_\sigma A = &[1-(\mathbf{e}_\alpha \cdot \mathbf{e}_\beta)^2]^{-1}[\mathbf{e}_\alpha \cdot \mathbf{e}_\beta \mathbf{e}_\beta \cdot (\mathbf{e}_\alpha \times \partial_\sigma \mathbf{e}_\alpha) + \mathbf{e}_\alpha \cdot (\mathbf{e}_\beta \times \partial_\sigma \mathbf{e}_\beta)] \\ &-[1-(\mathbf{e}_\alpha \cdot \mathbf{e}_\varepsilon)^2]^{-1}[\mathbf{e}_\alpha \cdot \mathbf{e}_\varepsilon \mathbf{e}_\varepsilon \cdot (\mathbf{e}_\alpha \times \partial_\sigma \mathbf{e}_\alpha) + \mathbf{e}_\alpha \cdot (\mathbf{e}_\varepsilon \times \partial_\sigma \mathbf{e}_\varepsilon)].\end{aligned} \tag{A.6}$$

By summing over all terms on the right-hand side of Eq. (17) we obtain (16) expressed in terms of the (non-symmetric) expression (A.1).

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