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Erratum

The Spectral Class of the Quantum-Mechanical Oscillator

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B. Levitan (Moscow State Univ.) has kindly pointed out two places at which the proofs are inadequate. The first occurs on p. 481 where it is stated that the contribution to $\int e_n f_n^0 \Delta q \, dx$ from $|x| \geqq n^{-1/6-}$ is rapidly vanishing. The estimates advanced do not support this, but B. Levitan says that $\|e_n f_n^0\|_2 \leqq n^{-1/2+}$ $(n \uparrow \infty)$, which suffices in view of the rapid vanishing of Δq. The proof is reported to be complicated. The second point occurs on p. 482 where the vanishing of the relative trace $H = \int [p(t,x,x) - p^0(t,x,x)]dx$ is said to prove the vanishing of the relative KDV invariants J_n: $n \geqq 0$ via the expansion $H = (4\pi t)^{-1/2}(J_0 t + J_1 t^2 + \ldots)$ $(t \downarrow 0)$. The idea is this. Let $Q = \int_0^t q[x(s)]ds$, in which $x(t)$: $t \geqq 0$ is the standard Brownian motion with infinitesimal operator $\partial^2/\partial x^2$. Then

$$\sqrt{4\pi t}H = \int E_x[e^{-Q} - e^{-Q^0} | x(t) = x]dx = \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \int E_x[(\Delta Q)^n e^{-Q^0} | x(t) = x]dx\,.$$

The expression is exact, producing an error $\leqq$ constant $\times\, t^n$ upon breaking it off after $n-1$ terms. The individual terms are now developed in powers of t. The computation is routine, the rapid vanishing of Δq providing the necessary domination to the formal manipulations.

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