Analysis, Manifolds and Physics

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Contemporary physical mathematics has much to offer the physicist as it provides well-defined concepts and techniques for the study of physical systems, helps in the very formulation of the laws of physical systems and brings a better understanding to physics. However, it is rare that the physicist has the time and motivation to learn the many facets of mathematics and use them in the analysis of physical phenomena.

The authors have, therefore, written this book to make contemporary mathematics more accessible and more useful to the physicist. Intended as a text for an advanced physical mathematics course, it brings together several branches of contemporary mathematics of interest to physics. Starting from simple and basic concepts, it leads to rigorous statements of recent results in analysis and differential geometry. The scope of the coverage is unusually wide and the material treated with more rigor than is customary in a mathematical physics text, because only then can the results be used correctly and fruitfully.

The first two chapters comprise a review of fundamental notions of algebra, topology, integration and analysis, and a description of the essentials of differential calculus and calculus of variations on Banach spaces. They have been included to spare the reader the task of looking up in several specialized books the definitions and basic theorems used in subsequent chapters. The third chapter treats differentiable manifolds, tangent bundles, groups of transformations, and Lie groups. Chapter IV deals with exterior differential forms, integration, and exterior differential systems. Riemannian structures are examined in Chapter V and Chapter VI is concerned with distributions. The final chapter comprises some selected topics in the theory of infinite dimensional manifolds including symplectic structures and hamiltonian systems, the theory of degree and the Leray-Schauder theory, the Morse theory, and integration on function spaces. Several problems of current interest are worked out at the end of each chapter which show how the concepts and the theorems introduced in the text can be used in physics.

Relatively simple but rigorous, with a wide range of applications, this book together with its references gives a pedagogical presentation of several fundamental trends in contemporary mathematics relevant to physics.
