Commun. math. Phys. 48, 89—96 (1976)

Communications in Mathematical Physics

Pure Massless Electrodynamics in Veltman's Gauge

A. Rouet

Centre de Physique Théorique, C.N.R.S. Marseille, F-13274 Marseille Cedex 2, France

Abstract. To show how the method developed by C. Becchi, R. Stora and the present author to prove Slavnov's identities in gauge theories works in Abelian cases including a nonlinear gauge without any discrete symmetry, a specific example is worked out, exhibiting the details of the technical procedure.

I. Introduction

Recently Becchi, Stora and the present author have developed a method for proving Slavnov identities in gauge theories [1] which relies on very general theorems of renormalization, more precisely a quantum version of the Schwinger action principle; the proof consists in an extensive use of consistency conditions. However, the proof given was restricted to massive semi-simple cases with linear gauge; a specific Abelian case was also studied [2], but we used a discrete symmetry to simplify the problem. Generalizations of this method to the massless case were studied by Lowenstein [3] and Becchi [4] (pure Yang-Mills theory) and Clark and the present author [5] (Georgi-Glashow model).

The generalization to cases including Abelian subgroups with linear gauges was given by Becchi, Blasi and Collina [6]; there the couplings of the Faddeev-Popov ghost related to the Abelian subgroup are assumed to be superrenormalizable. Here we give an example of what has to be done in a case including a nonlinear gauge, when no discrete symmetry simplifies the problem, and when the couplings of the Abelian Faddeev-Popov ghosts are not superrenormalizable. The example given is pure massless electrodynamics in a Veltman type gauge [7].

Section II is devoted to the definition of the model in the tree approximation; in Section III the quantum action principle is recalled; Section IV is devoted to the proof of the Slavnov identity at any order.

II. Definition of the Model in the Tree Approximation

Let us consider pure massless electrodynamics in the nonlinear gauge

$$\mathcal{G} = \partial_\mu a^\mu + \varrho a_\mu a^\mu ,$$

as proposed by Veltman [7]; c and $\bar{c}$ will denote the Faddeev-Popov ghosts.

The tree approximation Lagrangian reads:

$$\begin{aligned}\mathscr{L} = & -4^{-1}(\partial_\mu a_\nu - \partial_\nu a_\mu)(\partial^\mu a^\nu - \partial^\nu a^\mu) - (2\alpha)^{-1}(\partial_\mu a^\mu + \varrho a_\mu a^\mu)^2 \\ & -\alpha^{-1}(\bar{c}\,\Box c + 2\varrho\,\partial_\mu \bar{c} a^\mu c) \\ & + j_\mu a^\mu + \bar{\xi} c + \xi \bar{c} + \eta \mathscr{G} + \zeta \mathscr{H}\end{aligned} \tag{1}$$

with

$$\mathscr{H} = \Box \bar{c} + 2\varrho a_\mu \partial^\mu \bar{c},$$

η and ζ are external fields which are respectively assigned dimension 2 and 1; η carries no Faddeev-Popov charge, ζ carries the charge of the c.

This Lagrangian is invariant under the Slavnov transformation [1]:

$$\begin{aligned} & a_\mu \to \lambda \partial_\mu \bar{c} \\ & c \to \lambda \mathscr{G} \equiv \lambda(\partial_\mu a^\mu + \varrho a_\mu a^\mu) \\ & \bar{c} \to 0 \\ & \mathscr{G} \to \lambda \mathscr{H} = \lambda(\Box \bar{c} + 2\varrho a_\mu \partial^\mu \bar{c})\end{aligned} \tag{2}$$

where λ is an infinitesimal parameter, independent of x, which anticommutes with $c, \bar{c}, \xi, \bar{\xi}, \zeta$ and commutes with a_μ, j_μ, η.

To this invariance corresponds the Slavnov identity:

$$\begin{aligned}\mathscr{S} Z^c(j_\mu, \bar{\xi}, \xi, \eta, \zeta) = \int d^4x\, [& j_\mu \partial^\mu \delta/\delta\xi - \bar{\xi}\,\delta/\delta\eta \\ & + \eta\,\delta/\delta\zeta](x) Z^c(j_\mu, \bar{\xi}, \xi, \eta, \zeta) = 0\end{aligned} \tag{3}$$

where $Z^c(j_\mu, \bar{\xi}, \xi, \eta, \zeta)$ denotes the generating functional of connected Green functions.

Equation (3) can be written for the generating functional of the vertex functions $\Gamma(a^\mu, c, \bar{c}, \eta, \zeta)$, related to $Z^c(j_\mu, \bar{\xi}, \xi, \eta, \zeta)$ through a Legendre transformation:

$$\begin{aligned}\Gamma(a_\mu, c, \bar{c}, \eta, \zeta) = & \, Z^c(j_\mu, \bar{\xi}, \xi, \eta, \zeta) \\ & - \int d^4x [j_\mu a^\mu + \bar{\xi} c + \xi \bar{c}](x) \left| \begin{array}{l} \delta Z^c/\delta j_\mu = a^\mu \\ \delta Z^c/\delta \bar{\xi} = c \\ \delta Z^c/\delta \xi = \bar{c}\,, \end{array}\right.\end{aligned} \tag{4}$$

$$\begin{aligned}\mathscr{S}(\Gamma) \equiv \int d^4x [& \bar{c}(x)\partial_\mu(\delta/\delta a_\mu)(x)\Gamma + (\delta/\delta c)(x)\Gamma(\delta/\delta\eta)(x)\Gamma \\ & + \eta(x)(\delta/\delta\zeta)(x)\Gamma] = 0\,.\end{aligned} \tag{5}$$

III. The Quantum Action Principle

Let us first make precise the notation:

$\mathscr{L}(\phi)$ denotes the effective Lagrangian;

$Z^c(j, \eta)$ is the generating functional of the connected Green's functions depending on the sources j and the external fields η; the corresponding generating functional of the vertex functions is denoted $\Gamma(\phi, \eta)$;

$(\Delta Z^c)(j, \eta)$ denotes the generating functional of the connected Green's functions with the insertion Δ, where Δ is some integrated normal product of fields;

in the same way $(\Delta(x)Z^c)(j,\eta)$ denotes the generating functional of connected Green's functions with the insertion $\Delta(x)$, where $\Delta(x)$ is some normal product of fields at the point x; then the meaning of $(\Delta\Gamma)(\phi,\eta)$ and $(\Delta(x)\Gamma)(\phi,\eta)$ is unambigous;

at last $\mathcal{O}(\hbar^n, d)$ denotes some local operator of dimension d which is at least of order $\hbar^n$.

We are now in a position to write the two statements of the action principle. The first one, due to Lowenstein [8], concerns the derivation with respect to an external field $\eta(x)$ of dimension d_η:

$$\begin{aligned}&(\delta/\delta\eta)(x)Z^c(j,\eta) = [\Delta_\eta(x)Z^c](j,\eta)\\&\Delta_\eta(x) = (\delta/\delta\eta)(x)\int \mathcal{L}(\varphi(y))dy + \mathcal{O}(\hbar, 4-d_\eta)(x)\,.\end{aligned} \tag{6}$$

For a constant parameter λ, this reduces to:

$$\begin{aligned}&(\delta/\delta\lambda)Z^c(j,\eta) = \Delta_\lambda Z^c(j,\eta)\\&\Delta_\lambda = (\delta/\delta\lambda)\int \mathcal{L}(\varphi(y))dy + \mathcal{O}(\hbar, 4)\,.\end{aligned} \tag{7}$$

The second statement, due to Lam [9] concerns the infinitesimal variation of a quantized field:

Let us consider an infinitesimal variation of the quantized field

$$\delta\varphi(y) = M(\varphi, D\varphi)(x)\delta\omega(y) \tag{8}$$

where $M(\phi, D\phi)(x)$ is a polynomial of normal products in the fields $\phi(x)$ and in its derivatives $D\phi(x)$; to $M(\phi, D\phi)$ is assigned dimension d_M greater than or equal to the naive dimension of M; $d\phi$ denotes the canonical dimension of the field ϕ. The action principle reads:

$$\begin{aligned}&\int dy\{[\delta\mathcal{L}(\varphi(y))/\delta\varphi(x))Z^c](j,\eta) + j(y)\delta(y-x)\,[M(\varphi, D\varphi)(x)Z^c](j,\eta)\}\\&= [\mathcal{O}(\hbar M, 4+d_M-d_\varphi)(x)Z^c](j,\eta)\,.\end{aligned} \tag{9}$$

As a final remark, let us write the following useful identities:

$$\begin{aligned}&\Gamma(\varphi,\eta) = \int \mathcal{L}_{\text{eff}}(\varphi(x))dx + \mathcal{O}(\hbar, 4)\\&[\Delta(x)\Gamma](\varphi,\eta) = \Delta(x) + \mathcal{O}(\hbar\Delta, d_\Delta)\end{aligned} \tag{10}$$

where d_Δ is the dimension of the normal products of fields Δ.

IV. Radiative Corrections: Proof of the Slavnov Identity

For the sake of definiteness, let us say we are using the subtraction procedure for massless theory given by Lowenstein and Zimmermann [10]; in fact, nothing is changed if we use another renormalization prescription valid for massless cases [11] since the validity of the action principle in the form we wrote in the last section does not depend on this choice.

To simplify the notation, we shall from now on forget the inessential indices, subscripts, etc.; for instance, we shall denote

$$Z^c \equiv Z^c(j_\mu, \bar{\xi}, \xi, \eta, \zeta)\,.$$

Then, due to the action principle Equation (9), the Slavnov identity Equation (5) considered at any order in $\hbar$ becomes:

$$\mathscr{S}(\Gamma)=\Delta\Gamma \tag{11}$$

where Δ is an insertion of order $\hbar$ and strict dimension 5 (no insertion of dimension strictly less than 5 does appear because of the zero mass subtraction procedure [10, 3, 4]). We want to prove that there exists $\hat{\mathscr{L}}$ of dimension 4 such that:

$$\Delta=\mathscr{S}\hat{\mathscr{L}}+\mathcal{O}(\hbar\Delta). \tag{12}$$

Then it is possible to change the Lagrangian at any order $\hbar$ in such a way that we absorb the anomaly Δ at any order; in fact, if to the Lagrangian $\mathscr{L}$ corresponds (Eq. 11), to the Lagrangian $\mathscr{L}+\hat{\mathscr{L}}'$ corresponds:

$$\mathscr{S}(\Gamma)=\Delta\Gamma+\mathscr{S}(\hat{\mathscr{L}}')+\mathcal{O}(\hbar\hat{\mathscr{L}}'). \tag{13}$$

Because of Equation (12) this is soluble for $\hat{\mathscr{L}}'$ computed as a formal power series in $\hbar$ in such a way that we get

$$\mathscr{S}(\Gamma)=0 \tag{14}$$

at any order in $\hbar$.

Let us now prove that the consistency conditions imply Equation (12).

Let us first write

$$\Delta=\Delta^{\text{ext}}+\Delta^{\text{int}} \tag{15}$$

where Δ^{int} does not depend on the external fields η, ζ. The general form of Δ^{ext} is:

$$\begin{aligned}\Delta^{\text{ext}}&=c_0\zeta\bar{c}\partial_\mu\bar{c}a^\mu+c_0'\zeta\bar{c}\Box\bar{c}+c_1\eta\bar{c}a_\mu^2+c_2\eta\bar{c}\partial_\mu a^\mu+c_3\eta\partial_\mu\bar{c}a^\mu+c_4\eta\Box\bar{c}\\&\equiv\zeta\bar{c}\Delta^{\zeta}+\eta\bar{c}\Delta^{\eta_1}+\eta\Delta^{\eta_2}+\eta^2\bar{c}.\end{aligned} \tag{16}$$

Let us first remark that $\zeta\eta\bar{c}$ varies in $\eta^2\bar{c}+\zeta\mathscr{H}\bar{c}$. The second term can be absorbed in $\zeta\bar{c}\Delta^{\zeta}$ so that $\eta^2\bar{c}$ can be absorbed by modifying the lagrangian. Let us, following the general method [1], couple Δ to an external field β and consider the Lagrangian $\mathscr{L}+\beta\Delta$. Due to this modification of the Lagrangian, (Eq. 11) transforms into:

$$\mathscr{S}(\Gamma)=\delta\Gamma/\delta\beta-\beta(\mathscr{S}^{\Delta}(\mathscr{L})+\mathscr{S}(\Delta))+\mathcal{O}(\hbar\Delta). \tag{17}$$

The term $\mathscr{S}^{\Delta}\mathscr{L}$ takes into account the change in the definitions of $\delta/\delta\zeta$, $\delta/\delta\eta$ which appear in $\mathscr{S}$, due to the presence of the couplings $\beta\Delta^{\text{ext}}$.

In other words $\mathscr{L}$ was formally invariant under the Slavnov transformation Equation (2). Because of the definition of the couplings of η and ζ in the Lagrangian Equation (1) this invariance was translated in the Slavnov identity Equation (3). Now we keep the form of the Slavnov operator, but we change the couplings of η and ζ and the Slavnov transformation Equation (2) becomes

$$\begin{aligned}&a_\mu\to\lambda\partial_\mu\bar{c}+\mathcal{O}(\hbar\Delta)\\&c\to\lambda(\mathscr{G}+\beta\bar{c}\Delta^{\eta_1}+\beta\Delta^{\eta_2})+\mathcal{O}(\hbar\Delta)\\&\mathscr{G}\to\lambda(\mathscr{H}+\beta\bar{c}\Delta^{\zeta})+\mathcal{O}(\hbar\Delta).\end{aligned} \tag{18}$$

$\beta\mathscr{S}^{\Delta}\mathscr{L}$ is the variation of the Lagrangian under this new part of the transformation.

Equation (17) yields:

$$\begin{aligned}\mathscr{S}^2(\Gamma)|_{\beta=0} &= -\delta\mathscr{S}(\Gamma)/\delta\beta|_{\beta=0} = \mathscr{S}^\Delta(\mathscr{L}) + \mathscr{S}(\Delta) + \mathcal{O}(\hbar\Delta)|_{\beta=0} \\ &\equiv \Delta' + \mathcal{O}(\hbar\Delta)\end{aligned} \tag{19}$$

which can be written

$$\int (\delta/\delta c)\Gamma(\delta/\delta\zeta)\Gamma = \Delta' + \mathcal{O}(\hbar\Delta) \tag{20}$$

which is a perturbed version of order Δ' of:

$$\int (\delta/\delta c)\Gamma(\delta/\delta\zeta)\Gamma = 0 \tag{21}$$

itself a perturbation of

$$\int (\delta/\delta c)\mathscr{L}(\delta/\delta\zeta)\mathscr{L} = 0\,. \tag{22}$$

The solution of (Eq. 22) is known to be [1]

$$(\delta/\delta c)\mathscr{L} = \kappa(\delta/\delta\zeta)\mathscr{L} \tag{23}$$

where κ denotes from now on an arbitrary coefficient. This proves that

$$(\delta/\delta c)\Gamma = \kappa(\delta/\delta\zeta)\Gamma \tag{24}$$

is the solution of Equation (21) and the solution of Equation (20) is:

$$(\delta/\delta c)\Gamma = \kappa(\delta/\delta\zeta)\Gamma + R^\Delta \tag{25}$$

where R^Δ, which is of order Δ, can be written

$$R^\Delta = (\delta/\delta c)R^\Delta \tag{26}$$

since $(\delta\Gamma/\delta\zeta)$ is of the form $(\delta/\delta c)(\ )$.
Writing this solution Equation (25) in Equation (20) yields:

$$\kappa(\delta/\delta\zeta)\Gamma(\delta/\delta c)R^\Delta = \Delta' + \mathcal{O}(\hbar\Delta)\,. \tag{27}$$

The left-hand side is proportional to $\mathscr{S}^2(R^\Delta)$, from the definition of $\mathscr{S}^2$, and we get the consistency condition:

$$\kappa\mathscr{S}^2(R^\Delta) = \mathscr{S}^\Delta(\mathscr{L}) + \mathscr{S}(\Delta) + \mathcal{O}(\hbar\Delta)\,. \tag{28}$$

To this point we have followed the standard method given in [1]; we now need a specific analysis. From Equation (5) and Equation (16) we know that

$$\mathscr{S}^\Delta(\mathscr{L}) = ((\delta/\delta c)\mathscr{L})\,[\bar{c}\Delta^{\eta_1} + \Delta^{\eta_2}] + \eta\bar{c}\Delta^\zeta \tag{29}$$

with

$$(\delta/\delta c)\mathscr{L} = \mathscr{H} + \mathcal{O}(\hbar)\,. \tag{30}$$

The first term in the right-hand side of Equation (29) has the form

$$\mathscr{S}^2[c(\bar{c}\Delta^{\eta_1} + \Delta^{\eta^2})] + \mathcal{O}(\hbar\Delta)\ ;$$

its contribution to (Eq. 28) can be absorbed in the left-hand side, and (Eq. 28) reduces to:

$$\kappa\mathscr{S}^2(R'^\Delta) = \eta\bar{c}\Delta^\zeta + \mathscr{S}(\Delta) + \mathcal{O}(\hbar\Delta)\,. \tag{31}$$

Let us first look at the part of Equation (31) depending on the external fields:

$$\begin{aligned}\mathscr{S}^2(R)|^{\text{ext}} &= \eta\bar{c}\Delta^{\zeta} + [\eta\bar{c}\Delta^{\zeta} - \eta\bar{c}(2c_1 a_\mu \partial^\mu \bar{c} + c_2 \Box\bar{c})] + \mathcal{O}(\hbar\Delta) \\ &= \eta\bar{c}[2(c_0 - c_1)a_\mu \partial^\mu \bar{c} + (2c_0' - c_2)\Box\bar{c}] + \mathcal{O}(\hbar\Delta)\,. \end{aligned} \tag{32}$$

Equation (32) implies the relations

$$\begin{aligned} &c_0 - c_1 = \varrho\lambda \\ &2c_0' - c_2 = \lambda \end{aligned} \tag{33}$$

in such a way that the right-hand side is $\mathscr{S}^2(\lambda\eta\bar{c}c)$. We can now consider the part of Equation (31) depending only on the internal fields:

$$\mathscr{S}^2(R)|^{\text{int}} = \mathscr{S}(\Delta)|^{\text{int}} + \mathcal{O}(\hbar\Delta)\,. \tag{34}$$

Let us decompose Δ into

$$\begin{aligned} &\Delta^{\flat}: \mathscr{S}(\Delta^{\flat})|^{\text{int}} \neq 0 \\ &\Delta^{\natural}: \mathscr{S}(\Delta^{\natural}) = 0 \\ &\Delta^{\#}: \mathscr{S}(\Delta^{\#})|^{\text{int}} = 0\,; \qquad \mathscr{S}(\Delta^{\#})|^{\text{ext}} \neq 0\,. \end{aligned} \tag{35}$$

From (Eq. 34) we see that $\Delta^{\flat}$ is of the wanted form $\mathscr{S}(\hat{\mathscr{L}}^{\flat})$. Let us look at $\Delta^{\#}$: it consists of a part depending on the external fields such that $\mathscr{S}(\Delta^{\#})|^{\text{ext}} \neq 0$, and a part independent of these external fields which combines with the other to ensure $\mathscr{S}(\Delta^{\#})^{\text{int}} = 0$; since

$$\mathscr{S}(\Delta^{\text{ext}})|^{\text{ext}} = \mathscr{S}(\zeta\bar{c}\Delta^{\zeta} + \eta\bar{c}\Delta^{\eta_1} + \eta\Delta^{\eta_2})|^{\text{ext}} = \mathscr{S}(\eta\bar{c}\Delta^{\eta_1})|^{\text{ext}} \tag{36}$$

the first part is nothing but $\eta\bar{c}\Delta^{\eta_1}$ and

$$\Delta^{\#} = \eta\bar{c}\Delta^{\eta_1} + \Delta^0 \tag{37}$$

where Δ^0 does not depend on the external fields.

$$\mathscr{S}(\Delta^{\#})|^{\text{int}} = 0 = \mathscr{H}\bar{c}\Delta^{\eta_1} + \mathscr{S}\Delta^0 + \mathcal{O}(\hbar\Delta)\,. \tag{38}$$

But we have the relation

$$\mathscr{H}\bar{c}\Delta^{\eta_1} = \mathscr{S}^2(c\bar{c}\Delta^{\eta_1}) + \mathcal{O}(\hbar\Delta) \tag{39}$$

which ensures that either Δ^0 is invariant, and then does not contribute to $\Delta^{\#}$, or is of the wanted form $\mathscr{S}(\hat{\Delta}^0)$; (Eq. 38) reduces to

$$\mathscr{H}\bar{c}\Delta^{\eta_1} + \mathcal{O}(\hbar\Delta) = 0 \tag{40}$$

which implies $\Delta^{\eta_1} = 0$, or – from (Eq. 16) – $c_1 = c_2 = 0$; then Equation (33) implies $\Delta^{\zeta} = (\lambda/2)\mathscr{H}$ so that

$$\begin{aligned}\zeta\bar{c}\Delta^{\zeta} &= (\lambda/2)\zeta\bar{c}\mathscr{H} = \mathscr{S}^2((\lambda/2)\zeta\bar{c}c) + \mathcal{O}(\hbar\Delta) \\ &= \mathscr{S}(\hat{\mathscr{L}}^{\zeta}) + \mathcal{O}(\hbar\Delta)\,. \end{aligned} \tag{41}$$

Going on in our analysis, we are now concerned with the $\Delta^{\natural}$ part. For the piece depending on the external fields we stay with $\eta\Delta^{\eta^2}$ which has to satisfy:

$$\mathscr{S}(\eta\Delta^{\eta_2}) \equiv \mathscr{H}\Delta^{\eta_2} + \mathcal{O}(\hbar\Delta) = 0 \tag{42}$$

This implies that Δ^{η^2} is proportional to $\mathscr{H}$. But $\eta\mathscr{H} = \mathscr{S}^2(\eta c)$ is a variation.

At this stage we have proved the relation

$$\Delta^{\mathrm{ext}} = \mathscr{S}(\hat{\mathscr{L}}^{\mathrm{ext}}) + \mathcal{O}(\hbar\Delta)\,. \tag{43}$$

Following the analysis explained after Equation (12), by changing the Lagrangian at order Δ, we can reduce (Eq. 43) to $\Delta^{\mathrm{ext}} = \mathcal{O}(\hbar\Delta)$. Then $\mathscr{S}^{\Delta}(\mathscr{L}) = \mathcal{O}(\hbar\Delta)$, and Equation (28) reduces to

$$\mathscr{S}(\Delta) = \mathscr{S}^2(R) + \mathcal{O}(\hbar\Delta)\,. \tag{44}$$

We have already proved that the terms $\Delta^{\flat}$ assumed the form $\mathscr{S}(\mathscr{L}^{\flat})$; we have to consider now the terms $\Delta^{\natural}$ which do not depend on the external fields; we can write

$$\Delta^{\natural} = a_0\bar{c}\Delta_1 + a_1\bar{c}\partial_\mu\bar{c}ca^\mu + a_2\bar{c}\partial_\mu\bar{c}\partial^\mu c + a_3\bar{c}\,\Box\bar{c}c \tag{45}$$

where Δ_1 does not depend on $c, \bar{c}$.

Let us write that the part of $\mathscr{S}^2(\Delta^{\natural})$ with three $\bar{c}$ is zero:

$$(a_1 - 2\varrho a_3)\bar{c}\partial_\mu\bar{c}\,\Box\bar{c}a^\mu + a_2\bar{c}\partial_\mu\bar{c}\,\Box\partial^\mu\bar{c} + 2\varrho a_2\bar{c}\partial_\mu\bar{c}\,\Box\bar{c}a^\mu = 0 \tag{46}$$

which yields:

$$\begin{aligned} a_2 &= 0 \\ a_1 &= 2\varrho a_3 \end{aligned} \tag{47}$$

and $\Delta^{\natural}$ reduces to

$$\Delta^{\natural} = a_0\bar{c}\Delta_1 - a_3\bar{c}c\mathscr{H}\,. \tag{48}$$

Let us decompose Δ_1:

$$\begin{aligned} &\Delta_1 = \Delta_1^{\natural} + \Delta_1^{\flat} \\ &\mathscr{S}(\Delta_1^{\natural}) = 0 \\ &\mathscr{S}(\Delta_1^{\flat}) \neq 0 \end{aligned} \tag{49}$$

$(\Delta^{\natural}) = 0$ implies

$$a_0\mathscr{S}(\Delta_1^{\flat}) = a_3\mathscr{G}\mathscr{H} \Rightarrow a_0\Delta_1^{\flat} = (a_3/2)\mathscr{G}\mathscr{G}\,. \tag{50}$$

Since there exists just one possible term for $\Delta_1^{\natural}$, Equation (48) reads:

$$\Delta^{\natural} = a_0\bar{c}(\partial_\mu a_\nu - \partial_\nu a_\mu)(\partial^\mu a^\nu - \partial^\nu a^\mu) + a_3\bar{c}(\tfrac{1}{2}\mathscr{G}^2 + \mathscr{H}c)\,. \tag{51}$$

These two terms cannot be eliminated through the consistency condition, as we have seen; this difficulty is due to the group being Abelian.

To prove these terms do not exist, we shall use an alternative method: let us make an assumption about the couplings of the $\bar{c}$ in the effective Lagrangian, and suppose that there exist just the couplings $c\,\Box\bar{c}$ and $ca_\mu\partial^\mu\bar{c}$, excluding $(\partial_\mu ca_\mu\bar{c} - c\partial_\mu a^\mu\bar{c})$ and $ca_\mu a^\mu\bar{c}$. Then, besides the two terms of Equation (51), we get three possible anomalies which are the Slavnov variations of these new forbidden terms in the Lagrangian, and the Slavnov identity can be written:

$$\begin{aligned} \mathscr{S}(\Gamma) = {} & a_0\bar{c}(\partial_\mu a_\nu - \partial_\nu a_\mu)(\partial^\mu a^\nu - \partial^\nu a^\mu) + a_3\bar{c}(\tfrac{1}{2}\mathscr{G}^2 + \mathscr{H}c) + b_1(\bar{c}\partial_\mu\bar{c}\partial^\mu c + \bar{c}a_\mu\partial^\mu\mathscr{G} \\ & - \bar{c}\,\Box\bar{c}c - \bar{c}\partial_\mu a^\mu\mathscr{G}) + b_2(\tfrac{1}{2}\bar{c}a_\mu a^\mu\mathscr{G} + \bar{c}\partial_\mu\bar{c}a^\mu c) + \mathcal{O}(\hbar\Delta) \equiv \Delta + \mathcal{O}(\hbar\Delta)\,. \end{aligned} \tag{52}$$

Let us test this identity with respect to $(\tilde{\delta}/\delta\bar{c})(0)\tilde{\delta}_x(0)$, where δ_x denotes a product of derivations with respect to the photon fields, and put all the fields to zero. From (Eq. 5) we get:

$$\begin{aligned}&\int d^4x\,[(\tilde{\delta}/\delta\bar{c})(0)(\delta/\delta c)(x)\tilde{\delta}_{x_1}(p)\Gamma]\,[(\delta/\delta\eta)(x)\tilde{\delta}_{x_2}(p)\Gamma]\\&=(\tilde{\delta}/\delta\bar{c})(0)\tilde{\delta}_x(0)\Delta+\mathcal{O}(\hbar\Delta)\end{aligned}\tag{53}$$

where δ_{x_1}, δ_{x_2} is a partition of δ_x.

Because of our hypothesis on the couplings of $\bar{c}$, the first factor in the left-hand side is zero, due to $(\tilde{\delta}/\delta\bar{c})(0)$, and Equation (53) yields:

$$\begin{aligned}\mathcal{O}(\hbar\Delta)&=(\tilde{\delta}/\delta\bar{c})(0)\tilde{\delta}_x(0)\Delta\\&\equiv\tilde{\delta}_x(0)\,\{a_0(\partial_\mu a_\nu-\partial_\nu a_\mu)(\partial^\mu a^\nu-\partial^\nu a^\mu)+a_3(\mathcal{G}^2/2+\mathcal{H}c)\\&\quad+b_1(\partial_\mu\bar{c}\partial^\mu c+a_\mu\partial^\mu\mathcal{G}-\Box\bar{c}c-\partial_\mu a^\mu\mathcal{G})+b_2(\tfrac{1}{2}a_\mu a^\mu\mathcal{G}+\partial_\mu\bar{c}a^\mu c)\end{aligned}\tag{54}$$

taking for δ_x the derivation with respect to two, three and four photons yields four relations which imply

$$a_0=a_1=b_1=b_2=0\tag{55}$$

so that we have proved the Slavnov identity.

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Communicated by K. Symanzik

Received August 12, 1975