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Propagation of Shock Waves in Interacting Higher Spin Wave Equations

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Abstract. The derivation of characteristic surfaces for interacting higher spin wave equations is discussed in a shock wave formalism. Equations describing the propagation of shock waves along the bicharacteristics are established for several interacting systems.

1. Introduction

The programme of constructing Lagrangians to give a consistant field theoretical description of higher spin particles ($s \geqq 1$) was initiated by Fierz and Pauli [1]. The resulting Lagrange equations are then equivalent to the equations of motion of the field together with the constraints necessary to reduce the number of degrees of freedom of the field to the number specified by its spin. This method eliminates the algebraic inconsistencies which may arise when an interaction is introduced and the constraint equations are postulated independently of the equations of motion.

But it was noticed by Johnson and Sudarshan [2] that when the Rarita-Schwinger field for a spin-3/2 particle is coupled minimally to the electromagnetic field, a peculiar anomaly appears: namely, despite the addition of a relativistically invariant interaction term to the free Lagrangian, the equal time commutation relation between fields is not positive definite in all Lorentz frames. Subsequently, Velo and Zwanziger [3–5] showed that anomalies appear in the first quantized versions of the same, and other, interacting systems by demonstrating, for example [3], that the minimally coupled Rarita-Schwinger equation has characteristic surfaces which can be space-like for any non-vanishing value of the Maxwell tensor. Their method involves considering the Cauchy problem and using the definition of characteristic surfaces as those initial surfaces for which it has not a unique solution [6].

In this paper we use the fact that characteristic surfaces are surfaces across which there can exist discontinuities in the highest order derivatives appearing in the wave equation. This method was first used by Stell-

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macher [7] to analyse the propagation of gravitational waves. The method is described in Section 2. It will be seen that, once the basic formulae for the various orders of discontinuity have been established, the calculation of the characteristic surfaces becomes very straightforward. This analysis allows us also to establish the equations which govern the propagation of the discontinuities or shocks along the bicharacteristics. We show that even when the characteristic surface is space-like, the shock waves do in fact propagate. Several examples of the interacting spin-1 Proca field are considered in Section 3, and in Section 4 the linear Rarita-Schwinger equation for a spin-3/2 field minimally coupled to the electromagnetic field is discussed. In all the cases considered, there is a remarkable similarity both in the form of the equations which define the characteristic surfaces, and in the form of the equations describing the propagation of the shock waves.

2. Discontinuity Formulae

Let σ be a smooth hypersurface given in a region of Minkowski space-time. Here and in what follows, by smooth we mean differentiable of class $\mathscr{C}^n$, $n \geqq 3$. Let x^μ, $\mu = 0, 1, 2, 3$, be an inertial coordinate system and $z(x^\mu)$ a real-valued smooth function of x^μ regular in a neighbourhood $\mathscr{U}$ of σ and vanishing on σ. σ divides $\mathscr{U}$ into two regions $\mathscr{U}^+$ and $\mathscr{U}^-$ corresponding to $z \geqq 0$ and $z < 0$ respectively. Define $\xi_\mu = \partial_\mu z$. ξ_μ is nonvanishing in $\mathscr{U}$ and normal to σ.

Consider a function $\phi(x^\mu)$ (with values in a space to be specified in Sections 3 and 4) defined in $\mathscr{U}$ and smooth in the interior of $\mathscr{U}^+$ and $\mathscr{U}^-$. We wish to give general expressions for the possible discontinuities across σ in the first, second and third order derivatives of ϕ with respect to x^μ. We consider two cases. In the first case we suppose that ϕ is continuous across σ but has a discontinuity in a first and possibly higher derivatives. In the second case we suppose that ϕ is continuously differentiable across σ but has a discontinuity in a second and possibly higher derivatives.

Let ϕ be denoted by $\phi^\pm$ in the regions $\mathscr{U}^\pm$. By extending $\phi^\pm$ smoothly into $\mathscr{U}^\mp$, the discontinuity $[\phi] \equiv \phi^+ - \phi^-$ may be defined as a smooth function in $\mathscr{U}$. There exist therefore (uncountably many) smooth functions $\tilde{k}, \tilde{f}, \tilde{g}$ defined in $\mathscr{U}$ such that

$$[\phi] = z\tilde{k} + \frac{z^2}{2!}\tilde{f} + \frac{z^3}{3!}\tilde{g}\,. \tag{2.1}$$

We shall later in Sections 3 and 4 place restrictions on these functions. For a function $\tilde{k}$ defined in $\mathscr{U}$ let $\tilde{k}_{|\sigma}$ be its restriction to σ. Define k, f, g by

$$k = \tilde{k}_{|\sigma}\,, \quad f = \tilde{f}_{|\sigma}\,, \quad g = \tilde{g}_{|\sigma}\,. \tag{2.2}$$

Suppose now that $k \not\equiv 0$. From (2.1) we can calculate the discontinuities across σ in the first and second derivatives of ϕ in an arbitrary direction:

$$[\partial_\alpha \phi] \equiv (\partial_\alpha [\phi])_{|\sigma} = \xi_\alpha k\,, \tag{2.3}$$

$$[\partial_\beta \partial_\alpha \phi] \equiv (\partial_\beta \partial_\alpha [\phi])_{|\sigma} = \xi_\beta \xi_\alpha f + \xi_{(\alpha} \partial_{\beta)} \tilde{k} + \partial_\beta \xi_\alpha k\,. \tag{2.4}$$

Suppose that $k \equiv 0$. From (2.1) we can calculate the discontinuities across σ in the second and third derivatives of ϕ in an arbitrary direction:

$$[\partial_\beta \partial_\gamma \phi] \equiv (\partial_\beta \partial_\gamma [\phi])_{|\sigma} = \xi_\beta \xi_\gamma f\,, \tag{2.5}$$

$$[\partial_\alpha \partial_\beta \partial_\gamma \phi] \equiv (\partial_\alpha \partial_\beta \partial_\gamma [\phi])_{|\sigma} = \xi_\alpha \xi_\beta \xi_\gamma g + \xi_{(\alpha} \xi_\beta \partial_{\gamma)} \tilde{f} + \xi_{(\alpha} \partial_\beta \xi_{\gamma)} f\,. \tag{2.6}$$

The brackets on the indices indicate cyclic summation.

We shall use these formulae (2.3)–(2.6) in the next two sections to find the characteristic surfaces and shock propagation equations for various interacting higher spin wave equations.

3. Interacting Spin-1 Proca Field

We first consider the self-coupled vector boson. With a term proportional to W^4 in the Lagrangian, the wave equation is

$$R^\mu \equiv \partial_\lambda G^{\lambda\mu} + m^2 W^\mu + \lambda W^2 W^\mu = 0\,, \tag{3.1}$$

where

$$G^{\lambda\mu} = \partial^\lambda W^\mu - \partial^\mu W^\lambda\,.$$

A constraint is found by taking the divergence of R^μ. This gives

$$d \equiv a_{\alpha\beta} \partial^\alpha W^\beta = 0\,, \tag{3.2}$$

where we have defined the "supplementary metric" $a_{\alpha\beta}$ by

$$a_{\alpha\beta} = (m^2 + \lambda W^2)\, g_{\alpha\beta} + 2\lambda W_\alpha W_\beta\,. \tag{3.3}$$

Suppose that W^μ is smooth except for a possible discontinuity in its second derivative across a hypersurface σ, as described in Section 2. There exist therefore a vector field $\tilde{f}^\mu$ defined in a neighbourhood $\mathscr{U}$ of σ and a vector field g^μ defined on σ such that

$$[\partial_\beta \partial_\gamma W^\mu] = \xi_\beta \xi_\gamma f^\mu\,, \qquad f^\mu = \tilde{f}^\mu_{|\sigma}\,,$$

$$[\partial_\alpha \partial_\beta \partial_\gamma W^\mu] = \xi_\alpha \xi_\beta \xi_\gamma g^\mu + \xi_{(\alpha} \xi_\beta \partial_{\gamma)} \tilde{f}^\mu + \xi_{(\alpha} \partial_\beta \xi_{\gamma)} f^\mu\,.$$

Now consider the discontinuities of R^μ and $\partial_\alpha d$ across the hypersurface σ:

$$[R^\mu] = \xi^2 f^\mu - \xi^\mu f \cdot \xi = 0\,, \tag{3.4}$$

$$[\partial_\alpha d] = \xi_\alpha a_{\lambda\mu} \xi^\lambda f^\mu = 0\,. \tag{3.5}$$

Equations (3.4) and (3.5) are sufficient to determine the characteristic surfaces, but before doing so we examine the higher order discontinuity structure implied by (3.1) and (3.2) in order to describe the propagation of the discontinuity f^μ along the bicharacteristics. In fact we examine the quantities $[\partial_\alpha R^\lambda]$ and $[\partial_\alpha \partial_\beta d]$.

Choose $\tilde{f}$ to satisfy (3.4) and (3.5) in $\mathscr{U}$. We have then on σ

$$[\partial_\alpha R^\lambda] = \xi_\alpha \left\{ 2\frac{df^\lambda}{dr} + \partial_\mu \xi^\mu f^\lambda + \xi^2 g^\lambda - \partial^\lambda(\tilde{f}\cdot\xi) - \xi^\lambda(g\cdot\xi + \partial_\alpha \tilde{f}^\alpha)\right\} = 0\,, \tag{3.6}$$

$$[\partial_\alpha \partial_\beta d] = \xi_\alpha \xi_\beta \{a_{\lambda\mu}\xi^\lambda g^\mu + \partial^\lambda(a_{\lambda\mu}\tilde{f}^\mu)\} = 0\,. \tag{3.7}$$

We have defined $d/dr = \xi^\mu \partial_\mu$.

The characteristic surfaces fall into two classes: (i) $\xi^2 = 0$ and (ii) $\xi^2 \neq 0$.

With $\xi^2 = 0$, it follows from (3.4) that $\xi \cdot f = 0$, and from (3.5) that $f \cdot W = 0$ provided $W \cdot \xi \neq 0$. The vector f^μ must be space-like since it is normal to a light-like vector, and so may be written in the form

$$f^\mu = A n^\mu\,, \qquad n^2 = -1\,.$$

From (3.6) it is straightforward to derive the equation

$$\partial_\lambda (A^2 \xi^\lambda) = 0 \tag{3.8}$$

which expresses the conservation of the amplitude of the shock along the bicharacteristics of σ. Using (3.6) and (3.7) we find the transport equation for the vector n^λ:

$$\frac{dn^\lambda}{dr} = -\xi^\lambda \frac{dW\cdot n}{dr} \bigg/ W\cdot\xi\,, \tag{3.9}$$

provided $W \cdot \xi \neq 0$.

When $\xi^2 \neq 0$ we can write

$$f^\lambda = B\xi^\lambda\,, \qquad B = f\cdot\xi/\xi^2\,.$$

It follows that the characteristic surface is given by

$$a_{\alpha\beta}\xi^\alpha \xi^\beta = 0\,. \tag{3.10}$$

From (3.6) and (3.7), we derive then the conservation law

$$\partial_\lambda (B^2 a^{\lambda\mu}\xi_\mu) = 0\,. \tag{3.11}$$

We next consider the vector boson with an external symmetric tensor interaction. In this case, we have

$$R^\mu \equiv \partial_\lambda G^{\lambda\mu} + m^2 W^\mu + \lambda T^{\mu\nu} W_\nu = 0\,,$$

$$d \equiv \partial_\alpha (a^{\alpha\beta} W_\beta) = 0\,,$$

where

$$a_{\alpha\beta} = m^2 g_{\alpha\beta} + \lambda T_{\alpha\beta}\,. \tag{3.12}$$

The discussion here is completely analogous to that of the previous example, and it is possible to derive Eqs. (3.4) to (3.8) but with the supplementary metric $a_{\alpha\beta}$ given by (3.12). The only difference is that for light-cone characteristics, Eq. (3.9) is replaced by

$$\frac{dn^\lambda}{dr} = -\xi^\lambda \frac{dT^{\alpha\beta}}{dr} \xi_\alpha n_\beta / T^{\alpha\beta} \xi_\alpha \xi_\beta , \tag{3.13}$$

provided $T^{\alpha\beta}\xi_\alpha\xi_\beta \neq 0$.

As a last example, we consider the vector boson with minimal electromagnetic interaction. The replacement $\partial_\lambda \to D_\lambda = \partial_\lambda - ieA_\lambda$ introduces the minimal electromagnetic interaction, so that we have

$$R^\mu \equiv D_\lambda G^{\lambda\mu} + m^2 W^\mu = 0 ,$$

$$d \equiv a_{\alpha\beta} D^\alpha W^\beta = 0 ,$$

where now

$$a_{\alpha\beta} = m^2 g_{\alpha\beta} + ieF_{\alpha\beta} . \tag{3.14}$$

With the supplementary metric given by (3.14), Eqs. (3.4) and (3.5) show that there can only be light-cone characteristics, because of the anti-symmetry of $F_{\alpha\beta}$.

Equation (3.8) is established as before. The transport equation for n^λ is not well defined. We have only

$$\frac{Dn_{[\alpha}\xi_{\beta]}}{dr} = 0 . \tag{3.15}$$

The transport equation for n_α was not well defined in the previous two examples along those bicharacteristics whose tangent vector ξ^α belonged to the intersection of the Minkowski cone and the supplementary cone. In this example the two cones coincide.

In cases where minimal coupling to the electromagnetic field is being discussed, it is worth noting that Eqs. (2.3), (2.4), (2.5) and (2.6) are still valid when the derivative ∂_α is replaced by the covariant derivative D_α, and it is understood that

$$D_\alpha \xi_\beta \equiv \partial_\alpha \xi_\beta .$$

4. Minimally Coupled Rarita-Schwinger Equation

Before investigating the spin-3/2 equation, we examine, for the purposes of comparison, the minimally coupled Dirac equation which does not imply any constraints:

$$R \equiv (i\gamma^\alpha D_\alpha - m)\,\psi = 0 .$$

Suppose that ψ is smooth except for a possible discontinuity in its first derivative across a hypersurface σ as described in Section 2. There exist therefore a spinor field $\tilde{k}$ defined in a neighbourhood $\mathscr{U}$ of σ and a spinor field f defined on σ such that

$$[\partial_\alpha \psi] = \xi_\alpha k\,, \qquad k = \tilde{k}_{|\sigma}\,,$$

$$[\partial_\beta \partial_\alpha \psi] = \xi_\beta \xi_\alpha f + \xi_{(\alpha} \partial_{\beta)} \tilde{k} + \partial_\beta \xi_\alpha k\,.$$

It follows that

$$[R] = i\xi \cdot \gamma k\,,$$

so that if $k \neq 0$ then $\xi^2 = 0$. If we then choose $\tilde{k}$ such that $\xi \cdot \gamma \tilde{k} = 0$, we have

$$[\partial_\beta R] \equiv \xi_\beta (i\xi \cdot \gamma f + i\gamma \cdot D\tilde{k} - mk) = 0\,.$$

Putting $f = An$, $\bar{n}n = \pm 1$, we get

$$\partial_\alpha (A^2 \xi^\alpha) = 0\,, \qquad \frac{Dn}{dr} = 0\,.$$

We shall see how these are modified in the minimally coupled Rarita-Schwinger equation, where the wave equation is

$$R_\lambda \equiv (i\Gamma^\mu_{\lambda\varrho} D^\varrho - M_\lambda{}^\mu)\psi_\mu = 0\,. \tag{4.1}$$

$\Gamma^\mu_{\lambda\varrho}$ and $M_{\lambda\mu}$ are given by

$$\Gamma^\mu_{\lambda\varrho} = \delta^\mu_\lambda \gamma_\varrho - \delta^\mu_\varrho \gamma_\lambda + \sigma_{\lambda\varrho}\gamma^\mu\,,$$

$$M_{\lambda\mu} = -m\sigma_{\lambda\mu}\,.$$

Contracting (4.1) with D^λ and γ^λ we have

$$D^\lambda R_\lambda = \frac{e}{2} F^{\lambda\varrho} \Gamma^\mu_{\lambda\varrho} \psi_\mu - M_{\lambda\mu} D^\lambda \psi^\mu\,,$$

$$d \equiv \gamma^\lambda R_\lambda = 2i\sigma^{\lambda\mu} D_\lambda \psi_\mu + 3m\gamma^\lambda \psi_\lambda\,.$$

Using the fact that

$$F^{\lambda\varrho} \Gamma^\mu_{\lambda\varrho} = 2i\gamma^5 \gamma_\alpha F^{*\alpha\mu}\,,$$

(the * indicating the dual of $F^{\alpha\mu}$) and defining

$$b_{\alpha\beta} = g_{\alpha\beta} + \frac{2e}{3m^2} \gamma^5 F^*_{\alpha\beta}\,,$$

we get

$$d' \equiv 2iD^\lambda R_\lambda - m\gamma^\lambda R_\lambda = -3m^2 b_{\alpha\beta} \gamma^\alpha \psi^\beta\,.$$

Suppose as before that ψ^μ is smooth except for a possible discontinuity in its first derivative across a hypersurface σ. There exist therefore a spinor-vector field $\tilde{k}^\mu$ defined in a neighbourhood $\mathscr{U}$ of σ and a spinor-vector field f^μ defined on σ such that

$$[\partial_\alpha \psi^\mu] = \xi_\alpha k^\mu, \qquad k^\mu = \tilde{k}^\mu_{|\sigma},$$
$$[\partial_\beta \partial_\alpha \psi^\mu] = \xi_\beta \xi_\alpha f^\mu + \xi_{(\alpha}\partial_{\beta)}\tilde{k}^\mu + \partial_\beta \xi_\alpha k^\mu .$$

The discontinuity structure of R_λ, d, and $D_\alpha d'$ may now be examined, giving

$$[R_\lambda] = i\Gamma^\mu_{\lambda\varrho}\xi^\varrho k_\mu,$$
$$[d] = 2i\sigma_{\alpha\beta}\xi^\alpha k^\beta,$$
$$[D_\alpha d'] = -3m^2 \xi_\alpha b_{\lambda\mu}\gamma^\lambda k^\mu,$$

which lead to

$$\xi \cdot \gamma k_\lambda - \xi_\lambda \gamma \cdot k = 0, \tag{4.2a}$$
$$\xi \cdot k - \xi \cdot \gamma\gamma \cdot k = 0, \tag{4.2b}$$
$$b_{\lambda\mu}\gamma^\lambda k^\mu = 0. \tag{4.2c}$$

From (4.2a), we get

$$\xi^2 k_\lambda = \xi_\lambda \xi \cdot k,$$

and, by defining the supplementary metric $a_{\alpha\beta}$ by

$$a^\gamma_\alpha \equiv b_{\alpha\beta}b^{\gamma\beta} = \delta^\gamma_\alpha + \left(\frac{2e}{3m^2}\right)^2 F^*_{\alpha\beta}F^{*\gamma\beta},$$

it follows that

$$\xi^2 a_{\alpha\mu}\xi^\alpha k^\mu = 0. \tag{4.3}$$

There are two classes of characteristic surfaces: (i) $\xi^2 = 0$, in which case $\xi \cdot k = 0$ and $\xi \cdot \gamma\gamma \cdot k = 0$; (ii) $\xi^2 \neq 0$ in which case ξ_α satisfies the equation

$$a^{\alpha\beta}\xi_\alpha \xi_\beta = 0.$$

The propagation equations for the shock are, as before, found by examining higher order discontinuities. If we require $\tilde{k}_\lambda$ to satisfy (4.2) in $\mathscr{U}$, then by examining $[D^\alpha D^\lambda R_\lambda]$, $[D_\alpha D_\beta d']$ and $[D^\alpha R_\lambda]$ across σ we find

$$\sigma_{\lambda\mu}(\xi^\lambda f^\mu + D^\lambda \tilde{k}^\mu) = \frac{-ie}{m}\gamma^5 \gamma_\alpha F^{*\alpha\mu}k_\mu, \tag{4.4a}$$

$$b_{\lambda\mu}\gamma^\lambda f^\mu = 0, \tag{4.4b}$$

$$(i\gamma \cdot D - m)\tilde{k}^\lambda - \frac{m}{2}\gamma^\lambda \gamma \cdot k + i(\xi \cdot \gamma f^\lambda - \xi^\lambda \gamma \cdot f - D^\lambda(\gamma \cdot \tilde{k})) = 0. \tag{4.4c}$$

respectively. Using the fact that

$$i\gamma \cdot D(\gamma \cdot \xi \tilde{k}_\lambda - \xi_\lambda \gamma \cdot \tilde{k}) = 0,$$

(4.4c) becomes

$$i(2\xi \cdot Dk_\lambda + \partial_\alpha \xi^\alpha k_\lambda) - m\xi_\lambda \gamma \cdot k + \frac{m}{2}\gamma_\lambda \xi \cdot \gamma\gamma \cdot k$$
$$+ i(\xi^2 f_\lambda - \xi_\lambda \xi \cdot \gamma\gamma \cdot f - D_\lambda(\xi \cdot \gamma\gamma \cdot \tilde{k}) - \xi_\lambda \gamma \cdot D(\gamma \cdot \tilde{k})) = 0 .$$

In the case $\xi^2 = 0$ we may write

$$k_\lambda = A n_\lambda , \qquad \bar{n}_\lambda n^\lambda = \mp 1 .$$

The conservation equation for the amplitude of the shock is

$$\partial_\alpha (A^2 \xi^\alpha) = 0 , \tag{4.5}$$

and the transport equation for n_λ is

$$\frac{D n_\lambda}{d r} = \frac{-1}{a_{\alpha\beta}\xi^\alpha \xi^\beta} b_{\varrho\sigma} \frac{d b_{\mu\nu}}{d r} \gamma^\sigma \gamma^\nu \xi^\varrho n^\mu \xi_\lambda , \tag{4.6}$$

provided $a^{\alpha\beta}\xi_\alpha \xi_\beta \neq 0$.

If $\xi^2 \neq 0$ then

$$k_\lambda = B \xi_\lambda , \qquad B = \frac{1}{\xi^2} \xi \cdot k ,$$

and the conservation equation for the amplitude of the shock is

$$\partial^\mu (\bar{B} b_{\lambda\mu} \gamma^\lambda B) = 0 . \tag{4.7}$$

At each point we have two cones defined: the Minkowski cone which contains the normals to null hypersurfaces; and the supplementary cone which contains the normals to the anomalous characteristic hypersurfaces.

In the previous examples a choice can be made of the free parameters such that the causality requirement is satisfied; that is, such that the supplementary cone does not lie inside the Minkowski cone.

In the first example this condition is $\lambda \geqq 0$; (and $\lambda W^2 > -m^2$) in the second example it is that $\lambda T_{\alpha\beta}$ be a positive matrix.

This is not the case in this example. The supplementary metric may be written as

$$a_{\lambda\mu} = \left(1 - \left(\frac{e}{3m^2}\right)^2 F_{\alpha\beta} F^{\alpha\beta}\right) g_{\lambda\mu} - \left(\frac{2e}{3m^2}\right)^2 \tau_{\lambda\mu} , \tag{4.8}$$

where $\tau_{\lambda\mu}$ is the Maxwell stress-energy tensor. Causality here requires that

$$\tau_{\lambda\mu} \xi^\lambda \xi^\mu \leqq 0$$

for all ξ^λ in the supplementary cone $\left(\text{provided } 1 - \left(\frac{e}{3m^2}\right)^2 F_{\alpha\beta} F^{\alpha\beta} > 0\right)$. This is obviously impossible.

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