A Point Mass in a Einstein Universe

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Abstract. An exact solution of Einstein's equations is presented for a perfect fluid representing a point mass in a Einstein Universe.

Unlike De Sitter's and Friedman's cosmological solutions for which a generalization with a point mass is known to exist [1, 2], such corresponding generalizations was not yet found for the Einstein Solution. This gap is here filled.

Let us consider the line element given by

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} [1 - a(r-m)^2]^{-1} \, dr^2$$
$$- r^2(d\theta^2 + \sin^2\theta \, d\phi^2) + \left(1 - \frac{2m}{r}\right) dt^2 \,. \tag{1}$$

The calculated components of the energy-momentum tensors are

$$-8\pi(T_1^1 = T_2^2 = T_3^3) = \frac{-a(r-m)^2}{r^2}\,, \tag{2}$$

$$8\pi\varrho = \frac{a}{r^2}(3r - 5m)(r - m)\,. \tag{3}$$

It represents therefore an ideal fluid. For $a = 0$ the element reduces to that of the Schwarzschild's exterior solution. For $m = 0$ we have the Einstein universe.

The correct signature is obtained for

$$2m < r \quad \text{and} \quad (r-m)^2 < \frac{1}{a}\,. \tag{4}$$

The two inequalities are compatible for $m^2 a < 1$. Provided that this last inequality is satisfied it is possible to find a range of values for r so that (4) holds.

The density and the pressure are of opposite sign. However, whenever the inequalities (4) are satisfied we have in absolute value $|\varrho| > |p|$. As r approach the value $2m$, the absolute values of ϱ and p become equal.

Note added in proof. This solution cannot be reduced to the static particular case of the one given in reference [2].

References

1. Tolman, R. C.: Relativity thermodynamics and cosmology, p. 245. Equation 96.3 gives the line element of Schwarzschild-De Sitter solution. Oxford 1934.
2. McVittie, G. C.: Monthly Notices Roy. Astron. Soc. **93**, 325 (1933). It gives the line element for a point mass in a Friedman universe.

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