Communications in Mathmeatical Analysis
Volume 19, Number 1, pp. 1–5 (2016) ISSN 1938-9787
Special Issue www.math-res-pub.org/cma

SPECIAL ISSUE OF COMMUNICATIONS IN MATHEMATICAL ANALYSIS DEVOTED TO:

Wave Operators and Similarity for Long Range N-body Schrödinger Operators

by

HITOSHI KITADA

DEDICATED TO TOSIO KATO

Abstract: This Interview is a part of the Special Issue of Communications in Mathematical Analysis dedicated to late Prof. Tosio Kato on his 100th birthday. We extend our deepest thanks to Prof. Hitoshi Kitada for dedicating his paper "Wave Operators and Similarity for Long Range N-body Schrödinger Operators" to Prof. Tosio Kato. Further, we thank him for accepting to answer to our questions.

Picture of Prof. Hitoshi Kitada.

EDITED BY
Toka Diagana
Department of Mathematics, Howard University, Washington DC, 20059, USA
Managing Editor, Communications in Mathematical Analysis.

1 Short Biography of Prof. Hitoshi Kitada

Prof. Hitoshi Kitada was born in Tokyo, Japan. After graduating from the University of Tokyo Department of Mathematics, he went to Osaka City University to pursue his graduate studies in Mathematics. When he completed his Master's degree, he worked at the Japan's Ministry of Health and Welfare, as a technical official. Once the work was finished, he used to come home and to devote himself to two-body long-range scattering problem, which he had been thinking since he was a graduate student. In one year, he came up with a solution to the problem which consists of proving the asymptotic completeness for long-range scattering, and his results were published in 1977 and 1978 in [8] and [9]. In these days, the scattering theory for two-body short-range case was mostly completed at the beginning of 70's, but the theory for the two-body long-range case was undeveloped. Many mathematicians made attempts to prove the asymptotic completeness for two-body long-range scattering problem, including Shmuel Agmon, Lars Hörmander (*). Prof. H. Kitada became known for the solution he gave to this problem. Before the publication of his results, he moved to the University of Tokyo in 1977. He received his PhD degree in Mathematics in 1979 from University of Tokyo. In the 80s, he obtained an important outcome concerning the introduction of micro-local analysis in scattering theory. It should be noticed that Prof. Kitada is the one who introduced and adapted the technique of micro-local analysis to scattering theory. In March 1983 and in 1984-85, he was invited to conduct some research activities in a stimulating atmosphere at the California Institute of Technology. Thereafter, he had several opportunities to visit and stay at European and American universities/institutes. Those research visits led him to new investigations. In 1992, the Institute of Mathematics to which he belonged was reformed to become a part of the Graduate School of Mathematical Sciences at the University of Tokyo. During the same year, he obtained the idea that a quantum system is characterized by its own dynamics and introduced the notion of quantum local time proper to each quantum system, which was published in [10] and has recently attracted physicists (e.g., [3], [12]). He, since 2012, became a member of the Metasciences Academy.

(*) Later Hörmander gave a proof in his book [2] in 1985.

2 The Interview with Prof. Hitoshi Kitada

Diagana: Thank you so much Professor Hitoshi Kitada for accepting my request to interview you within the framework of the special issue of Communications in Mathematical Analysis dedicated to late Professor Tosio Kato upon his 100th birthday. I also seize this opportunity in thanking you from the bottom of my heart for your continued support to this journal. What does the mathematical work of Tosio Kato represent for you?

Kitada: The work of Tosio Kato is vast and covers a very broad area from abstract theory of functional analysis to the concrete problem of nonlinear partial differential equations. He was originally a physicist, when he found and solved a mathematical problem: a criterion for the selfadjointness of perturbed Hamiltonian, known now as Kato-Rellich theorem. The perspective which is suggested by the work he left extends to the far future. We learn from his book, Perturbation Theory for Linear Operators [5], that he is an extremely precise and rigorous mathematician, which agrees with my impression of his personality that I had when I met him for the first time in 1976 in Tokyo. We can see this also in the work

[6], [7], in which he with Prof. Shigetoshi Kuroda have given an exquisite theory for the existence and asymptotic completeness of wave operators for the short-range two-body Schrödinger operators based on abstract functional analytic formulation of the problem. The abstract part of my work on two-body long-range problem [8], [9] is developed utilizing the ideas in these works. Also the smooth operator technique introduced in [4] for some non-selfadjoint operators in abstract functional analytic setting gives an important method in scattering theory, especially in N-body scattering theory, and plays an important role in the present work. Except for the area of recently developing micro-local analysis, Kato's work covers almost all important areas of mathematical analysis at his age, and the influence extends to the present age and to the far future. To sum up, what the mathematical work of Tosio Kato represents for me is the abstract functional analytic treatment of the problems in mathematical analysis.

Diagana: Do you mind describing the direct links that exist between your work and that of Kato?

Kitada: Speaking of the links between Kato's work and my work, what influenced me most has been the ones on quantum scattering theory like Kato-Kuroda's papers [6], [7], and the work [4] on smooth operators. The most impressive work I read as a graduate student was [4], which gave me a very real dream that I had obtained a proof of the asymptotic completeness for two-body long-range problem by the smooth operator technique given in [4]. After a few years since the dream I found a complicated proof of the problem in 1977-78 ([8], [9]). Much later in 2011, my dream in youth came true in the work [11], where micro-localization was extended to the localization in the extended phase space. An extension of this extended micro-localization introduced in [11] to many-body case enabled me to apply smooth operator technique to the N-body problem, which led me to the present work. To sum, the direct link between Kato's work and my work is the simple and beautiful Kato's smooth operator technique given in his paper [4].

Diagana: What have you been able to achieve in your paper?

Kitada: The purpose of the present work is to consider and extend what Kato called similarity [4] i.e. the asymptotic completeness of wave operators, to the N-body long-range case. The extension of the extended micro-localization in [11] to the N-body case gives the similarity for positive long-range pair potentials decaying like $(1+|x|)^{-\delta}$ $(\delta > 1/2)$ at $|x| \to \infty$. As a byproduct, it makes the proof of similarity for the short-range case considerably simple.

Diagana: What are the possible applications of your work?

Kitada: For the area of scattering theory, a direct application of the present result is the mathematical warranty for the important physical intuition that the total probability conserves for those potentials. As well a problem raised along with the present work is the necessity of the investigation of Yafaev channel, which seems to be dominant for the longer-range case with $\frac{1}{2} > \delta > 0$. Not only seeing such direct applications and accompanying problems, but also returning to the origin of the problem might open further perspectives. In fact my first motivation of the present work was entirely in different direction of considering classical particles following Hamilton's canonical equation. I derived a curious conclusion; even looking a contradictory conclusion, that orbit of every particle is bounded

even if pair potentials are repulsive, from the assumption that the density function satisfies the equation of continuity. I was struggling with the paradox and asked Prof. Jacob Møller for his opinion. He suggested me that I missed that the density must be the density in three-dimensional space in considering particles' density, not in higher dimensional phase space. Namely my initial target was not directly toward the problem considered in the present work, but a physical problem whose inappropriate formulation created a paradox. The problem which was behind this physical problem is a seemingly irrelevant question: whether or not the universe (consisting of N particles) could be bounded in an appropriate sense in space either in classical or quantum mechanical context with assuming some universal interaction among particles. This problem is related with the exponential decay of eigenfunctions of N-body Hamiltonian in the quantum case and has been settled in [1] as that the non-threshold eigenfunctions decay exponentially. The remaining problem thus is about the classical case stated above. Owing to Jacob's suggestion I noticed that the problem in classical case depends on the properties of interactions. Having this instruction in mind, I returned to the original context and started considering classical scattering, in which I found an extension of the extended micro-localization in [11] to N-body case, as written in section 5 of the present paper. This naturally led me to the quantum case and produced the present paper. The method of extended micro-localization I gave for N-body quantum problem in the present work will be applicable in various problems related with many-body scattering theory such as the analysis of scattering amplitude for many-body case. Not only such direct applications in mathematical physics, the idea behind the extended micro-localization will be also useful and find fruitful applications in various areas of mathematical analysis. Further, the notion of scattering spaces introduced here to treat the long-range pair potentials will be useful in the investigation of Yafaev channel.

Diagana: Thank you so much.

REFERENCES

[1] R. Froese and I. Herbst, Exponential bounds and absence of positive eigenvalues for N-body Schrödinger operators, Commun. Math. Phys. 87 (1982), pp. 429-447.

[2] L. Hormander, The Analysis of Linear Partial Differential Operators IV Fourier Integral Operators, Springer-Verlag, Berlin Heidelberg New York Tokyo, 1985.

[3] J. Jeknic-Dugic, M. Arsenijevic and M. Dugic, A local-time-induced pointer basis, Proc. R. Soc. A 470 (2014) 20140283

[4] T. Kato, Wave operators and similarity for some non-selfadjoint operators, Math. Annalen 162 (1966), pp. 258-279.

[5] T. Kato, Perturbation Theory for Linear Operators, Springer-Verlag, Berlin, Heidelberg, New York 1976.

[6] T. Kato and S. T. Kuroda, Theory of simple scattering and eigenfunction expansions, Functional Analysis and Related Fields, Springer-Verlag, Berlin, Heidelberg, and New York, 1970, pp. 99-131.

[7] T. Kato and S. T. Kuroda, The abstract theory of scattering, Rocky Mount. J. Math. 1 (1971), pp. 127-171.

[8] H. Kitada, Scattering theory for Schrödinger operators with long-range potentials I, abstract theory, J. Math. Soc. Japan 29 (1977), pp. 665-691.

[9] H. Kitada, Scattering theory for Schrödinger operators with long-range potentials II, spectral and scattering theory, J. Math. Soc. Japan, 30 (1978), pp. 603-632.

[10] H. Kitada, Theory of Local Times, Nuovo Cimento, 109B (3) (1994), pp. 281-302.

[11] H. Kitada, A remark on simple scattering theory, Commun. Math. Anal. 11 (2011), No. 2, pp. 124-138.

[12] P. L. Simeonov, Yet another time about time Part I: An Essay on the Phenomenology of Physical Time. http://www.sciencedirect.com/science/article/pii/S0079610715001285