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Differences of Composition Operators on Weighted Banach Spaces of Holomorphic Functions Defined on the Unit Ball of a Complex Banach Space

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Abstract

We investigate differences of composition operators acting between weighted spaces of holomorphic functions defined on the open unit ball of a Banach space. We give necessary and sufficient conditions for such operators to be bounded resp. compact.

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1 Introduction

Let $\mathbb{D}$ denote the open unit disk in the complex plane and $H(\mathbb{D})$ the collection of all analytic functions on $\mathbb{D}$. A bounded and continuous function $v : \mathbb{D} \to (0,\infty)$ (*weight*) induces a weighted space of holomorphic functions given by

$$H_v^\infty := \left\{ f \in H(\mathbb{D});\ \|f\|_v := \sup_{z\in\mathbb{D}} v(z)|f(z)| < \infty \right\}.$$

Endowed with norm $\|.\|_v$ this is a Banach space. Such spaces arise naturally in a variety of research topics, such as functional analysis, partial differential equations and convolution equations as well as in distribution theory. The spaces themselves became of interest and have been investigated by several authors. Among others we refer to the articles [1], [2], [3], [4], [18], [19]. These studies motivated Garcia, Maestre and Rueda to study such spaces

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in a more general setting, see [13]. They considered a complex Banach space X, its open unit ball B_X and the collection $H(B_X)$ of all holomorphic functions $f : B_X \to \mathbb{C}$ in order to analyze spaces of the following type

$$H_v(B_X) := \left\{ f \in H(B_X);\ \|f\|_v = \sup_{x \in B_X} v(x)|f(x)| < \infty \right\},$$

where $v : B_X \to (0, \infty)$ denotes a bounded and continuous function (*weight*). Endowed with the weighted sup-norm $\|.\|_v$, this is a Banach space as in the case described above.

An analytic self-map ϕ of $\mathbb{D}$ induces through composition a linear composition operator

$$C_\phi : H(\mathbb{D}) \to H(\mathbb{D}),\ f \mapsto f \circ \phi.$$

Since such operators appear in many problems and since they link - in the classical setting of the Hardy space H^2 (see [10] and [24]) - operator theoretical questions with classical results in complex analysis their study has a long and rich history.
In [6] Bonet, Domański, Lindström and Taskinen characterized boundedness and compactness of operators

$$C_\phi : H_v^\infty \to H_w^\infty,\ f \mapsto f \circ \phi$$

in terms of the inducing symbol ϕ as well as the involved weights v and w. The work of Bonet, Domański, Lindström and Taskinen gave rise to the investigations of Garcia, Maestre and Sevilla-Peris regarding boundedness and compactness of composition operators in the setting of weighted spaces $H_v(B_X)$ of holomorphic functions defined on the unit ball of a Banach space X, that is of operators

$$C_\phi : H_v(B_Y) \to H_w(B_X),\ f \mapsto f \circ \phi,$$

where $\phi : B_Y \to B_X$ is analytic, v and w are weights and Y is another Banach space. Inspired by the article [14] and the paper of Bonet, Lindström and Wolf [7] which deals with boundedness and compactness of differences of composition operators

$$C_\phi - C_\psi : H_v^\infty \to H_w^\infty,\ f \mapsto f \circ \phi - f \circ \psi$$

we give necessary and sufficient conditions for an operator

$$C_\phi - C_\psi : H_v(B_Y) \to H_w(B_X),\ f \mapsto f \circ \phi - f \circ \psi$$

to be bounded resp. compact.

2 Basics on weights and weighted spaces

This section is devoted to an overview of basic facts on weights and weighted spaces in the setting of a complex Banach space X and its open unit ball B_X. Further and deeper information can be found in the articles [13] and [14]. We start with collecting some basic definitions. We say that a set $A \subset B_X$ is *B_X-bounded* if there exists $0 < r < 1$ such that $A \subset rB_X$ and write

$$H_b(B_X) := \{f \in H(B_X);\ f \text{ bounded on the } B_X\text{-bounded sets }\}.$$

As said before, in this article we are interested in operators acting on spaces of the following type:

$$H_v(B_X) = \left\{ f \in H(B_X);\ \|f\|_v := \sup_{x \in B_X} v(x)|f(x)| < \infty \right\}.$$

With the norm $\|.\|_v$, the space $H_v(B_X)$ is a Banach space.
Now, our studies require some extra conditions on the weights. Thus, a weight v is called *radial* if $v(\lambda x) = v(x)$ for every $\lambda \in \mathbb{C}$ with $|\lambda| = 1$ and every $x \in B_X$.
A weight v satisfies Condition I if $\inf_{x \in rB_X} v(x) > 0$ for every $0 < r < 1$. If v enjoys Condition I, then $H_v(B_X) \subset H_b(B_X)$. In case of finite-dimensional Banach spaces X all weights on B_X have Condition I. In the sequel we will always assume that a weight v satisfies the Condition I.
An important tool when dealing with weights and weighted spaces are the so calles *associated weights*. In the setting of the unit disk in the complex plane this concept was introduced by Anderson and Duncan in [1] and thoroughly studied by Bierstedt, Bonet and Taskinen in [3]. The generalization to the present setting has been done by Garcia, Maestre and Sevilla-Peris in [14]. Given any weight v we consider the so called *associated weight* $\tilde{v}$, given by

$$\tilde{v}(z) = \frac{1}{\sup\{|f(z)|; \|f\|_v \leq 1\}}.$$

By [14] Proposition 1.1 such an associated weight has the following properties:

1. $0 < v \leq \tilde{v}$ and $\tilde{v}$ is bounded and continuous, i.e. $\tilde{v}$ is a weight.

2. $\tilde{v}$ is radial and decreasing whenever v is so.

3. $\|f\|_v \leq 1 \Longleftrightarrow \|f\|_{\tilde{v}} \leq 1$.

4. For every $x \in B_X$ there is $f_x \in H_v^\infty$ with $\|f\|_v \leq 1$ such that $|f_x(x)| = \frac{1}{\tilde{v}(x)}$.

Very important for us are the *norm-radial* weights, that is weights v with $v(x) = v(y)$ for every $x, y \in B_X$ with $\|x\| = \|y\|$. We need some extra condition on the weight that - in a sense - is an analogon to the Lusky condition $(L1)$ which appeared during his studies on the isomorphism classes of H_v^∞, see [18]. We say that v satisfies condition (A) if and only if

$$(A) \quad \text{there are } 0 < r < 1 \text{ and } C < \infty \text{ such that}$$
$$\frac{v(z)}{v(p)} \leq C \text{ for every } z, p \in B_X \text{ with } d(z, p) \leq r.$$

Here $d(z, p)$ denotes the generalized pseudohyperbolic distance of two points $z, p \in B_X$, that is

$$d(z, p) := \sup\{\rho(h(z), h(p));\ h : B_X \to \mathbb{D} \text{ holomorphic}\},$$

where ρ is the *pseudohyperbolic distance*

$$\rho : \mathbb{D}\times\mathbb{D} \to [0,1),\ (z,p) \mapsto \left|\frac{z-p}{1-\overline{p}z}\right|.$$

Now, for fixed p let φ_p denote the Möbius transformation that interchanges p and 0, that is,

$$\varphi_p(z) = \frac{p-z}{1-\overline{p}z} \text{ for every } z \in \mathbb{D}.$$

Hence $\rho(z,p) = |\varphi_p(z)|$ for every $z, p \in \mathbb{D}$. In the setting of H_v^∞ condition (A) is given as above with ρ instead of d and equivalent to the condition $(L1)$, see [11] and [7].

3 A useful lemma

The following lemma turns out to be extremely helpful when treating differences of composition operators. In the setting of the unit disk it was proved by Bonet, Lindström and Wolf in [7].

Lemma 3.1. *Let v be a norm-radial weight on B_X and let $f \in H_v(B_X)$. Assume that there are $0 < r < 1$ and $C < \infty$ such that for all $z, p \in B_X$ with $d(z,p) \leq r$ we have that $\frac{v(z)}{v(p)} \leq C$. Then*

$$|f(z) - f(p)| \leq 4Cd(z,p)\frac{\|f\|_v}{rv(p)}$$

for all $z, p \in B_X$ with $d(z,p) \leq \frac{r}{2}$.

Proof. Let $z, p \in B_X$ with $z \neq p$ such that $d(z,p) \leq \frac{r}{2}$. Then there is $\varepsilon \in \mathbb{D}$ such that $1+\varepsilon \in \mathbb{D}$ and

$$h : \mathbb{D} \to B_X,\ h(t) = (t-\varepsilon)p + (1-(t-\varepsilon))z$$

is holomorphic. Moreover, $h(\varepsilon) = z$ and $h(1+\varepsilon) = p$. We get

$$|f(z) - f(p)| = |f(h(\varepsilon)) - f(h(1+\varepsilon))| = |(f\circ h)(\varepsilon) - (f\circ h)(1+\varepsilon)|.$$

Now, put $g_{1+\varepsilon}(t) = (f\circ h)(\varphi_{1+\varepsilon}(t))$ for every $t \in \mathbb{D}$. Since $\varphi_{1+\varepsilon}(\varphi_{1+\varepsilon}(\varepsilon)) = \varepsilon$ and $\varphi_{1+\varepsilon}(0) = 1+\varepsilon$ we obtain

$$|f(z) - f(p)| = |g_{1+\varepsilon}(\varphi_{1+\varepsilon}(\varepsilon)) - g_{1+\varepsilon}(0)|.$$

For $|\varepsilon| = \rho(\varphi_{1+\varepsilon}(\varepsilon), 1+\varepsilon) \leq r$ we get

$$|g_{1+\varepsilon}(\varepsilon)| = \frac{|f(h(\varphi_{1+\varepsilon}))|}{v(h(\varphi_{1+\varepsilon}(\varepsilon))}v(h(\varphi_{1+\varepsilon}(\varepsilon)) \leq \frac{\|f\|_v}{v(h(\varphi_{1+\varepsilon}(\varepsilon))}\frac{v(p)}{v(p)} \leq C\frac{\|f\|_v}{v(p)}.$$

Next, we obtain the following estimate

$$\begin{aligned}|f(z) - f(p)| = |f(h(\varepsilon)) - f(h(1+\varepsilon))| = |g_{1+\varepsilon}(\varphi_{1+\varepsilon}(\varepsilon)) - g_{1+\varepsilon}(0)| &= |\varphi_{1+\varepsilon}(\varepsilon)|\left|\int_0^1 \frac{\partial}{\partial t} g_{1+\varepsilon}(t)\, dt\right| \\ = |\varphi_{1+\varepsilon}(\varepsilon)|\frac{1}{2\pi}\left|\int_{|\xi|=r}\frac{g_{1+\varepsilon}(\xi)}{(\xi-\Theta)^2}\, d\xi\right| &\leq |\varphi_{1+\varepsilon}(\varepsilon)|\frac{rC\|f\|_v}{(r-|\varphi_{1+\varepsilon}(\varepsilon)|^2)}\frac{1}{v(p)} \\ = |\varphi_{1+\varepsilon}(\varepsilon)|\frac{4C\|f\|_v}{rv(h(1+\varepsilon)} &= |\varphi_{1+\varepsilon}(\varepsilon)|\frac{4C\|f\|_v}{rv(p)} \\ \leq 4C\sup\{\rho(k(z),k(p));\ k : B_X \to \mathbb{D} \text{ holomorphic }\}\frac{\|f\|_v}{rv(p)} &= 4Cd(z,p)\frac{\|f\|_v}{rv(p)}\end{aligned}$$

since $|\Theta| \leq |\varphi_{1+\varepsilon}(\varepsilon)| \leq \frac{r}{2}$. Hence the claim follows. □

Lemma 3.2. *Let v be a norm-radial weight on B_X with condition (A). For every $f \in H_v(B_X)$ there is a constant $C_v > 0$ (depending on the weight) such that*

$$|f(z) - f(p)| \leq C_v \|f\|_v d(z,p) \max\left\{\frac{1}{v(z)}, \frac{1}{v(p)}\right\}.$$

Proof. By Lemma 3.2 we have that

$$|f(z) - f(p)|v(z) \leq \frac{4C}{s}\|f\|_v d(z,p)$$

for every $z, p \in B_X$ with $d(z,p) \leq \frac{s}{2}$. If $d(z,p) > \frac{s}{2}$ then

$$|f(z) - f(p)| \min\{v(z), v(p)\} \leq 2\|f\|_v \leq \frac{4}{s}\|f\|_v d(z,p).$$

Hence

$$|f(z) - f(p)| \min\{v(z), v(p)\} \leq C_v \|f\|_v d(z,p)$$

for every $z, p \in B_X$ and the claim follows. □

4 Boundedness

Now, we obtain the first of our main results which is analogous to the result of Bonet, Lindström and Wolf in [7].

Theorem 4.1. *Let v and w be weights such that v is norm-radial and satisfies condition (A). Moreover let $\phi, \psi : B_X \to B_Y$ be holomorphic maps. Then the following are equivalent:*

(a) $C_\phi - C_\psi : H_v(B_Y) \to H_v(B_X)$ *is well-defined and continuous.*

(b) $\sup_{x \in B_X} w(x) \max\left\{\frac{1}{\tilde{v}(\phi(x))}, \frac{1}{\tilde{v}(\psi(x))}\right\} d(\phi(x)), \psi(x)) < \infty$

Proof. We start with showing that (a) implies (b) and assume to the contrary that (b) does not hold. In this case there must be a sequence $(x_n)_n \subset B_X$ such that $\|x_n\|_X \to 1$ and

$$w(x_n) \max\left\{\frac{1}{v(\phi(x_n))}, \frac{1}{v(\psi(x_n))}\right\} d(\phi(x_n), \psi(x_n)) \geq n$$

for every $n \in \mathbb{N}$.
Next, we put

$$f_n(y) := g_n(y)\varphi_{k(\phi(x_n))}(k(y)) \text{ for every } y \in B_Y,$$

where $g_n \in H_v(B_Y)$, $\|g_n\|_v \leq 1$ and $|g_n(\phi(x_n))| = \frac{1}{\tilde{v}(\phi(x_n))}$ and $k : B_Y \to \mathbb{D}$ is an arbitrary but fixed holomorphic map. Then $\|f_n\|_v \leq 1$ for every $n \in \mathbb{N}$ and

$$c \geq w(x_n)|f_n(\phi(x_n)) - f_n(\psi(x_n))| = \frac{w(x_n)}{\tilde{v}(\phi(x_n))}\rho(h(\phi(x_n)), h(\psi(x_n)) \geq n$$

which is a contradiction.
Conversely, we assume that (b) holds and want to show (a). An application of Lemma 3.2 gives for every $f \in H_v(B_Y)$

$$\|(C_\phi - C_\psi)f\|_w = \sup_{z\in\mathbb{D}} w(z)|f(\phi(z)) - f(\psi(z))|$$
$$\leq C_v\|f\|_v d(\phi(z),\psi(z)) \max\left\{\frac{1}{v(\phi(z))}, \frac{1}{v(\psi(z)))}\right\}$$

and the claim follows. □

5 Compactness

First, we need the following lemma which can be derived easily from Lemma 3.1 in [14].

Lemma 5.1. *Let $C_\phi - C_\psi : H_v(B_Y) \to H_w(B_X)$ be continuous. Then the following are equivalent:*

(a) $C_\phi - C_\psi$ is compact.

(b) Each bounded sequence $(f_n)_n \subset H_v(B_Y)$ such that $f_n \to 0$ with respect to the compact-open topology τ_0 satisfies that $\|(C_\phi - C_\psi)(f_n)\|_w \to 0$.

Proposition 5.2. *If $\lim_{\|x\|\to 1} w(x) \max\left\{\frac{1}{\tilde{v}(\phi(x))}, \frac{1}{\tilde{v}(\psi(x))}\right\} d(\phi(x),\psi(x)) = 0$ and $\phi(rB_X)$ and $\psi(rB_X)$ are relatively compact for every $0 < r < 1$, then $C_\phi - C_\psi : H_v(B_Y) \to H_w(B_X)$ is compact.*

Proof. Let us suppose that the operator $C_\phi - C_\psi$ is not compact. By Lemma 5.1 there must be a τ_0-null sequence $(f_n)_n \subset H_v(B_Y)$ with $\|f_n\|_v \leq 1$ for every $n \in \mathbb{N}$ such that $(\|(C_\phi - C_\psi)(f_n)\|_w)_n$ does not converge to 0. W.l.o.g. we may assume that there is $\alpha > 0$ such that

$$\sup_{x\in B_X} w(x)|f_n(\phi(x)) - f_n(\psi(x))| = \|(C_\phi - C_\psi)(f_n)\|_w > \alpha$$

for every $n \in \mathbb{N}$. Next, we choose a sequence $(x_n)_n \subset B_X$ with $w(x_n)|f_n(\phi(x_n))| \geq \alpha$ for every $n \in \mathbb{N}$. We suppose that we can find a subsequence $(x_{n_k})_k$ such that $\lim_k \|x_{n_k}\| = 1$. Now, fix $\varepsilon > 0$. Then there is $k_0 \in \mathbb{N}$ with

$$w(x_{n_k}) \max\left\{\frac{1}{\tilde{v}(\phi(x_{n_k})}, \frac{1}{\tilde{v}(\psi(x_{n_k})}\right\} d(\phi(x_{n_k}),\psi(x_{n_k})) < \frac{\varepsilon}{C_v}$$

for every $k \geq k_0$. Hence by Lemma 3.2 we obtain

$$\alpha \leq w(x_{n_k})|f_{n_k}(\phi(x_{n_k})) - f_{n_k}(\psi(x_{n_k}))|$$
$$\leq C_v w(x_{n_k}) \max\left\{\frac{1}{\tilde{v}(\phi(x_{n_k}))}, \frac{1}{\tilde{v}(\psi(x_{n_k}))}\right\} d(\phi(x_{n_k}),\psi(x_{n_k}))\|f\|_v \leq \varepsilon.$$

This contradicts the fact that $\alpha > 0$. Therefore there exists $0 < s < 1$ such that $\|x_n\| \leq s$ for every $n \in \mathbb{N}$. Since $(\phi(x_n))_n \subset \phi(sB_X)$ and $(\psi(x_n))_n \subset \psi(sB_X)$ and both are relatively compact, there exists $n_1 \in \mathbb{N}$ with

$$\sup_{y\in\phi(sB_X)} |f_n(y)| < \frac{\varepsilon}{2M} \text{ and } \sup_{y\in\psi(sB_X)} |f_n(y)| < \frac{\varepsilon}{2M} \text{ for every } n \geq n_1,$$

where $M := \sup_{x\in B_X} w(x)$. Therefore $|f_n(\phi(x_n)) - f_n(\psi(x_n))| < \frac{\varepsilon}{M}$ for every $n \geq n_1$ and

$$\alpha \leq w(x_n)|f_n(\phi(x_n)) - f_n(\psi(x_n))| < \varepsilon.$$

Again we obtain a contradiction. Thus, the claim follows. □

Proposition 5.3. *Let v and w be norm-radial weights that satisfy condition (A). Moreover, let $C_\phi - C_\psi : H_v(B_Y) \to H_w(B_X)$ be compact, then*

$$\lim_{\min\{||\phi(x)||,||\psi(x)||\}\to 1} w(x)\max\left\{\frac{1}{\tilde{v}(\phi(x))}, \frac{1}{\tilde{v}(\psi(x))}\right\} d(\phi(x),\psi(x)) = 0.$$

Proof. We suppose to the contrary that $w(x)\max\left\{\frac{1}{\tilde{v}(\phi(x))}, \frac{1}{\tilde{v}(\psi(x))}\right\} d(\phi(x),\psi(x))$ does not converge to 0 when $\min\{||\phi(x)||, ||\psi(x)||\} \to 1$. Then there must be a sequence $(x_n)_n \subset B_X$ such that $\lim_{n\to\infty} \min\{||\phi(x_n)||, ||\psi(x_n)||\} = 1$ and $c > 0$ such that

$$w(x_n)\max\left\{\frac{1}{\tilde{v}(\phi(x_n))}, \frac{1}{\tilde{v}(\psi(x_n))}\right\} d(\phi(x_n),\psi(x_n)) \geq c$$

for every $n \in \mathbb{N}$. Now, we have to study the following cases:
Case 1: We have that

$$w(x_n)\max\left\{\frac{1}{\tilde{v}(\phi(x_n))}, \frac{1}{\tilde{v}(\psi(x_n))}\right\} d(\phi(x_n),\psi(x_n)) = w(x_n)\frac{1}{\tilde{v}(\phi(x_n))} d(\phi(x_n),\psi(x_n)).$$

In this case, for every $n \in \mathbb{N}$ we can find $f_n \in H_v(B_Y)$ with $||f_n||_v \leq 1$ such that

$$|f_n(\phi(x_n))| = \frac{1}{\tilde{v}(\phi(x_n))}.$$

Furthermore, we put $g_n(y) := f_n(y)\varphi_{k(\psi(x_n))}(k(y))$ for every $y \in B_Y$, where $k : B_Y \to \mathbb{D}$ is an arbitrary but fixed holomorphic map. Since $(||\phi(x_n)||)_n \to 1$ we may assume that

$$||\phi(x_n)|| > \left(1 - \frac{1}{n}\right)^{\frac{1}{n}} \text{ for every } n \in \mathbb{N}.$$

We choose $y_n^* \in Y^*$ with $||y_n^*|| = 1$ such that $|y_n^*(\phi(x_n))| > (1 - \frac{1}{n})^{\frac{1}{n}}$ and we define

$$h_n(y) := y_n^*(y)^n g_n(y) \text{ for every } y \in B_Y.$$

This yields

$$\sup_{y\in B_Y} v(y)|y_n^*(y)|^n|g_n(y)| \leq \sup_{y\in B_Y} v(y)||y||^n|g_n(y)| \leq \sup_{y\in B_Y} v(y)||y||^n|g_n(y)| \leq ||g_n||_v \leq 1.$$

Hence $(h_n)_n \subset H_v(B_Y)$ and the sequence is bounded. Since the unit ball of $H_v(B_Y)$ is τ_0-bounded, for any compact set $K \subset B_Y$ we can find $M > 0$ such that

$$\sup_{y\in K}|g_n(y)| \leq M \text{ for every } n \in \mathbb{N}.$$

Since K is compact, there must be $0 < r < 1$ such that $K \subset rB_Y$. Hence

$$\sup_{y\in K} |h_n(y)| = \sup_{y\in K} |y_n^*(y)|^n |g_n(y)| \le M \sup_{y\in K} \|y\|^n \le Mr^n.$$

Thus $(h_n)_n \subset H_v(B_Y)$ is bounded and τ_0-convergent to 0. By Lemma 5.1, the sequence $(\|(C_\phi - C_\psi)(h_n)\|_w)_n$ must tend to 0. On the other hand we have for every $n \in \mathbb{N}$:

$$\|(C_\phi - C_\psi)(h_n)\|_w \ge w(x_n)|h_n(\phi(x_n)) - h_n(\psi(x_n))|$$
$$= \frac{w(x_n)}{\tilde{v}(\phi(x_n))}\rho(k(\phi(x_n)), k(\psi(x_n)))|y_n^*(\phi(x_n))|^n \ge c\frac{n-1}{n}.$$

Since $k : B_Y \to \mathbb{D}$ was an arbitrary holomorphic mapping, this yields a contradiction.
Case 2: We have that

$$w(x_n)\max\left\{\frac{1}{\tilde{v}(\phi(x_n))}, \frac{1}{\tilde{v}(\psi(x_n))}\right\} d(\phi(x_n), \psi(x_n)) = w(x_n)\frac{1}{\tilde{v}(\psi(x_n))} d(\phi(x_n), \psi(x_n)).$$

We proceed analogously to Case 1. The only change is that we take $f_n \in H_v(B_Y)$ with $\|f_n\|_v \le 1$ such that $|f_n(\psi(x_n))| = \frac{1}{\tilde{v}(\psi(x_n))}$ and $g_n(y) := f_n(y)\varphi_{k(\phi(x_n))}(k(y))$ for every $y \in B_Y$ where $k : B_Y \to \mathbb{D}$ is an arbitrary but fixed holomorphic map. □

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