

## BOOK REVIEWS

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*The linearization method in hydrodynamical stability theory*, by V. I. Yudovich. Translated from Russian by J. R. Schulenberger. Translations of Mathematical Monographs, vol. 74, American Mathematical Society, Providence, R.I., 170 pp., \$74.00. ISBN 0-8218-4528-4

V. I. Yudovich is a well-known researcher in the mathematical theory of the Navier-Stokes equations in the USSR. In the 1960s he proved a number of results in stability theory and in the bifurcation of steady state and time periodic solutions of the Navier-Stokes equations. In this monograph he develops the fundamental mathematical theory and gives extensive proofs of the basic stability theorems.

In Chapter I the  $L_p$  theory of the Navier-Stokes equations is developed. A summary of interpolation theory, singular integral operators, and the Calderon-Zygmund theory is given. Then  $L_p$  estimates for elliptic and parabolic equations and the linearized Navier-Stokes equations are given. The  $L_p$  theory of the projection  $\Pi$  onto the solenoidal vector fields is given and of the resolvent of the Stokes operator.

In Chapter II the linearized stability theory of the Navier-Stokes equations is developed. One of the main results is that if  $p > 3$ , the  $L_p$  norm of the initial perturbation from equilibrium is sufficiently small, and the spectrum of the linearized stability operator is contained in the right half plane, then the solution is regular, exists for all time, and tends to equilibrium as  $t \rightarrow \infty$ .

The regularity theorem of the Navier-Stokes equations has never been completely resolved. It is known that weak solutions exist for all time; but it has never been proved for general initial data, even

for  $C^\infty$  initial data, that the solutions are regular for all time. Using energy estimates, Shinbrot and Kaniel (*Arch. Rational Mech. Anal.* **21** (1966), 270–285) proved that if the initial values of the solution are sufficiently small in the Dirichlet norm, *viz.*,

$$\|u\| = \left( \int_{\Omega} \sum_{i,j} \left( \frac{\partial u_i}{\partial x_j} \right)^2 dx \right)^{1/2},$$

then the solution is regular for all time and tends to zero. These methods could be extended to consider the perturbations about a smooth equilibrium solution; but using the  $L_p$  theory of the Navier-Stokes equations, Yudovich has given a significant strengthening of this result.

In Chapter III the stability of time periodic solutions of the Navier-Stokes equations is investigated. This chapter contains a careful treatment of Floquet theory for infinite-dimensional dissipative systems. The stability results are proved by constructing the monodromy operator, which takes the solution at time 0 to the solution at time  $T$ . ( $T$  = period of the periodic motions.) The Floquet multipliers are the eigenvalues of the linearized monodromy operator. The periodic motion is stable if all the Floquet multipliers are less than one in modulus and unstable if at least one of the multipliers is greater than one in modulus.

In the unstable case the stable and unstable invariant manifolds are also constructed.

In the case of time periodic solutions of an autonomous system, which is the situation which arises when time periodic solutions bifurcate from an unstable equilibrium solution, one of the Floquet multipliers is always 1, due to the time invariance of the problem. In this case, which is treated in §6, one obtains only orbital stability.

The monograph is restricted entirely to the subject of stability theory and does not treat the closely related topics of loss of stability and bifurcation. This is somewhat surprising, since a number of the early rigorous results in this direction are due to Yudovich. The book contains an excellent bibliography of the Russian literature, but is somewhat spotty with regard to the western literature, especially in the subject of bifurcation theory. A useful supple-

ment to Yudovich's book is D. D. Joseph's treatise, *Stability of Fluid Motion*, Springer-Verlag, 1976.

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*J Contractive matrix functions, reproducing kernel Hilbert spaces and interpolation*, by Harry Dym. American Mathematical Society, Providence, R. I., 147 pp., \$22.00. ISBN-0-8218-0722-6

The primary focus of this book is a collection of interpolation problems for matrix-valued functions. The classical versions of these problems for scalar functions are associated with the names of Nevanlinna–Pick, Caratheodory–Fejer, and Nehari. The Nevanlinna–Pick problem is to find a function analytic on the unit disk mapping the disk into itself which takes prescribed values at a finite number of prescribed points in the unit disk. In the Caratheodory-Fejer problem one seeks an analytic function mapping the unit disk into the right half plane with prescribed values for the first few Taylor coefficients at the origin. In the Nehari problem one seeks a function on the unit circle with supremum norm over the unit circle at most one which has its Fourier coefficients with negative indices equal to a prescribed sequence of numbers. The work of Nevanlinna, Pick, Caratheodory, and Fejer was in the early part of this century while that of Nehari was somewhat later. The solution to all these problems splits into two parts. First, existence of a solution is equivalent to the positive semidefiniteness of a certain matrix or the contractivity of a certain operator built from the data of the problem. Secondly, when this condition holds, either the solution is unique or there are infinitely many solutions which can be described as the image of some linear fractional map applied to the set of analytic functions mapping the unit disk into its closure.

Interest in the matrix-valued versions of all these problems has become particularly intense in the last ten to fifteen years, in large part due to the advent of a new approach in engineering, called *H*-infinity control theory. In control theory, matrix-valued functions arise as transfer functions of physical systems. Input-output