mentor, by telling him: "Poincaré said that geometry is the science of correct reasoning from incorrectly drawn figures; for you it is the other way around."

In the end it doesn't much matter why Ulam shied away from technical mathematics; what matters is that we see him as he was, a very human hero who succeeded in turning a weakness into major strength. Rota's scientific and psychological portrait, suffused with love, pain, understanding, and admiration, succeeds in bringing him to life.

The unusual and unusually beautiful design and artwork, including three portraits of Ulam by Jeff Segler, enhance the value of this volume and the pleasure it gives; the editor, Nicky Cooper, has earned our gratitude.

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Kleinian groups, by Bernard Maskit. Grundlehren der Mathematischen Wissenschaften, vol. 287, Springer-Verlag, Berlin, Heidelberg, New York, 1988, xiii + 326 pp., $77.50. ISBN 3-540-178746-9

Colleagues and friends of Bernie Maskit can turn to almost any page of his book and immediately recognize his characteristic style. The focus is on that part of the field where his own contributions are most strongly felt. Before turning to the book itself, we will make some general comments.

In the complex analysis we learn early about Möbius transformations; the conformal automorphisms of the (Riemann) 2-sphere; and their classification into elliptic, parabolic, and loxodromic/hyperbolic. Each is the composition of two or four reflections in circles which following Poincaré leads easily to its natural extension to a conformal automorphism of the 3-ball. Besides giving rise to the orientation preserving conformal automorphisms of the ball, this procedure also establishes that the totality of these extensions gives the full group of orientation preserving isometries of hyperbolic 3-space $\mathbf{H}^3$.

From an algebraic point of view, each Möbius transformation corresponds to a 2×2 complex matrix of determinent one uniquely determined up to sign. Introducing homogeneous coordinates into its action on the 2-sphere, we find that it becomes a linear transformation, a 4×4 matrix in $SO(1, 3)$, which preserves the Lorentz metric. From this viewpoint, the projective (Klein) model of hyperbolic space can be nicely constructed.

A Kleinian group is a nonelementary discrete group of Möbius transformations, that is, a discrete subgroup of $PSL(2, \mathbf{C})$, or a discontinuous group of isometries in $\mathbf{H}^3$. An elementary group (after Jorgensen) is one with the property that any two elements of infinite order have a common fixed point on the Riemann sphere, which is the sphere $\partial\mathbf{H}^3$ at infinity for $\mathbf{H}^3$. The set of accumulation points of the orbit of a point in $\mathbf{H}^3$ under a Kleinian group G is a closed perfect set on $\partial\mathbf{H}^3$ which either is the whole thing or is nowhere dense. In the latter case, the complementary components are called the region of discontinuity $\Omega(G)$. The quotient $\mathscr{M}(G) = \mathbf{H}^3 \cup \Omega(G)/G$ is a manifold with the induced neighborhood system if there are no elliptic elements; if there are elliptic elements branch lines appear and $\mathscr{M}(G)$ is called an orbifold. If $\Omega(G) \neq \phi$ there is a boundary which inherits a complex structure, but not a metric structure as does the interior. The Kleinian manifold or orbifold $\mathscr{M}(G)$ is represented and visualized by a choice of Dirichlet region, or fundamental polyhedron. All the secrets are hidden in that and are not easily found. For example, the fundamental result [10] that finitely generated groups are finitely presented is a consequence of purely topological analysis. Likewise, all the secrets are hidden in the limit set in case it is fractal (not a circle or the whole sphere) since then G has finite index in its stabilizer.

If a polyhedron has a finite number of faces, $\mathscr{M}(G)$ is "essentially" compact, that is, compact except for "cusps." This condition, which is independent of the choice of base point for the Dirichlet region, is called geometric finiteness. There is a new theory, the theory of automatic groups [2, 3] that shows in particular that the word problem for geometrically finite groups can be solved by computer. It may take a lot of memory to find the finite state automata that do the job. Once found for a group, the elements can be listed by computer without duplication.

With Dehn's lemma and the loop theorem of Papakyriakopoulis and more directly the work of Waldhausen, it has been possible to approach the study of Kleinian groups by studying the associated

Kleinian 3-manifolds [7, 8]. Building on earlier work of Haken, Waldhausen showed in particular that Kleinian manifolds can be systematically cut along internal surfaces until obtaining in the end a union of balls. Reversing the steps, the balls can be put together by successively identifying certain regions on their boundaries until ending up with the original manifold. The simplest version of this construction is that of a solid torus (doughnut) cut along a single cross-sectional disk resulting in a topological ball, and conversely the identification of two disjoint disks in the boundary of a ball resulting in a solid torus. More generally, two disjoint disks in the boundary of a 3-manifold can be identified to obtain a new 3-manifold whose fundamental group is an HNN extension of the fundamental group of the original. The original disk reappears inside the new manifold.

From the 2-dimensional point of view, the geometric process of "combining" Kleinian groups was already known to Klein. A geometric realization of the HNN extension described above goes as follows. Suppose Δ_1 and Δ_2 are disjoint closed round disks such that $T\Delta_i \cap \Delta_i = \phi$ for all $T \neq id$ in G. Choose any Möbius transformation S that sends the exterior of Δ_1 onto the interior of Δ_2. Then G and S generate a new Kleinian group with the required structure.

The approach to Kleinian groups by combination theorems is identified with Maskit, because of his intensive development of this approach in the 60s and early 70s. The goal was to show that all Kleinian groups can be built up from simple pieces by the methodology of the combination theorems. For the case of "function groups," that is, groups which leave invariant a connected open subset of the Riemann sphere, this was soon established to be the case. (This strange name was assigned by Ford because this is the case that there are functions automorphic under the full group.) From the 3-dimensional point of view, in the case of a Kleinian manifold, there is a boundary component such that the inclusion map of its fundamental group into the fundamental group of the 3-manifold is surjective; thus it is quite special.

The trouble with the combination theorems as they stood was that very specific geometric conditions had to be satisfied before groups could be joined together, and except in the simplest cases there was no way of ensuring that these could be satisfied. Indeed, recalling discussions involving Leon Greenberg and Peter Scott in

Maryland in 1973, it could be shown that the constructions then possible could not yield certain topological types of 3-manifolds (that later events showed to have hyperbolic structures).

What now places the theory of Kleinian groups on center stage in 3-dimensional topology is the astounding theory of Thurston concerning the geometrization and uniformization of 3-manifolds and orbifolds [11]. For instance, assume that M is a compact, orientable 3-manifold with nonempty boundary whose fundamental group $\pi_1(M)$ does not have an Abelian subgroup of finite index. Thurston proved that the interior of M is homeomorphic to the interior of a Kleinian manifold if (and only if) every embedded 2-sphere bounds a ball, and every mapping of the torus T^2 into the interior of M which induces an injection $\pi_1(T^2) \to \pi_1(M)$ is homotopic to a map into a torus boundary component of M (in particular, every embedded torus whose fundamental group injects is parallel to a boundary component).

The hyperbolic structure thus found has finite volume if and only if all the boundary components are tori, and then it is uniquely determined; this case covers most knot and link complements in the 3-sphere. Other results cover "most" closed manifolds and orbifolds. The proof involves showing that the decomposition of a 3-manifold into balls which is possible by work of Haken and then Waldhausen can be reversed geometrically. A key part of the proof is the "skinning lemma" [12] (recently simplified by McMullan [6]), that the groups involved can be deformed to the position that desired boundary indentifications become merely a matter of identifying congruent sets.

Maskit's book, *Kleinian groups*, approaches the subject mainly from the point of view of the action on the Riemann sphere. The chapter headings are: I. Fractional Linear Transformations; II. Discontinuous Groups in the Plane; III. Covering Spaces; IV. Groups of Isometries; V. The Geometric Basic Groups; VI. Geometrically Finite Groups; VII. Combination Theorems; VIII. A Trip to the Zoo; IX. B-groups; X. Function Groups. Useful exercises, some with references to the literature, are at the end of each chapter.

There is a brief discussion of half-rotations (called half-twists in the book) and the matrix Lie product, which were introduced by Jorgensen for his studies involving relations between the algebra of matrices and 3-dimensional geometry. Dirichlet fundamental polyhedra are described. There is a complete proof of Poincaré's theorem that a hyperbolic polyhedron whose faces are paired by isometries satisfying rational edge relations and a condition at points at infinity is a fundamental polyhedron for the group generated by the face pairing transformations. There is a chapter of interesting and illuminating examples. The last third of the book is taken up with the complete analysis of the structure of B-groups and more generally all function groups with signature. This culminates the previous discussions and includes a wonderful new proof of Maskit's planarity theorem by Gromov that characterizes regular planar coverings.

Many other topics could not be included. The structure theory of 3-dimensional hyperbolic manifolds and orbifolds and associated 3-manifold topology is not discussed. The relation between Hausdorff dimension of the limit set, exponent of convergence of Poincaré series, spectral theory of the hyperbolic Laplacian, and ergodic theory, which is also very interesting, is not covered [see 9]. Ahlfors's finiteness theorem (from which the contemporary theory originated in 1964) is not proved—there are largely topological proofs now available [5]—nor are the methods for getting estimates for the various quantities appearing in that theorem. Mostow's rigidity theorem [see 13], and more generally deformation theory and moduli spaces, are not covered. In particular the story of quasi-Fuchsian space and its close connection to Teichmüller theory is not related. However, the theory of quasi-Fuchsian groups, which was initiated by Bers and Maskit, is covered without use of quasiconformal mappings. Degenerate groups are found by using one complex parameter families of deformations given by "sliding and bending" (called quakebends in [4]).

This is a carefully written monograph. As a text, it could profitably be used by students who want to undertake a deeper study of Kleinian groups. Some prior knowledge of surface topology, hyperbolic geometry, and the theory of Fuchsian groups may be helpful as these subjects are only briefly reviewed. It will certainly be a valuable reference as the authoritative account of one of the principal approaches to the field.

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Lectures on the asymptotic theory of ideals, by David Rees. Cambridge University Press, Cambridge (London Math Society, Lecture Notes #113), 1988, 200 pp., $24.95. ISBN 0-521-31127-6

Commutative ring theory was born in the early part of this century, a child of Emmy Noether and Wolfgang Krull (David Hilbert filling in as grandpa). It grew up on a tough block, living between algebraic number theory and algebraic geometry. Those two have always been bigger and brasher, and maybe tried to bully it a bit.