is not likely to be a book that the average mathematician would want on his shelf. Every chapter has a bibliography. However, there are some inaccurate references, e.g., an unappropriate quotation on p. 384 (they quote only one author of a statement proved in a joint work in Amer. J. Math. **100** (1978), 727–746). Students who have finished a first course in commutative algebra and are interested in this subject can profit a great deal in studying this book.

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Buildings, by K. S. Brown, Springer-Verlag, New York, Berlin, Heidelberg, 1989, viii + 215 pp., $39.00. ISBN 0-387-96876-8

Lectures on buildings, by M. A. Ronan, Perspectives in Mathematics, No. 7, Academic Press, Orlando, 1989, xii + 216 pp., $27.95. ISBN 0-12-594750-X

Buildings were invented by Jacques Tits to provide a unified geometric setting for the study of groups of Lie type, especially the "exceptional" ones. They are (usually) simplicial complexes, formed by splicing together copies of the "Coxeter complex" associated with a Coxeter group (such as the Weyl group of a simple Lie group). After a while the subject teems with architectural language: chambers ("rooms"), apartments, walls, panels, galleries, blueprints, foundations, etc. But Tits is no ordinary architect. His buildings have some of the flavor of M. C. Escher's drawings: e.g., any two apartments are required to share at least one chamber.

Over the years buildings have proven their conceptual usefulness in a broad spectrum of group-theoretic and homological investigations, and have been applied and generalized (by Tits and others) in a number of interesting directions unforeseen in the earlier work. See [7], for example.

It is impossible to do full justice to the subject in a few pages. But fortunately there exist a number of accessible surveys—notably, the review by C. W. Curtis **[5]** of Tits' influential monograph **[10]**, the address by Tits **[11]** at the 1974 International Congress, and the new survey by M. A. Ronan **[8]**.

By now I should at least have given a precise definition of the term "building," but this is not a completely straightforward matter. For one thing, there is no universal agreement on what level of generality is desirable (e.g., whether Coxeter groups of infinite rank should be allowed). Moreover, different sets of axioms are used by different people, even when the end result is essentially the same. Most important, one needs to know something about Coxeter complexes before launching into the general study of buildings. (Of the two books under review, Ronan starts off more briskly, defining "building" on p. 27, while Brown gets to a formal definition only on p. 76.)

As defined by Bourbaki **[3]**, a Coxeter group W of rank n is an abstract group generated by a set S of elements of order 2 (with $|S| = n$), subject only to the most obvious relations: those specifying the order—possibly infinite—of each product of a pair of generators. Important special cases are Weyl groups and other finite reflection groups (acting in a real Euclidean space) and affine Weyl groups (Weyl groups extended by their root lattices). The "parabolic" subgroups (subgroups conjugate to those generated by subsets of S) play an important role as stabilizers of various sets in a naturally defined geometric representation of W. This is easily visualized for a finite reflection group, if one uses the reflecting hyperplanes to form a simplicial decomposition of the unit sphere. This is the idea of the Coxeter complex, which can be specified abstractly just in terms of the system of parabolic subgroups (and their cosets). When W is finite, the Coxeter complex has the homotopy type of a sphere; otherwise it is contractible.

A building is a simplicial complex which is a union of "apartments," each of them a copy of some fixed Coxeter complex. A "chamber" is a simplex of maximum dimension, and (as already noted) any two apartments are required to share at least one chamber. It is also required that two apartments containing a given chamber be isomorphic under an isomorphism which fixes every one of their common simplexes. It turns out that a building of "spherical" type (with apartments coming from finite Coxeter groups) has the homotopy type of a bouquet of spheres; all other buildings are contractible.

While Coxeter complexes are intimately tied to Coxeter groups, the relations between buildings and groups are more problematic. From a group with a "BN-pair" ("Tits system" in the terminology of Bourbaki) one gets a building based on the system of "parabolic" subgroups. The latter are defined relative to the Bruhat decomposition, which in turn depends on the "Weyl group" of the BN-pair, always a Coxeter group. The given group then acts naturally on the building. In the other direction, one can start with a building and try to locate a suitable group acting on it. In important special cases (cf. again **[10]**) one may be able to go back and forth, in the spirit of the fundamental theorem of projective geometry.

Buildings associated with groups of Lie type (algebraic, analytic, p-adic, finite, Kac-Moody,...) have been especially well studied, including those of spherical type (using the Coxeter complex of the Weyl group) and those of "Euclidean" or "affine" type (using the affine Weyl group instead). The latter buildings were suggested by work of N. Iwahori and H. Matsumoto **[6]** on algebraic groups over p-adic fields; they have since been studied in great depth and generality by F. Bruhat and Tits **[4]**. Buildings of both types have interacted beautifully with the study of the cohomology of arithmetic and related groups **[1, 2]**, e.g., the Euclidean buildings play for p-adic groups the role of symmetric spaces in the theory of real Lie groups. Moreover, in passing to a compactification (where group cohomology can be better calculated), the original spherical building of the underlying algebraic group (ignoring the p-adic valuation) miraculously appears "at infinity." In rank 1, a Euclidean building is a tree, a case of special interest to combinatorial group theorists, who get a lot of mileage out of studying discrete groups by their actions on trees.

The books of Brown and Ronan are the first (apart from the research-level monograph of Tits **[10]**) to attempt a systematic development of the subject. They have some superficial resemblance, having similar titles and being of similar length and provenance (graduate courses given around 1987). Both are carefully written and liberally supplied with exercises. But in fact their aims and scope differ markedly. One symptom of this (mentioned above) is Brown's more leisurely build-up to the definition of buildings. Another symptom: Each book includes roughly 60 references, but only a quarter of these occur in both books. In spite of some inevitable overlap in coverage, the books are in fact largely complementary.

Roughly speaking, Brown is more interested in describing special cases of buildings (spherical and Euclidean), in order to show how the applications to group cohomology come about, whereas Ronan is more concerned with *classification* problems, emphasizing group actions on buildings.

Brown's first three chapters take the reader on a guided tour of finite reflection groups, general Coxeter groups, and Coxeter complexes, with emphasis on detailed examples and motivation. Then three longer chapters introduce buildings and related groups, focusing mainly on the spherical and Euclidean types, and featuring classical linear groups as examples. The introduction to Bruhat-Tits theory (fixed point theorem, etc.) is especially helpful, since the literature of the subject is so formidable.

Brown is careful but informal, and always user-friendly ("most people don't learn about double cosets until they need to, so let's take a moment to review them$\cdots$"). He also provides a good dose of history. In his final chapter he sketches concisely some of the main applications of buildings to the cohomology of groups, his own source of interest in the subject. This draws mainly on the work of A. Borel and J.-P. Serre **[1, 2]**. (The author points out an overstatement on line 10 of p. 192, where it should not be asserted that L acts on X.)

Ronan begins with a suitable notion of "chamber system," then discusses Coxeter groups and complexes in an efficient but somewhat abstract style. His definition of building is a recent one (due to Tits), which requires the existence of a suitable "W-distance function." This differs from the definition Brown gives, by not mentioning apartments explicitly; but the apartments soon appear. BN-pairs and their buildings provide key examples. This material is covered clearly but rapidly, without the sort of lengthy motivational sections found in Brown. One of the early chapters (largely independent of the rest of the book) discusses local properties and coverings of chamber systems, following Tits **[12]**.

The next few chapters proceed systematically toward the classification and construction of spherical buildings of rank at least 3 (when W has connected Coxeter diagram). This is a streamlined version of what was done by Tits **[10]**, emphasizing the "Moufang property," a transitivity condition for certain groups of automorphisms which may fail if the rank is 1 or 2. A key ingredient is a method of Ronan and Tits **[9]** for constructing Moufang buildings.

The last two chapters of the book are devoted to the more recent description and classification of affine (Euclidean) buildings of rank at least 4, along the lines of Tits **[13]**, including the study of the spherical building at infinity. One associates with an affine building a family of "root groups with valuation" and a resulting affine BN-pair. The upshot is that all (locally finite) buildings of this type come in a reasonable way from algebraic groups. In the course of this classification, the earlier classification of spherical buildings plays a key role. Finally, there are appendices on various related matters: Moufang polygons, nondiscrete buildings, etc.

Both of these fine books belong in every research library. Brown's approach will appeal to those looking for a gentle introduction to the "classical" theory of buildings (and to those who share his interest in cohomology of discrete groups), but Ronan's book will do more to initiate the already motivated reader into current research on buildings and groups.

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Fourier analysis, by T. W. Körner. Cambridge University Press, Cambridge, 1988, xii + 591 pp., $95.00. ISBN 0-521-25120-6

When Arthur Dent, fortified by a prophylactic glass of beer, set off in ultimate pursuit of Life, the universe and everything, he was armed with only a towel. He was no braver than Tom Körner. This is truly an ambitious voyage through Fourier Analysis. Tom has real armour as a harmonic analyst of considerable personal accomplishment. Yet in both cases, the coping mechanism is the same—a kind of gentle English silliness which amuses, irritates, and finally enchants.

Let's go back to the beginning. The declared assignment is to provide a shop window for Fourier Analysis in a textbook which can be understood by a British undergraduate who possesses that knowledge which can be "supposed after two years of study." (A word of warning: the author teaches at the University of Cambridge where quite a lot is supposed to happen.) It follows that there are some bread and butter issues on which we must agree. What precise mathematical background is to be assumed, how do we organize the material so as to incorporate historical perspective, and which subject matter do we choose from the vast treasure house of Fourier Analysis?

The first practical decision on mathematical background concerns Lebesgue integration. Although Hardy wrote in 1922 that "No account of the theory of Fourier's series can possibly satisfy the imagination if it takes no account of the ideas of Lebesgue; the loss of elegance and of simplicity of statement is overwhelming" there still appears to be great reluctance to introduce these ideas early. Dr. Körner goes out of his way to avoid the Lebesgue integral (although he is obliged to define a null set in order to state Carleson's convergence theorem) and, it must be admitted, does so in a thoroughly sensible way. He concentrates wherever possible on continuous functions with finite integrals and even labels that class