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K. SUNDARESAN
CLEVELAND STATE UNIVERSITY

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Equimultiplicity and blowing up, by M. Herrmann, S. Ikeda and U. Orbanz, with an appendix by B. Moonen. Springer-Verlag, Berlin, Heidelberg, New York, 1988, xvii + 629 pp., $99.50. ISBN 3-540-15289-x

This book is intended as a special course in commutative algebra. It deals with two main subjects: the first is the notion of equimultiplicity and the second is the algebraic study of various graded rings in relation to blowing up. This topic arises directly from the resolution of algebraic and complex-analytic singularities. In 1964 Heisuke Hironaka solved the problem of the resolution of singularities of an algebraic variety over a field of characteristic zero. This problem of resolution of singularities, simply put, is to prove (or disprove):

(RES) *For any algebraic variety X over an algebraically closed field k there exists a proper map $f\colon Y \to X$, with Y nonsingular (smooth), such that f is an isomorphism over some open dense subset U of X (i.e. f maps $f^{-1}(U)$ isomorphically onto U).*

But even as it stands, (RES) poses a fundamental question, which is unanswered when k is of characteristic > 0 and X has dimension ≥ 4. (RES) was proved for curves over $\mathbb{C}$ in the last century by Kronecker, Max Noether, and others. Several geometric proofs of (RES) for X a surface over $\mathbb{C}$ were proposed by Italian geometers. The first completely rigorous proof was given by Walker in 1936. What he did, basically, was to show how to patch together "local" resolutions constructed by Jung in 1908. (H. W. E. Jung (1876–1953) was full professor of mathematics at Martin-Luther-University of Halle-Wittenberg. His method has influenced many other works on resolution.) Purely algebraic proofs of resolution for surfaces over fields of characteristic zero were given by Zariski.

His work on resolution in the late 30s and early 40s culminated in a memoir, in which he proved (RES) for surfaces, with f obtained by successively blowing up points and nonsingular curves of maximal multiplicity ("theorem of Beppo Levi," but see the new insight into this theorem discussed in **[5]**), and then deduced (RES) for three-dimensional X. In his great 1964 paper, Hironaka introduced the general notion of blowing up, including the notion of monoidal transformation in algebraic geometry and in analytic geometry. Moreover, Hironaka's proof also uses the notions of equimultiplicity and normal flatness of a scheme along a subscheme. Therefore Hironaka discussed the above-mentioned theorem of B. Levi-Zariski which asserts an almost canonical process of resolving the singularities of an arbitrary algebraic (or analytic) surface embedded in a three-dimensional algebraic (or analytic) manifold. In the theorem of B. Levi-Zariski, the equimultiplicity plays an important role as a condition to be imposed on the centers of monoidal transformations. In the resolution of singularities in arbitrary dimensions, Hironaka takes the normal flatness as the condition to be imposed on the centers of monoidal transformations, which turns out to be equivalent to the equimultiplicity in the hypersurface case as that of B. Levi-Zariski.

This is a new principle of the resolution of singularities in general, modelled on the theorem of B. Levi-Zariski. A first purely algebraic study of this principle was given in the monograph [2]. After publishing this book the reviewer rediscovered the unpublished thesis of Everett C. Dade **[1]** (see Math. Nachr. **89** (1979) on p. 286, Remark 1). This thesis then has influenced other works on equimultiplicity and blowing up (see, for example, J. Lipman **[4]** or some papers of the authors of the book under review). Since that time, the reviewer estimates that approximately 20 papers have appeared which involve this concept of equimultiplicity and blowing up or its generalizations. The authors have now produced a book surveying these algebraic developments on multiplicities, Hilbert functions, blowing up and some related topics. It involves the use of some of the most difficult techniques of modern commutative algebra, particularly the theory of graded structures and of local cohomology, and the use of local duality of graded modules.

It is now time to describe the structure of this book. It consists of nine chapters and a nice appendix by B. Moonen on a geometric interpretation of equimultiple ideals in complex-analytic geometry.

Chapter 1 contains an account of basic facts about multiplicities, Hilbert functions and reductions of ideals. Chapter 2 contains some general facts about graded rings that arise in connection with blowing up. The presentation uses also ideas of the above-mentioned thesis of E. C. Dade. Chapter III gives various characterizations of quasi-unmixed local rings. Recently it has been recognized that a useful tool for these characterizations are asymptotic sequences, which are somewhat analogous to regular sequences for the characterization of local Cohen-Macaulay rings.

Chapter IV presents various notions of equimultiplicity. For a hypersurface and a regular subvariety there exists a natural notion of equimultiplicity, and there are different directions of generalization:

(a) to the nonhypersurface case,

(b) to nonregular subvarieties. In these more general situations there are weaker and stronger notions, all of which specialize to equimultiplicity in the original case. The authors mention three essentially different algebraic generalizations of equimultiplicity together with a numerical description of each condition. This chapter also describes a main technical tool, a certain graded homomorphism first studied by Hironaka in proving (RES) and later on generalized by A. Grothendieck and in [2].

Chapter V investigates Cohen-Macaulay properties under blowing up by using conditions of Chapter IV. Chapter VI indicates that the new conditions of Chapter IV are useful in the study of a certain numerical behaviour of local rings R under blowing up an ideal I of R such that R/I need not be regular. This chapter collects known results of B. M. Bennett, B. Singh and of [2] published between 1970 and 1977. Some parts of the chapter are inspired by Dade's thesis [1].

Chapter VII gives a self-contained compendium of local cohomology and local duality of graded modules including a brief description of derived functors. However, the local duality (see p. 313) is of quite a simple form, that is, without using a dualizing complex.

Chapter VIII touches the theory of Buchsbaum local rings by presenting results on so-called generalized Cohen-Macaulay rings. The theory is developed in a unified manner according to Shiro Goto and Ngo Viet Trung. (The appendix in [6] gives various possibilities to generalize the concept of Buchsbaum rings.)

Finally, Chapter IX is concerned with applications of local cohomology to the Cohen-Macaulay behaviour of Rees algebras.

To sum up, this book is a solid treatment of many areas of the subject. It is clearly and essentially self-contained written. Good examples demonstrate the concept of the book.

The book has some weaknesses. It is a special course in purely commutative algebra of a rather technical nature. It does not contain concrete applications in algebraic geometry or in other fields of mathematics. This is not likely to be a book that the average mathematician would want on his shelf. Every chapter has a bibliography. However, there are some inaccurate references, e.g., an unappropriate quotation on p. 384 (they quote only one author of a statement proved in a joint work in Amer. J. Math. **100** (1978), 727–746). Students who have finished a first course in commutative algebra and are interested in this subject can profit a great deal in studying this book.

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Wolfgang Vogel
Martin Luther-Universität

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Buildings, by K. S. Brown, Springer-Verlag, New York, Berlin, Heidelberg, 1989, viii + 215 pp., $39.00. ISBN 0-387-96876-8

Lectures on buildings, by M. A. Ronan, Perspectives in Mathematics, No. 7, Academic Press, Orlando, 1989, xii + 216 pp., $27.95. ISBN 0-12-594750-X

Buildings were invented by Jacques Tits to provide a unified geometric setting for the study of groups of Lie type, especially the "exceptional" ones. They are (usually) simplicial complexes, formed by splicing together copies of the "Coxeter complex" associated with a Coxeter group (such as the Weyl group of a simple Lie group). After a while the subject teems with architectural language: chambers ("rooms"), apartments, walls, panels, galleries, blueprints, foundations, etc. But Tits is no ordinary architect. His buildings have some of the flavor of M. C. Escher's drawings: e.g., any two apartments are required to share at least one chamber.

Over the years buildings have proven their conceptual usefulness in a broad spectrum of group-theoretic and homological investigations, and have been applied and generalized (by Tits and others) in a number of interesting directions unforeseen in the earlier work. See [7], for example.