42. W. Nef, *Die Funktionentheorie der partiellen Differentialgleichungen zweiter Ordnung* (*Hyperkomplexe Funktionentheorie*), Bull. Soc. Firbourgeoise Sci. Nat. **37** (1944), 348–375. (Also see papers on this topic by W. Nef in Comment. Math. Helv., vols. 15, 16 and 17.)

43. I. R. Porteous, *Topological geometry*, Van Nostrand Reinhold, London, 1969.

44. M. Riesz, *Clifford numbers and spinors*, Chaps. I–IV, Inst. Fluid Dynamics and Appl. Math., Univ. of Maryland, College Park, Md., 1958.

45. J. Ryan, *Clifford analysis with generalized elliptic and quasi elliptic functions*, Applicable Anal. **13** (1982), 151–171.

46. ______, *Complexified Clifford analysis*, Complex Variables Theory Appl. **1** (1982), 119–149.

47. M. Sato, *Theory of hyperfunctions*. I and II, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **8** (1959/1960), 139–193; 387–436.

48. L. Schwartz, *Théorie des distributions*, Hermann, Paris, 1966.

49. B. Simon, *The* $P(\phi)_2$ *Euclidean* (*quantum*) *field theory*, Princeton Univ. Press, Princeton, N. J.. 1974.

50. F. Sommen, *Representation of distributions in* $\mathbf{R}^3$ *by regular functions of a quaternion variable*, Simon Stevin **53** (1979), 131–150.

51. ______, *Hypercomplex Fourier and Laplace transforms*. I, Illinois J. Math. **26** (1982), 332–352.

52. ______, *Some connections between Clifford analysis and complex analysis*, Complex Variables Theory Appl. **1** (1982), 97–118.

53. ______, *Hypercomplex Fourier and Laplace transforms*. II, Complex Variables Theory Appl. **1** (1983), 209–238.

54. V. Souček, *Complex-quaternionic analysis applied to spin-*(1/2) *massless fields*, Complex Variables Theory Appl. **1** (1983), 327–346.

55. R. F. Streater and A. S. Wightman, *PCT, spin and statistics, and all that*, Benjamin, New York, 1964.

56. A. Sudbery, *Quaternionic analysis*, Math. Proc. Cambridge Philos. Soc. **85** (1979), 199–225.

57. H. G. Tillmann, *Randverteilungen analytischer Funktionen und Distributionen*, Math. Z. **59** (1953), 61–83.

58. ______, *Distributionen als Randverteilungen analytischer Funktionen*. II, Math. Z. **76** (1961), 5–21.

59. ______, *Darstellung der Schwartzschen Distributionen durch analytische Funktionen*, Math. Z. **77** (1961), 106–124.

60. ______, *Darstellung vektorwertiger Distributionen durch holomorphe Funktionen*, Math. Ann. **151** (1963), 286–295.

61. V. S. Vladimirov, *Methods of the theory of functions of many complex variables*, M. I. T. Press, Cambridge, Mass., 1966.

62. ______, *Generalized functions in mathematical physics*, "Mir", Moscow, 1979.

63. A. I. Zayed, *Hardy space theory for functions with values in a Clifford algebra*, 1983 (preprint). (See the abstract *Boundary behavior of hypercomplex functions with values in a Clifford algebra*, Abstracts Amer. Math. Soc. **4** (1983), 71.)

Note: Two new papers in Clifford analysis have just appeared in Complex Variables Theory Appl. **2** (1983). R. Delanghe is a member of the editorial board of this new journal.

RICHARD D. CARMICHAEL

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 1, July 1984

0273-0979/84 $1.00 + $.25 per page

Cohomology of groups, by Kenneth S. Brown, Graduate Texts in Mathematics, Vol. 87, Springer-Verlag, New York, 1982, x + 306 pp., $28.00. ISBN 0-3879-0688-6

The cohomology theory of abstract groups is a tool kit, in much the same way as is representation theory. One of its attractions is its breadth: the methods derive from algebra and topology and often connect with algebraic number theory. A good example is the theory of Euler characteristics, one of the highlights of the present book and one to which its author has made brilliant contributions. It may be of interest to trace one thread in the development of this topic.

In 1951 Philip Hall gave a course of lectures in Cambridge on general group theory in which a certain numerical function made a fleeting appearance. If G is a group containing a finitely generated free abelian subgroup of finite index, then one may define the rank $r(G)$ unambiguously to be the rank of any one such free abelian subgroup. Hall contrasted this rank function with another, defined whenever G is a finite extension of a finitely generated free group H. If n is the rank of H, then the rational number

$$\delta(G) = 1 + (n-1)/|G:H|$$

is independent of the choice of H in G. Hall suggested that "δ-groups" might repay further study alongside the "groups with rank". For example, if A, B are rank groups, so is $A \times B$ and $r(A \times B) = r(A) + r(B)$; while if A, B are δ-groups, so is $A * B$ and $\delta(A * B) = \delta(A) + \delta(B)$.

Ten years later, C. T. C. Wall (1961) independently came to the same conclusion as Hall, but by way of topology and in a more general context. Let the group G be a finite extension of a group H whose classifying space X is a finite complex. If $\chi(X)$ is the classical Euler characteristic of X, then

$$\chi(G) = \chi(X)/|G:H|$$

is independent of H. The rational number $\chi(G)$ is called the Euler characteristic of G. If G happens to be a δ-group, then $\chi(G) = 1 - \delta(G)$. The additivity formula for δ generalizes to one for free products with amalgamation and takes the following form: if $G = A *_C B$, then

$$\chi(G) = \chi(A) + \chi(B) - \chi(C)$$

(provided all χ's are defined). For example, $\chi(\mathrm{SL}_2(\mathbf{Z})) = -1/12$, because $\mathrm{SL}_2(\mathbf{Z}) = C_4 *_{C_2} C_6$, where C_n denotes a cyclic group of order n.

There is another reason why $\mathrm{SL}_2(\mathbf{Z})$ has an Euler characteristic: its commutator subgroup is free of rank 2 and has index 12. Viewed in this way, $\mathrm{SL}_2(\mathbf{Z})$ is the tip of an enormous iceberg: it is an example of an arithmetic group and all such groups are of the type considered by Wall. The situation is as follows. Let G be an algebraic subgroup of GL_n defined over $\mathbf{Q}$ and Γ an arithmetic subgroup of $G(\mathbf{Q})$ (meaning that $\Gamma \cap G(\mathbf{Z})$ has finite index in both Γ and $G(\mathbf{Z})$). Then Γ is a discrete subgroup of the Lie group $G(\mathbf{R})$. If K is a maximal compact subgroup of $G(\mathbf{R})$, then $X = G(\mathbf{R})/K$ is diffeomorphic to a euclidean space of dimension say d. Now Γ acts on X and the stabilizers of points are finite subgroups.

There exists in Γ a torsion-free subgroup, say H, of finite index. It follows that the action of H on X is free and so the cohomological dimension cd H of H is at most d. If X/H is compact, then cd H equals d; but even when X/H is not compact, a formula for cd H was found by Borel and Serre (1974), provided G is semisimple and connected. They did this by enlarging X and the action of H to a manifold with corners $\bar{X}$ so that X is the interior of $\bar{X}$ and $\bar{X}/H$ is compact. Then $\bar{X}/H$ has the homotopy type of a finite complex. The number cd H is independent of the choice of H inside Γ, whence Γ has an Euler characteristic in the sense of Wall.

Number theory enters through the work of Harder (1971). The Gauss-Bonnet measure on X lifts to a unique invariant measure μ on $G(\mathbb{R})$. Harder proved the deep theorem that $\chi(\Gamma) = \mu(G(\mathbb{R})/\Gamma)$. This leads to an explicit fromula for $\chi(G(\mathbb{Z}))$ in terms of values of the zeta-function. For example, $\chi(\mathrm{SL}_2(\mathbb{Z})) = \zeta(-1)$ and since $\zeta(-1) = -1/12$, this gives a third way of arriving at the Euler characteristic of $\mathrm{SL}_2(\mathbb{Z})$.

Serre (1971) utilized Harder's results and integrality properties of $\chi(\Gamma)$ to establish integrality results about the values of the zeta-function of a number field. His very influential article contains the first coherent account of the basic facts on Euler characteristics of abstract groups. Brown's important paper (1974) is a direct outgrowth of Serre's work. In this paper Brown extends the domain of definition of the Euler characteristic function to groups that are virtually FP. These are groups G that contain a subgroup H of finite index, where H is of type FP: this means that $\mathbb{Z}$, as trivial H-module, admits an H-projective resolution of finite length with all terms finitely generated (or, equivalently, if H is finitely presented, that the complex $K(H,1)$ is finitely dominated). The definition of $\chi(G)$ is exactly the same as in Wall's case, but the fact that the resulting rational number is independent of H is nontrivial, a consequence of the result of Swan that a finitely generated projective module over a finite group is locally free.

There is a remarkable integrality theorem of Brown that provides information about the finite subgroups of G in terms of $\chi(G)$. If G is virtually FP and d is the lowest common multiple of the orders of the finite subgroups of G, then $d\chi(G)$ is an integer. As a very special case, if H is a free group of rank n, normal in G, of index a power of a prime p and $p \nmid n - 1$, then G splits over H. No direct proof of this result is known.

All the above material occurs in the second half (Chapters 7 to 10) of Brown's book and much of it in Chapter 9 on Euler characteristics, where the integrality theorem and related results are proved. For the proofs, an understanding of finiteness conditions (such as cd and FP) is necessary and these, together with a welcome detailed account of their topological significance, are the subject of Chapter 8. A crucial ingredient in the proofs is a representation theory on CW-complexes, pioneered by Quillen. The resulting equivariant homology is studied in Chapter 7 and involves spectral sequences. These are still found forbidding by many people who are otherwise kindly disposed to homological algebra. I urge them to take the trouble to study this chapter thoroughly: they will be rewarded with a good grasp of how spectral sequences arise and are used in group theory. I quarrel only with the author's advice (on p. 162), after announcing he will omit certain proofs, that "the reader can either supply the missing proofs (which are routine) or consult any text which treats spectral sequences". The first alternative could be rather discouraging for a student who meets these matters for the first time, since the proofs cannot, in the nature of things, be routine to him.

Chapters 7 to 10 are excellent. The author shows faultless judgment in what to prove, what to summarize and what to omit; he takes the reader frequently to the frontiers of research. For the record, I note three items that can be updated: p. 219, in Example 8, the work of Alperin and Shalen has now appeared (1982); p. 223, Eckmann and Linnell (1983) have completed the classification of Poincaré duality groups of dimension 2; p. 241, the Bass conjecture has been established by Linnell (1983) for all locally residually finite groups (Lemma 4.1). Further, Exercise 2 on p. 252 presents a theorem of Gottlieb that establishes a fascinating link between the Euler characteristic of a group and its commutator structure; a recent paper of Gottlieb (1983) carries this further.

I am not so happy with the first half of the book. This contains all the material one would normally expect in a course on group cohomology. The author's declared intention is to develop the subject from the beginning for students who know a little algebra and a little topology and to "take neither a purely algebraic nor a purely topological approach". This is laudable, but I fear that the balance has tipped decidedly in favour of topology. The treatment could easily frighten off students with the "wrong" background who need to know about group cohomology; this would be a great pity since it is precisely such students who would gain most from seeing topology at work in a group-theoretic setting. In fact, the knowledge of topology needed to read Brown's book is not extensive; a preliminary study of a text such as Massey (1967) is quite sufficient. Students with the "right" background suffer in a different way, by being denied many simplifying insights from group theory. For example, Hopf's description of H_2G is a completely elementary result if done algebraically, but this is far from evident in the topological derivation given in Chapter 2 (though it is true that there are exercises in Chapter 4 from which the algebraic proof can be extracted). Again, the standard resolution is a natural enough object topologically but it has no significance in group theory; there exist resolutions that have. Doing extension theory by manipulations with factor sets ignores the existence of underlying constructions which explain why it works; an account of extension theory from the group theoretic point of view is given in D. J. S. Robinson's book (1982).

The emphasis on topology in the early chapter of Brown's book can be justified on historical grounds and, more to the point, by the fact that topology is indispensable for the work reported on in the last chapters. The book fills an important gap in the literature. I hope it will be studied by all who wish to understand group cohomology; they will be richly rewarded.

References

R. C. Alperin and P. B. Shalen, *Linear groups of finite cohomological dimension*, Invent. Math. **66** (1982), 89–98.

A. Borel and J.-P. Serre, *Corners and arithmetic groups*, Comment. Math. Helv. **48** (1974), 244–297.

B. Eckmann and P. Linnell, *Poincaré duality groups of dimension two*. II, Comment. Math. Helv. **58** (1983), 111–114.

K. S. Brown, *Euler characteristics of discrete groups and G-spaces*, Invent. Math. **27** (1974), 229–264.

D. H. Gottlieb, *Transfers, centers, and group cohomology*, Proc. Amer. Math. Soc. **89** (1983), 157–162.
G. Harder, *A Gauss-Bonnet formula for discrete arithmetically defined groups*, Ann. Sci. Ecole Norm. Sup. (4) **4** (1971), 409–455.
P. Linnell, *Decomposition of augmentation ideals and relation modules*, Proc. London Math. Soc. **47** (1983), 83–127.
W. S. Massey, *Algebraic topology: an introduction*, Harcourt, Brace and World, 1967; reprinted, Graduate Texts in Math., vol. 56, Springer-Verlag, 1977.
D. J. S. Robinson, *A course in the theory of groups*, Graduate Texts in Math., No. 80, Springer-Verlag, 1982.
J.-P. Serre, *Cohomologie des groupes discrets*, Ann. of Math. Stud. **70** (1971), 77–169.
C. T. C. Wall, *Rational Euler characteristics*, Proc. Cambridge Philos. Soc. **57** (1961), 182–183.

K. W. GRUENBERG

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 1, July 1984

0273-0979/84 $1.00 + $.25 per page

Schauder bases in Banach spaces of continuous functions, by Zbigniew Semadeni, Lecture Notes in Mathematics, vol. 918, Springer-Verlag, Berlin, 1982, v + 135 pp., $9.80. ISBN 3-5401-1481-5

Let K be a metrizable compact set and let $C(K)$ denote the space of continuous functions on K endowed with the supremum norm. $C(K)$'s are, together with the Hilbert space, the most "popular" Banach spaces, applied and studied in practically every branch of analysis. For example, recall that every separable Banach space is linearly isometric to a subspace of, e.g. $C([0,1])$; a stochastic process can be thought of as a probability measure on $C([0,T])$. (More specifically, let us mention Ciesielski's construction of the Brownian motion using the Faber-Schauder basis of $C([0,1])$; see [**2**]; more about this later.)

Both for numerical and theoretical purposes the need arises to consider approximations of continuous functions. This is done by using *Schauder bases* (such as spline bases, etc.). Recall that a sequence (f_n) of elements of a Banach space X is called a Schauder basis iff every element f of X can be uniquely written as the sum (convergent in X) $\Sigma_{n\in\mathbf{N}} t_n f_n$, t_n being scalars. The partial sums $S_N f = \Sigma_{n=1}^N t_n f_n$ are then the successive approximations of f.

It has been known for quite a while (some 30 years) that every $C(K)$-space has a Schauder basis; moreover, a complete isomorphic classification of $C(K)$'s as Banach spaces has been available for almost as long [**1**, **5**, **7**, **8**]. More recently, it has been shown that in fact any separable space from a larger class of $\mathscr{L}^\infty$-spaces (or $\mathscr{L}^p$, $1 \leqslant p \leqslant \infty$) has a basis [**6**]; this, however, is not discussed in the book. The above contrasts with negative results for general Banach spaces: some of them do not even possess the much weaker approximation property (Enflo [**3**]).

The question of existence being settled, the emphasis switches to looking for bases with "nice" properties and to studying specific bases. To these the bulk