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Operator inequalities, by Johann Schröder, Mathematics in Science and Engineering, vol. 147, Academic Press, New York, 1980, xvi + 367 pp., $39.50.

There exists an extensive literature on the theory of differential inequalities relative to initial value problems in finite and infinite dimensional spaces, including random differential inequalities [**2, 3, 5, 6–8, 9**]. This theory is also known as the theory of comparison principle. The corresponding theory of differential inequalities related to boundary value problems of ordinary and partial differential equations has also developed substantially [**1, 4, 5, 9**]. The treatment of this general theory of differential inequalities is not for its own sake. The essential unity is achieved by the wealth of its applications to various qualitative and quantitative problems of a variety of dynamical systems. This theory can be applied employing as a candidate a suitable norm or more generally a Lyapunov-like function, to provide an effective mechanism for investigating various problems. It is therefore natural to expect the development of an abstract theory so as to bring out the unifying theme of various theories of inequalities. The present book is an attempt in this direction.

As the title suggests, this book is concerned with inequalities that are described by operators which may be matrices, differential operators, or integral operators. As an example the inverse-positive linear operators M, may be described by the property that $Mu \geqslant 0$ implies $u \geqslant 0$. These are operators M which have a positive inverse M^{-1}. For an inverse-positive operator M one can derive estimates of $M^{-1}r$ from properties of r without knowing the inverse M^{-1} explicitly. This property can be used to derive a priori estimates for solutions of equations $Mu = r$. There are important applications as well. For example, if M is inverse-positive, an equation $Mu = Nu$ with a nonlinear operator N may be transformed into a fixed-point equation $u = M^{-1}Nu$, to which then a monotone iteration method or other methods may be applied, if N has suitable properties. Moreover, for inverse-positive M the eigenvalue problem $M\phi = \lambda\phi$ is equivalent to $T\phi = \kappa\phi$ with $\kappa = \lambda^{-1}$ and $T = M^{-1} \geqslant 0$. Thus the theory of inverse-positive operators is closely related to the eigenvalue theory of positive operators, in particular to the Perron-Frobenius theory on positive eigenelements.

Main topics covered in this book are the theory of inverse-positive linear operators with applications to M-matrices, the boundary maximum principle, oscillation theory, theory of eigenvalues, and convergence proofs. A corresponding theory of inverse-monotone nonlinear operators is also given. Abstract terms are employed in developing the theory. However, abstract results are not considered as ends in themselves, but as means to obtain results for concrete problems. For example, to treat inverse-positive linear operators, first an abstract theory is developed, and then this theory is applied to matrices and differential operators. A theory on abstract inverse-monotone linear operators is also formulated, but most results on concrete problems are derived directly. Since the variety of possible estimates and the variety of nonlinear operators which may be of practical interest is so immense, the book concentrates on investigating some problems by a simpler direct approach in detail and describing certain methods in abstract terms so as to provide the tools for obtaining results on problems not considered here in an analogous way. Described in a more general way, the basic results have the form of input-output statements, where properties of an unknown element are derived from known properties of the image Mv of this element v under an operator M. Since here properties of elements in the domain of the operator are derived from properties of elements in its range, one can speak also of range-domain statements and range-domain implications. Such implications can formally be written as $Mv \in C \rightarrow v \in K$. The implication $Mv \leqslant Mw \rightarrow v \leqslant w$, which describes inverse-monotone operators, is a special case. By properly applying such implications, together with Schauder's fixed-point theorem or other results of existence theory to a given equation, one can often prove the existence of a solution that lies in a certain set K.

As is natural, the topics of this monograph and their presentation correspond to the scientific interest of the author. The topics presented intersect with some well-established theories and touch others. The discussion related to partial differential operators is essentially omitted because of limitations of the size of the book. However, it is pointed out that most of the results which have the form of range-domain implications can be carried over without any essential difficulty to elliptic-parabolic operators of the second order. This also applies to the theory of inverse-monotone operators as well as to pointwise norm estimates. Of course, results for partial differential equations, including existence statements, are, in general, more difficult to achieve.

The book is accessible to readers having acquaintance with the theory of differential inequalities. It should be pointed out that the book gives special emphasis to differential operators related to boundary value problems only. The contents are well motivated and illustrated. The knowledge of the material of this volume will well prepare one to start further work in this fascinating area.

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Stochastic filtering theory, by G. Kallianpur, Springer-Verlag, New York, Heidelberg, Berlin, 1980, xvi + 316 pp.,$29.80.

An important problem in statistical communication theory is the separation of random signals from random noise. These phenomena are modelled by stochastic processes s_t, n_t called respectively the signal and noise processes. The signal cannot be observed directly; instead, at time t the sum

$$z_t = s_t + n_t \tag{1}$$

is observed. Roughly speaking, the filtering problem is to make a "best" estimate for s_t given observations z_τ for times $\tau \leqslant t$. Closely related problems are to best estimate s_T when $t < T$ (the prediction problem) and for $T < t$ (the data smoothing problem). By "best" estimate $\hat{s}_t$ let us mean an estimate minimizing the mean squared error $E(s_t - \hat{s}_t)^2$, with $E(-)$ denoting expected value. Pioneering work on the filtering problem was done by Wiener and Kolmogorov during the 1940s. In that work, the filtering problem was considered in the frequency domain, by taking Fourier transforms of z_t, s_t, n_t. The problem was reduced to solving an integral equation of Wiener-Hopf type.

Linear filtering theory took a distinctive new direction around 1960, stimulated by two key papers by Kalman [**9**] and Kalman and Bucy [**10**]. In their approach the filtering problem is considered in the time domain (rather than the frequency domain), and state space representations are introduced. The signal is expressed as a linear function of an N-dimensional state vector X_t,