and computations are presented on a generous scale. Quadratic fields, cyclotomic fields, units, class numbers, discriminants and differents are among the topics treated with meticulous care. The fundamental theorem–every ideal is uniquely a product of prime ideals–arrives on p. 387 (but in fairness it should be noted that the delay in getting there is due partly to strict adherence to the local-global plan).

At the end of every major episode there is a parallel treatment of the function field case. Where there is a big difference there is appropriate added material, e.g. the Riemann-Roch theorem.

It is trite but true: Every number-theorist should have this book on his or her shelf.

In closing I shall maintain the tradition of the reviewing craft by recording the typos I noticed: pages 99, 309, 388, 575, 616; lines -3, -6, 16, 7, 21; quadratic, been, is, fields, Rogers.

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Combinatorial problems and exercises, by László Lovász, North-Holland, Amsterdam, 1979, 551 pp., $26.75.

Perhaps combinatorics is no longer deemed to be the slum of topology but it still has a remarkable polarising effect on mathematicians. The practitioners of combinatorics tend to idolise it as the only truly interesting branch of mathematics, while people not active in combinatorics are likely to have no respect for it and dismiss it as a collection of scattered results and trivial artificial problems. This highly unsatisfactory situation cannot be blamed entirely on the youth of the subject, though it is certainly one of the reasons. Those of us who work in combinatorics are also at fault, for most of our journals do publish more than their fair share of below par papers. Furthermore, as combinatorics fails to command the respect of the majority of the mathematical community, some combinatorialists feel entitled to disregard the huge developments in the main branches of mathematics.

There are signs that these lean years for combinatorics will soon be over. This is the hope expressed by Lovász in the Preface of *Combinatorial problems and exercises*. "Having vegetated on the fringes of mathematical science for centuries, combinatorics has now burgeoned into one of the fastest growing branches of mathematics–undoubtedly so if we consider the number of publications in this field, its applications in other branches of mathematics and in other sciences, and also, the interest of scientists, economists and engineers in combinatorial structures. The mathematical world has been attracted by the successes of algebra and analysis and only in recent years has it become clear, due largely to problems arising from economics, statistics, electrical engineering and other applied sciences, that combinatorics, the study of finite sets and finite structures, has its own problems and principles.

These are independent of those in algebra and analysis but match them in difficulty, practical and theoretical interest and beauty."

One must add that even today the last sentence clearly identifies Lovász as a mathematician firmly on the side of combinatorics. Nonetheless it is undeniable that combinatorics is slowly establishing itself as a serious branch of mathematics. This is due partly to the best known missionary of combinatorics, Paul Erdös, who introduced a great number of young people to combinatorics. Furthermore, the general mathematical public cannot but be impressed by some of the fruits of combinatorial research, for instance the effectiveness of the many versions of Ramsey's theorem and the formidable theorem of Szemerédi about arithmetic progressions. Finally, in recent years a number of monographs have appeared with real mathematical content, aimed at establishing various branches of combinatorics as legitimate and rich areas of mathematics.

Both the title and the structure of *Combinatorial problems and exercises* remind one of Halmos' admirable *Hilbert space problem book*. However, in the latter case one of the achievements of the author was to decompose the theory into attractive and tractable problems, while in the present case the challenge would have been in presenting a cohesive whole, for few people need convincing that combinatorics is a collection of problems and exercises. Having said this, one must admit that the most amusing and thorough (though rather time-consuming) way of learning combinatorics is to ponder on some aspect of it for a while and then look up the necessary key ideas.

The book covers a vast amount of ground from basic enumeration problems through a rich variety of topics in graph theory to hypergraphs. The exercises vary from those a student is likely to come across in an exam, to problems the reader cannot possibly be expected to solve without a fair amount of thought and effort. The following selection of problems will convey something of the book's flavour.

In the second chapter, on sieves, the cornerstone of a long sequence of problems built on each other is the following result of Rényi. Let $c_1, \ldots, c_n$ be real numbers and let $f_1, \ldots, f_k$ be polynomials in n events $A_1, \ldots, A_n$ of a probability space (Ω, P). Thus $B_i = f_i(A_1, \ldots, A_n)$ is an expression in $A_1, \ldots, A_n$ involving unions, intersections and complements. Then the assertion is that

$$\sum_1^k c_i P(B_i) \geqslant 0$$

holds in every probability space and for every $A_1, \ldots, A_n$ provided it holds whenever each A_j is either Ω or $\varnothing$. In order to prove this simple but beautiful and very useful result, express each f_i as union of atoms, that is sets of the form $\bigcap_{l=1}^n C_l$, where each C_l is either A_l or $\bar{A}_l$, the complement of A_l. It suffices to show that for each nonempty atom C the coefficient of $P(C)$ in $\sum_1^k c_i P(B_i)$ is positive. Suppose $C = (\bigcap_1^m A_l) \cap (\bigcap_{m+1}^n \bar{A}_l) \neq \varnothing$ and set $A_1' = \cdots = A_m' = \Omega$, $A_{m+1}' = \cdots = A_n' = \varnothing$. Then $C \subset B_i$ iff $B_i' = f_i(A_1', \ldots, A_n') = \varnothing$. Consequently the coefficient of $P(C)$ in $\sum_1^k c_i P(B_i)$ is

$$\sum_{B_i \supset C} c_i = \sum_1^n c_i P(B_i'),$$

which is positive by assumption.

A consequence of the above result of Rényi is the formula of K. Jordan. Let $A_1, \ldots, A_n$ be events as before, for $\varnothing \neq I \subset \{1, 2, \ldots, n\}$ set

$$A_I = \bigcap_{i \in I} A_i \quad \text{and} \quad A_\varnothing = \Omega$$

and put

$$\sigma_j = \sum_{|I|=j} P(A_I).$$

Then the probability that exactly q of the A_i occur is

$$\sum_{j=q}^n (-1)^{j+q} \binom{j}{q} \sigma_j.$$

To see this, note that by Rényi's theorem we may assume that $A_1 = \cdots = A_m = \Omega$ and $A_{m+1} = \cdots = A_n = \varnothing$. Then $\sigma_j = \binom{m}{j}$ and the sum

$$\sum_{j=q}^n (-1)^{j+q} \binom{j}{q} \binom{m}{j}$$

is easily seen to be 0 if $q > m$ and 1 if $q \leqslant m$.

Another notable consequence of Rényi's result is Brun's sieve. Let $f(k) \geqslant 0$ be any integer-valued function defined on $[1, n] = \{1, 2, \ldots, n\}$ and set

$$F = \{I \subset [1, n]: |I \cap [1, k]| \leqslant 2f(k), k = 1, \ldots, n\}.$$

Then

$$P\left(\bigcap_1^n \bar{A}_i\right) \leqslant \sum_{I \in F} (-1)^{|I|} P(A_I).$$

As an application of further sieve formulae one has the following result which is very refreshing in a book on combinatorics. If x is large enough, then the number of primes in the sequence $l, k + l, \ldots, k(x-1) + l$ $(0 < l < k)$ is not greater than $3kx/(\phi(k)\log x)$, where ϕ is Euler's function. (In fact, by the prime number theorem for arithmetic progressions, if k and l are relatively prime then the number of primes in this sequence is asymptotic to $kx/(\phi(k)\log x)$.)

The chapter on the connectivity of graphs is particularly rich in beautiful results. Let a and b be vertices of a directed graph. A *flow* f from a to b is a nonnegative function defined on the edges satisfying Kirchhoff's current law at every vertex $x \neq a, b$. Thus the amount of current leaving x is equal to the amount entering it: $\Sigma_y f(x, y) = \Sigma_z f(z, x)$. The *value* of the current is $v(f) = \Sigma_y f(ay) = \Sigma_z f(zb)$. With each directed edge xy we associate a nonnegative number $c(x, y)$, called the *capacity* of the edge. What is the maximal value of a flow from a to b if the current flowing through an edge cannot be more than the capacity of the edge? An answer to this question is provided by the so called Max-flow min-cut theorem, due to Ford and Fulkerson. Define a *cut* to be a subset of the edges after whose removal there is no positive-valued flow from a to b. The *capacity* of a cut is the sum of the capacities of the edges. Then it is clear that the capacity of a cut is at least as large as the capacity of a flow. The Ford-Fulkerson theorem claims that in fact the maximal value of a flow is equal to the minimal value of a cut. This simple but very useful result has numerous beautiful applications.

The reader is guided to recent and less known deep results, such as the following ones due to Mader, Watkins and Halin. Let G be a k-connected graph with at least $k + 2$ vertices and let A be a k-element separating set of vertices. Let G_1 be a component of $G - A$ and suppose A and G_1 are chosen so that G_1 has the minimal number of vertices. Then if B is any k-element separating set of vertices, either B contains all the vertices of G_1 or it contains none. In the first case G_1 has at most $k/2$ vertices. As a consequence of this result one finds that if G is critically k-connected (that is no edge can be omitted if one wants to keep G k-connected) then G has a vertex of degree k.

Probably the hardest problems the reader will meet are in the chapter on Ramsey theory. It is worth noting that Van der Waerden's classical theorem on arithmetic progressions in a partition of the integers appears as an unstarred exercise. "There exists a number $w = w(k, m)$ such that if the natural numbers $1, 2, \ldots, w$ are k-colored then there is a monochromatic arithmetic progression of length m."

The book is meant to be "useful to those students who intend to start research in graph theory, combinatorics or their applications, and for those who feel that combinatorial techniques might help them with their work in other branches of mathematics, management science, engineering and so on". Notwithstanding this laudable aim, it is certain that only readers with previous experience in combinatorics will attempt anything more than the most basic exercises. Moreover, most readers will use the book largely as a reference work and, rather than spend time solving hard problems, will either omit the proof or turn straight to the solution. For example, the solution to problem 4.29*, which asks for the number of ways of covering a chessboard by dominoes, runs to several pages of somewhat offputting formulae and also relies heavily on previous results; only the most dedicated and persistent reader is likely to complete it by himself. These minor reservations are far outweighed, though, by the merits of the book. *Combinatorial problems and exercises* contains an abundance of deep, interesting and stimulating results, many of which are due to the author himself. The material is attractively presented and easy to read. Throughout the book Lovász makes it clear that he is a mathematician of good taste and in command of his field. His work, which will take its place among the best texts in combinatorics, will certainly help to establish the respectability and worth of the subject.

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